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# NONLINEAR ANALYSIS OF ROTORDYNAMIC INSTABILITIES IN HIGH-SPEED TURBOMACHINERY

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## ABSTRACT

A nonlinear approach based on the method of averaging has been developed to study unstable lateral vibrations of rotors in high-speed turbomachinery. The method makes use of an extended concept of bearing dynamic coefficients which are defined by applying small perturbations to the dynamic equilibrium positions. It has been applied to determine the stability threshold of flexible rotors supported on short and tilting-pad journal bearings respectively. Stability performance of systems supported on short journal bearings is improved by the presence of mass unbalance. However, for flexible rotors supported on tilting-pad journal bearings, increased unbalance can lead to lower rotordynamic stability margins. This has the significant implication that stability estimates from linear analysis are not always conservative. The present method provides a computationally more efficient way to better understand the nonlinear vibration behaviour of high-speed turbomachinery.

## 1. INTRODUCTION

Linearized bearing dynamic coefficients are widely used in rotordynamic analysis for calculating critical speeds, unbalance response and stability [Lund et al.(1965), Gunter (1966), Ehrich (1992)]. The concept of such coefficients is based on a linear assumption of infinitesimally small motions about the static equilibrium position [Lund (1987)]. This assumption is however not always valid, e.g. when the journal motion relative to the bearing clearance is fairly large due to strong dynamic forces. It is well known that the dynamic characteristics of hydrodynamic bearings is highly nonlinear for large amplitude motions. Nevertheless, linear analysis has been found to give reasonable results for the prediction of unbalance response orbits with amplitudes upto about 30-40% of bearing clearance [Holmes (1970), Lund (1987)].

Despite the general success of linear analysis, it fails to cope with motions with amplitudes which approach the bearing clearance, as well as a number of nonlinear phenomena such as instability limit cycle, subharmonic vibrations, etc. These nonlinear cases are often handled by means of the time-marching numerical integration method [Adams (1980)]. This method is very versatile and accurate for nonlinear analysis but its major drawback is the substantial computational time needed. Various approximate methods have been applied to reduce the computational effort for nonlinear rotordynamic analysis [Nataraj and Nelson (1989), Hwang and Shiau (1991), Choy et al.(1992)]. Most of these approximate methods are aimed at steady-state problems where it takes a long time for the numerical integration results to converge.

For the stability analysis of rotor-bearing systems with unbalance, Lund and Nielsen (1980) applied the method of averaging to rigid rotors supported on short journal bearings. The method makes use of the time-averaged bearing dynamic coefficients along the unbalance response orbit. It is computationally efficient and gives results which agree with those calculated by the time-marching numerical integration method. In this study, the method is further developed to handle flexible rotors which are more realistic models of modern high-speed turbomachinery. Besides the bearing-induced oil whirl/whip problems [Muszynska (1986)], the method is applied to investigate the stability threshold of rotors which are supported on tilting-pad journal bearings and excited by aerodynamic cross-coupling forces. The aerodynamic cross-coupling represents the destabilizing influence from fluid-mechanical interactions, e.g. impeller/diffuser clearances, seals in high-speed turbomachinery [Ehrich&Child (1984)].

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Together with the extended method of averaging, the concept of quasi-linear bearing dynamic coefficients is introduced. In contrast with linearized coefficients which depend only on the static equilibrium position, the quasi-linear coefficients are defined with reference to dynamic equilibrium positions, i.e. points along the unbalance response orbit. These quasi-linear coefficients are functions of the journal displacements and velocities at each dynamic equilibrium position. Thus, they require a modified way of calculation in the bearing design procedure which conventionally evaluates bearing dynamic coefficients by perturbing the static equilibrium conditions.

In this paper the theory behind the extended method of averaging and quasi-linear bearing dynamic coefficients is briefly presented. A more comprehensive treatment can be found in Chan (1992). Results are then compared with predictions by the time-marching numerical integration method to verify the validity and accuracy of the present method. The assumptions and limitations of the present method for nonlinear rotordynamic analysis are also discussed.

## 2. THEORY

### 2.1 Quasi-Linear Bearing Dynamic Coefficients

The concept of dynamic coefficients is extremely useful in defining the dynamic characteristics of journal bearings and is extended here to eliminate the assumption of small motions. Since unbalance is the most common cause of large amplitude vibrations in rotor-bearing systems, the response and stability of the motion under unbalance forces are studied. The rotor is forced to whirl along a certain orbit due to unbalance and, under steady state conditions, the rotor can be described as in a state of dynamic equilibrium at each point along the orbit. Contrary to the static equilibrium position, the rotor possesses not only displacements but also velocities at the dynamic equilibrium position. The dynamic characteristics of the bearing depend on both the journal displacements and velocities relative to the bearing, and a new type of dynamic coefficients is obtained by linearizing the forces about the dynamic equilibrium position. The ensuing analysis by using these 'quasi-linear' bearing dynamic coefficients can account for the nonlinear effect in an approximate manner. This is different from a nonlinear analysis by time-marching numerical integration where bearing dynamic coefficients are not used.

The calculation of static and dynamic characteristics of hydrodynamic journal bearings can be found in Chan (1992). The two types of bearings considered here are short journal bearings and tilting-pad journal bearings. There exists no major difference with respect to the determination of dynamic coefficients about the static or dynamic equilibrium position for the case of short journal bearings, because the analytical formulae derived for these bearing coefficients have already allowed for the journal velocities. On the other hand, the dynamic coefficients of tilting-pad journal bearings were conventionally calculated by perturbing the journal about the static equilibrium position. This perturbation procedure has to be extended to take into consideration the journal velocities at the dynamic equilibrium position.

The bearing dynamic characteristics are represented by eight sets of quasi-linear dynamic coefficients for one orbit. Each set of these quasi-linear dynamic coefficients are expanded into Fourier series. The respective Fourier series coefficients, i.e. terms related to various harmonics, are used in the subsequent rotordynamic analysis to determine the response and stability of the system. Examples of unbalance orbits and the corresponding variation of quasi-linear dynamic coefficients are shown in Figs. 1 and 2 where the number of chosen points (time steps) along the unbalance orbit is 60. Fig. 1 illustrates the quasi-linear dynamic coefficients of short journal bearings. The unbalance response orbit of a flexible rotor supported on short journal bearings is shown in Fig. 1(a) where the displacements are normalized with respect to the bearing clearance C, i.e.

$$\bar{x} = \frac{x}{C} \qquad \bar{y} = \frac{y}{C} \tag{1}$$

The dimensionless stiffness and damping coefficients are shown in Figs. 1(b) and 1(c) respectively, and they are defined by

$$\bar{K}_{ij} = \frac{CK_{ij}}{W} \qquad \bar{C}_{ij} = \frac{C\Omega C_{ij}}{W} \tag{2}$$

where W is the static load and $\Omega$ is the rotational speed. Fig. 2 shows the corresponding parameters for tilting-pad journal bearings. It should be mentioned that the dynamic coefficients of tilting-pad journal bearings are reduced coefficients which depend on the whirl frequency [White & Chan (1992)].

### 2.2 Method of Averaging

The basic idea of the method of averaging [see e.g. Schmidt and Tondl (1986)] is briefly reviewed. For a nonlinear single degree-of-freedom (SDOF) system, the equation of motion can be written as

$$\ddot{y} + y = f(y) \tag{3}$$

where f(y) represents the nonlinear function. The solution may be assumed in the form

$$y = a(\tau)\cos[\tau + v(\tau)] = a(\tau)\cos\phi(\tau) \tag{4}$$

where $a(\tau)$ and $\phi(\tau)$ are respectively slowly varying amplitude and phase functions of dimensionless time $\tau$. The equation of motion is rearranged to yield the standard form of two differential equations of first order, i.e.

$$\begin{cases} \dot{a} = g(a,\phi) \\ \dot{\phi} = h(a,\phi) \end{cases} \tag{5}$$

The stationary solution is then obtained by averaging the terms on the right hand sides (containing the phase $\phi$) over one period and equating them to zero. Thus, the nonlinear equation of motion is solved approximately and the solution method is known as the method of averaging.

The detailed mathematical formulation of the extended method can be found in Chan (1992). Here a brief description of the calculation procedure is given. The calculation procedure can be divided into three main stages: (1) matrix reduction, (2) response calculation and (3) stability determination.

(1) Matrix Reduction:

The equations of motion including the mass and stiffness matrices are established in terms of all the rotor degrees of freedom in the x direction,

$$M\ddot{x} + Kx = F_x + f_x \tag{6}$$

where $F_x$ is the bearing force vector and $f_x$ is the mass unbalance force vector respectively in the x direction. Note that the nonlinearity lies in the bearing force vector $F_x$ which will be represented as a function of the quasi-linear bearing stiffness and damping coefficients ($K_{ij}$ & $C_{ij}$).

Then the matrices are partitioned into two groups. One group (the master co-ordinates) contains all the journal degrees of freedom at the bearing stations, the other group (the slave co-ordinates) consists of all the remaining rotor degrees of freedom. The purpose of the partition is to find the linear relationship between these two groups of co-ordinates. Note that their relationship does not depend on the nonlinear bearing forces. The rotor degrees of freedom are then expressed in terms of the journal degrees of freedom. The derivation here assumes that the system contains two bearings. Hence, the original equations are reorganized so that only two equations contain the nonlinear bearing forces on the right hand side. These two equations are extracted for applying the method of averaging and the reduced stiffness and mass matrices of order 2x2 are obtained. The matrix equation is given by

$$M_r\ddot{x}_r + K_r x_r = F_x + C_r f_x \tag{7}$$

(2) Response Calculation:

The equations of motion for x and y directions are combined.

$$\begin{bmatrix} M_r & 0 \\ 0 & M_r \end{bmatrix} \begin{pmatrix} \ddot{x}_r \\ \ddot{y}_r \end{pmatrix} + \begin{bmatrix} K_r & 0 \\ 0 & K_r \end{bmatrix} \begin{pmatrix} x_r \\ y_r \end{pmatrix} = \begin{pmatrix} F_x \\ F_y \end{pmatrix} + \begin{bmatrix} C_r & 0 \\ 0 & C_r \end{bmatrix} \begin{pmatrix} f_x \\ f_y \end{pmatrix} \tag{8}$$

By applying the method of averaging, the unknown journal displacements are expressed in terms of eight slowly varying parameters $x_{1c}$, $x_{1s}$, $x_{2c}$, $x_{2s}$, $y_{1c}$, $y_{1s}$, $y_{2c}$, $y_{2s}$ as given by

$$\begin{aligned} x_1 &= x_{1c}\cos\tau - x_{1s}\sin\tau \\ x_2 &= x_{2c}\cos\tau - x_{2s}\sin\tau \\ y_1 &= y_{1c}\cos\tau - y_{1s}\sin\tau \\ y_2 &= y_{2c}\cos\tau - y_{2s}\sin\tau \end{aligned} \tag{9}$$

These 8 slowly varying parameters can be physically interpreted as the in-phase and out-of-phase components of the journal displacements $x_1$, $x_2$, $y_1$ and $y_2$ at the bearing positions. Similar to the two parameters a and $\phi$ for an SDOF system, the 8 parameters here represent the solution to the equations of motion for a reduced system with 4 degrees-of-freedom, i.e. the journal displacements $x_1$, $x_2$, $y_1$ and $y_2$.

Then a set of eight homogeneous equations of first order involving the slowly varying parameters is set up (cf. Eqn. 5). Averaging is applied to these equations which contain the nonlinear bearing forces, and hence the quasi-linear bearing dynamic coefficients are utilized here in the computation. A Newton-Raphson iteration scheme is needed to estimate the 8 slowly varying parameters, and the unbalance whirl orbit amplitudes are obtained by computing the journal displacements for a number of time intervals over one period.

(3) Stability Determination:

The following variational equations of motion are formed

$$\begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix} \begin{pmatrix} \Delta\ddot{x} \\ \Delta\ddot{y} \end{pmatrix} + \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} \Delta F_x \\ \Delta F_y \end{pmatrix} \tag{10}$$

The time-averaged (quasi-linear) bearing dynamic coefficients are substituted for the bearing forces and hence yield the bearing damping and stiffness matrices, i.e.

$$\begin{pmatrix} \Delta F_x \\ \Delta F_y \end{pmatrix} = -\begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix} \begin{pmatrix} \Delta\dot{x} \\ \Delta\dot{y} \end{pmatrix} - \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} \tag{11}$$

A total matrix equation of second order (derivatives in x and y) is formed by combining Eqns. 10 and 11. It is then converted to a first order matrix equation in order to evaluate the eigenvalues. The system is stable if the real parts of all eigenvalues are negative and otherwise unstable. It should be mentioned that the quasi-linear bearing dynamic coefficients are utilized in both the iteration process of the response calculation and the eigenvalue calculation of the stability determination stage.

## 3. ANALYSIS RESULTS

Results obtained in this paper are based on flexible rotor models. In order to obtain generalized data for comparison, two rotor models are used. The first one is the well known Jeffcott rotor model where there is only one disk mounted on a massless elastic shaft which is supported by bearings at its two ends. The disk mass is M and the simply supported shaft stiffness is K. This is the rotor model used in Figs. 3 to 6. The second rotor model is the symmetrical three-disk rotor where there is one disk at each of the two bearing positions in addition to the midspan rotor. The ratio of bearing disk mass m to midspan disk mass M introduces an additional parameter to the system. This three-disk model is used in Figs. 7 to 12.

The stability threshold of flexible rotors supported on short journal bearings is first considered. The threshold depends on factors such as the shaft stiffness and the amount of unbalance, etc. In Fig. 3 the stability threshold speed curves of rotors with different shaft stiffnesses are shown. The stability threshold speed is normalized as

$$\bar{\Omega} = \frac{\Omega}{\Omega_G} \qquad \Omega_G = \sqrt{\frac{g}{C}} \tag{12}$$

where g is the acceleration due to gravity and the normalized shaft stiffness is given by the dimensionless parameter

$$\bar{K} = \frac{CK}{gM} \tag{13}$$

where M is the disk mass. Fig. 4 shows the stability threshold speed curves for rotors of a particular shaft stiffness but carrying different amounts of unbalance. The dimensionless unbalance parameter is given by

$$\rho = \frac{r_u}{C} \tag{14}$$

where $r_u$ is the radius of unbalance.

For rotors supported on tilting-pad journal bearings, instability problems can be caused by aerodynamic cross-coupling forces. The stability threshold of such rotor-bearing systems can be expressed in terms of the same threshold speed parameter as above. Fig. 5 shows the threshold speed curves for two different levels of aerodynamic cross-coupling which is non-dimensionalized as

$$\bar{Q} = \frac{Q}{K} \tag{15}$$

where K is the shaft stiffness. Alternatively, the stability threshold can be displayed by plotting the threshold cross-coupling parameter which is the level of aerodynamic cross-coupling when the system is on the borderline between stable and unstable conditions. Fig. 6 is a plot of threshold cross-coupling curves for rotors of different shaft stiffnesses.

For all the above results, the Jeffcott rotor model has been used. On the other hand, the flexible rotors employed in the following figures are the three-disk rotor model with a m/M ratio of 0.5. Figures 7 to 9 give the threshold cross-coupling curves for rotors carrying different amounts of unbalance. The normalized shaft stiffness (Eqn. 12) varies from 100 in Fig. 7 to 10 in Fig. 8 and 1 in Fig. 9. It is noted that the modified Sommerfeld number $\sigma$ is used for short bearing cases and the conventional Sommerfeld number S is employed for tilting-pad bearing cases. They are defined by the equations

$$\sigma = \frac{1}{8}\frac{\mu \Omega D L}{W}\left(\frac{L}{C}\right)^2 \qquad S = \frac{\mu N D L}{W}\left(\frac{R}{C_p}\right)^2 \tag{16}$$

The whirl/speed ratio curves are shown in Fig. 10 for rotors of a particular shaft stiffness but carrying different amounts of unbalance.

Finally, the results obtained by the time-marching numerical integration method are shown in Figs. 11 and 12 to verify the predictions of the method of averaging. These results can be compared with those intersection points along the line S=0.46 in Fig. 9 which gives the threshold cross-coupling curves for the same rotor-bearing system configuration. Fig. 11 shows the vibration orbit and frequency spectrum for a rotor with negligibly small unbalance ($\rho$=0.004) and subjected to a normalized aerodynamic cross-coupling of 0.10. The corresponding orbit and spectrum for the same rotor with a larger unbalance ($\rho$=0.4) under an aerodynamic cross-coupling of 0.06 are plotted in Fig. 12.

## 4. DISCUSSION

It can be seen from Figs. 1 and 2 that the quasi-linear bearing dynamic coefficients are varying along the unbalance response orbit. They are not only functions of the journal displacements but also functions of the journal velocities. That is, even if the journal passes through the same position, but with different velocities, the dynamic coefficients do not have the same values. In contrast with short journal bearings which have cross-coupled stiffness coefficients in the same order of magnitude as the direct coefficients, the cross-coupled stiffness coefficients of tilting-pad journal bearings are in general an order of magnitude smaller than their direct counterparts. Moreover, the cross-coupled stiffness coefficients $K_{xy}$ and $K_{yx}$ of tilting-pad journal bearings are equal, and their variation over one cycle with almost equal positive and negative contributions yields average values close to zero. When only the time-averaged coefficients are considered in the stability analysis, the destabilizing effect of bearing cross-coupled stiffness is negligible in the case of tilting-pad journal bearings.

For rotors supported on short journal bearings, the bearing cross-coupled stiffness will generate an instability at high shaft rotational speeds commonly referred to as oil whirl/whip [Muszynska (1986)]. The parameter of interest is usually the stability threshold speed, i.e. the speed above which the rotor will be dynamically unstable. As shown in Fig. 3, the threshold speed of flexible rotors is dependent on the shaft stiffness. The more flexible the shaft is, the lower the threshold speed. This is because the natural frequency of the rotor is lower for a more flexible shaft and instability occurs often when the rotational speed is above two times the rotor's natural frequency. When the rotor is unbalanced, the stability threshold is affected by the unbalance response since the dynamic characteristics of the bearings is nonlinear. The effect depends on the shaft stiffness but, broadly speaking, the stability threshold speed is increased when there is unbalance in the rotor. The larger the mass unbalance is, the higher the threshold speed as shown in Fig. 4. This is in agreement with predictions based on the rigid rotor model [Lund and Nielsen (1980)]. The improvement in stability characteristics for unbalanced rotors can be qualitatively explained by an increase in the natural frequency due to a larger effective bearing direct stiffness, especially at high Sommerfeld numbers or lightly loaded conditions.

For rotors supported on tilting-pad journal bearings, instability problems arise often because of fluid-mechanical interaction forces which can be represented by aerodynamic cross-coupling coefficients. The stability threshold speed of such rotor-bearing systems depends on the level of aerodynamic cross-coupling. The higher the level of aerodynamic cross-coupling is, the lower the threshold speed as shown in Fig. 5. Alternatively, the maximum level of aerodynamic cross-coupling which the system can endure before becoming dynamically unstable is a measure of the stability performance of these rotor-bearing systems. That a system can stand a larger threshold cross-coupling implies that the system is more stable against aerodynamic excitation. Similar to the short bearing cases, the shaft stiffness influences the stability threshold. As can be observed from Fig. 6, the system threshold cross-coupling is smaller when the shaft is more flexible. This can be explained by the fact that the stabilizing influence in the system comes from the bearing damping. The damping effect is reduced if the shaft becomes more flexible and the journal motions at the bearings get smaller in amplitude. Note that the threshold cross-coupling curves are calculated at a fixed operating speed. They change with different operating speeds but the general trend is similar.

One of the most interesting results discovered in this study is the effect of unbalance on the stability of flexible rotors supported on tilting-pad journal bearings. The stability behaviour of unbalanced rotors also depends on the shaft stiffness. For relatively stiff shafts ($CK/gM=100$ in Fig. 7), the presence of mass unbalance improves the stability threshold, i.e. the unbalanced system can endure a higher level of aerodynamic cross-coupling without turning unstable. For medium shaft stiffness ($CK/gM=10$ in Fig. 8), the effect is more difficult to generalize. A small amount of mass unbalance can slightly improve the stability threshold but a large amount can have the opposite effect. When the shaft is fairly flexible ($CK/gM=1$ in Fig. 9), the addition of mass unbalance causes a deterioration in the stability threshold. In effect, the unbalanced rotor becomes unstable at a lower level of aerodynamic cross-coupling than a balanced rotor. This is important since the linearized analysis then does not always yield a conservative estimate of the stability boundary. And in practice, it will be advantageous to minimize the residual unbalance in view of the stability consideration.

An useful diagnosis tool in vibration monitoring is the spectrum analysis of the rotor vibration. The characteristics of unstable vibrations due to aerodynamic cross-coupling forces is revealed by the ratio of whirl frequency to rotational speed. It has been found that the whirl frequency in such instability problems is typically subsynchronous, i.e. the whirl/speed ratio is less than unity and usually varies from 0.3 to 0.8. This is also the reason why the subsynchronous behaviour of tilting-pad journal bearings is of interest [White and Chan (1992)].

The accuracy of the method of averaging is dependent on such factors as the slowly varying parameters, number of averaging points in one cycle, deviation from the assumption of quasi-harmonic response, etc. It varies from case to case and cannot be established generally. Nevertheless, the validity of the results in this study has been confirmed by cross-checking with results from the time-marching numerical integration method, for example, results shown in Figs 11 and 12. By comparing Figs. 11 and 12, the stability threshold of the unbalanced flexible rotor is obviously lower than that of the well balanced rotor. They clearly demonstrate the negative effect of mass unbalance on stability under such conditions. The operating conditions for the rotor-bearing system in Figs. 11 and 12 are close to the line S=0.46 in Fig. 9. The good agreement of the normalized threshold cross-coupling at the intersection points in Fig. 9 with those in Figs. 11 and 12 also verifies the accuracy of the present method.

## 5. CONCLUSIONS

The following conclusions can be drawn from the results in this study:

(1) The concept of quasi-linear bearing dynamic coefficients is explained and utilized in the method of averaging which accounts for the effect of bearing nonlinearity in an approximate manner. Their usefulness is reflected in the stability prediction using the method of averaging.

(2) The method of averaging has been extended to calculate the stability threshold of flexible rotors supported on short journal bearings. The effect of shaft stiffness on the stability threshold has been investigated and it is found that the more flexible the shaft is, the lower the threshold speed. The stability threshold is found to be improved by the presence of mass unbalance on the rotor. The threshold speed becomes higher when the amount of unbalance increases. This is explained by an increase in system natural frequency due to a larger effective bearing direct stiffness.

(3) The method of averaging is also applied to investigate the stability threshold of rotors which are supported on tilting-pad journal bearings and subjected to aerodynamic cross-coupling. The threshold speed of such systems decreases when the cross-coupling becomes larger. The threshold cross-coupling is used as an alternative measure of stability performance for such systems. The stability threshold is lower for rotors with higher shaft flexibility because the effective bearing damping effect, which stabilizes the rotor-bearing system, is reduced by a smaller journal amplitude. The presence of mass unbalance is not necessarily beneficial to the stability of such systems. For fairly flexible rotors, increasing the unbalance can lead to a deterioration of the stability threshold.

(4) It has been shown that the main characteristics of rotordynamic instabilities due to aerodynamic cross-coupling is the subsynchronous vibration frequency, i.e. a whirl/speed ratio of less than unity. This is an useful diagnosis tool in the vibration monitoring of high-speed turbomachinery.

(5) The results from the time-marching numerical integration method confirm the validity and accuracy of the predictions by using the method of averaging. Therefore, the method of averaging offers a computationally efficient method for nonlinear rotordynamic analysis.

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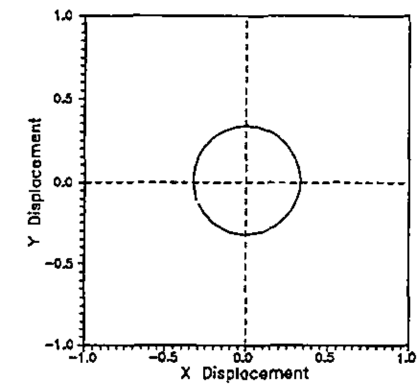
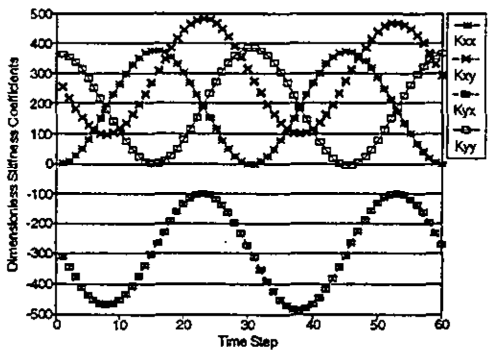
## NOMENCLATURE

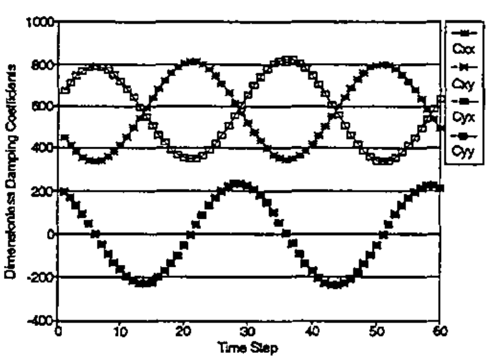
| | |
|---|---|
| $a(\tau)$ | Slowly varying amplitude function |
| $C_{ij}$ | Bearing damping coefficients |
| C | Bearing clearance |
| $C_p$ | Pad circle clearance |
| $C_r$ | Coefficient matrix in Eqn. 7 |
| D | Bearing diameter |
| $F_x, F_y$ | Bearing force vectors |
| $f_x$ | Mass unbalance force vector (in x direction) |
| g | Constant = acceleration due to gravity |
| K | Shaft stiffness or shaft stiffness matrix |
| $K_r$ | Reduced shaft stiffness matrix |
| $K_{ij}$ | Bearing stiffness coefficients |
| L | Bearing length |
| LOP | Load-on-pad tilting-pad journal bearings |
| M | Disk mass or shaft mass matrix |
| $M_r$ | Reduced shaft mass matrix |
| m | Bearing disk mass or bearing preload |
| N | Rotational speed in rev/sec |
| Q | Aerodynamic cross-coupling coefficient |
| R | Bearing radius |
| $r_u$ | Radius of unbalance |
| S | Sommerfeld number |
| W | Static bearing load |
| x | Displacement in vertical direction |
| $x_r$ | Journal displacement vector (in x direction) |
| $x_1$, $x_2$ | Journal displacements at bearings (in x direction) |
| $x_{1c}, x_{1s}$ | Slowly varying parameters in Eqn. 9 |
| y | Displacement in horizontal direction |
| $y_1$, $y_2$ | Journal displacements at bearings (in y direction) |
| $y_{1c}, y_{1s}$ | Slowly varying parameters in Eqn. 9 |
| $\epsilon_0$ | Static eccentricity ratio |
| $\mu$ | Lubricant viscosity |
| $\rho$ | Dimensionless unbalance parameter |
| $\sigma$ | Modified Sommerfeld number for short bearings |
| $\tau$ | Dimensionless time |
| $\phi(\tau)$ | Slowly varying phase function |
| $\Omega$ | Rotational speed in rad/sec |
| $\Omega_G$ | Normalizing factor in Eqn. 12 |



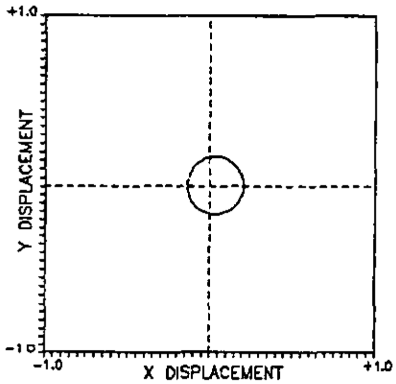

(a) Orbit Plot



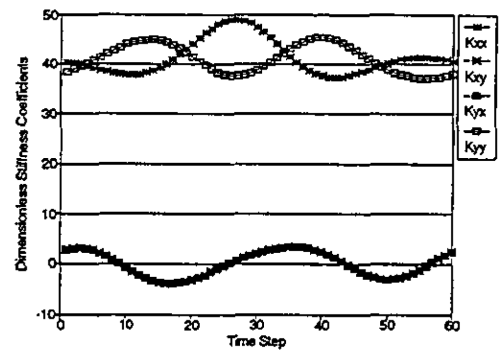

(b) Stiffness Coefficients



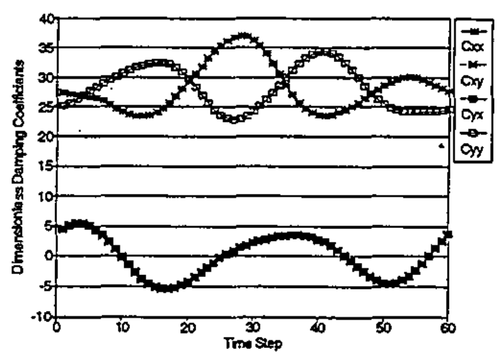

(c) Damping Coefficients




(a) Orbit Plot




(b) Stiffness Coefficients




(c) Damping Coefficients

**Figure 1** Variation of quasi-linear dynamic coefficients of short journal bearing ($\epsilon_0$=0.008, $\rho$=0.3)

**Figure 2** Variation of quasi-linear dynamic coefficients of tilting-pad journal bearing ($\epsilon_0$=0.05, $\rho$=0.2)

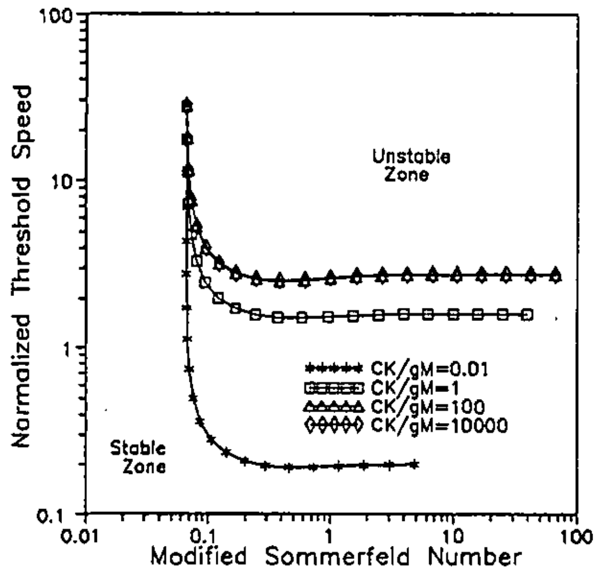



**Figure 3** Stability threshold speed curves of balanced rotors on short journal bearings

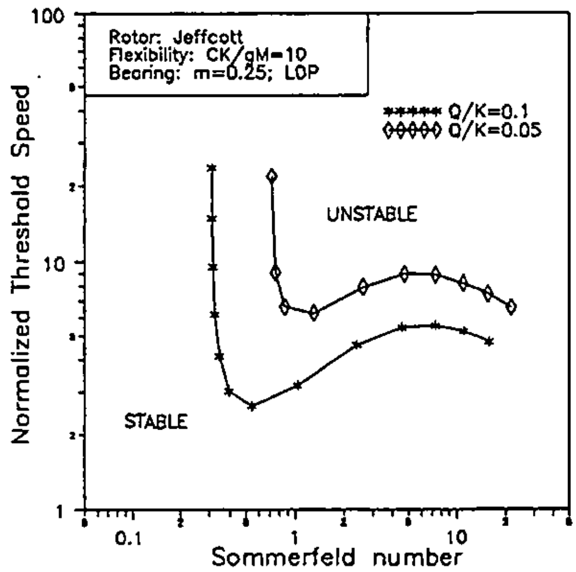



**Figure 5** Stability threshold speed curves of balanced rotors on tilting-pad journal bearings

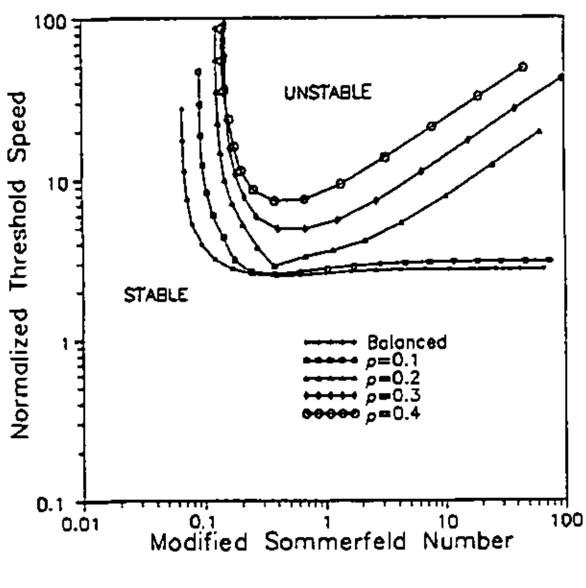



**Figure 4** Stability threshold speed curves of unbalanced rotors (CK/gM = 10000) on short journal bearings

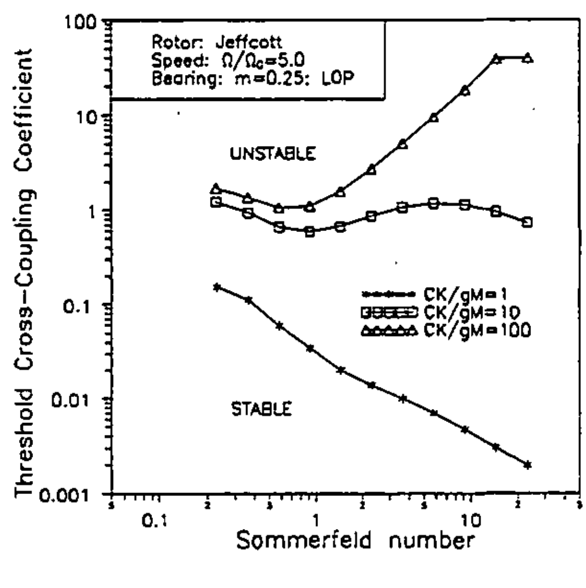



**Figure 6** Stability threshold cross-coupling curves of balanced rotors on tilting-pad journal bearings

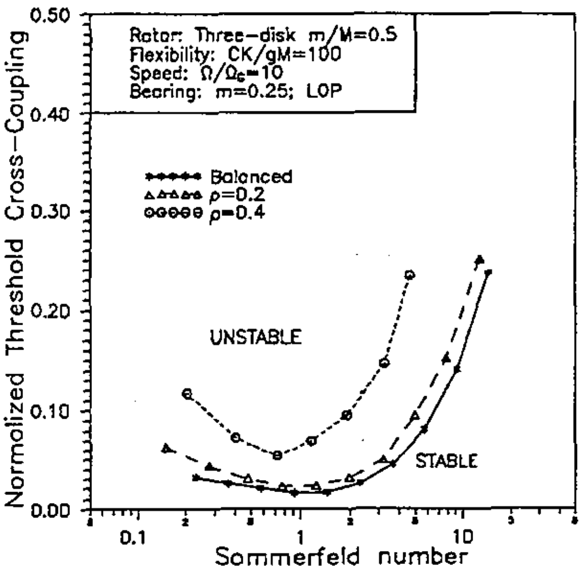



**Figure 7** Stability threshold cross-coupling curves of unbalanced rotors (CK/gM = 100) on tilting-pad journal bearings

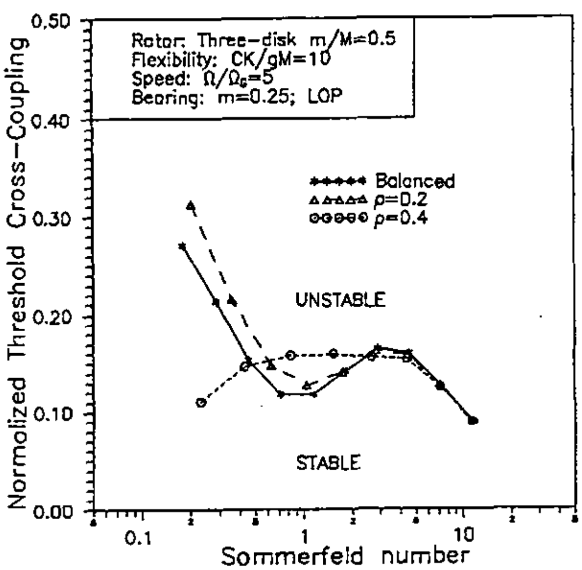



**Figure 8** Stability threshold cross-coupling curves of unbalanced rotors (CK/gM = 10) on tilting-pad journal bearings

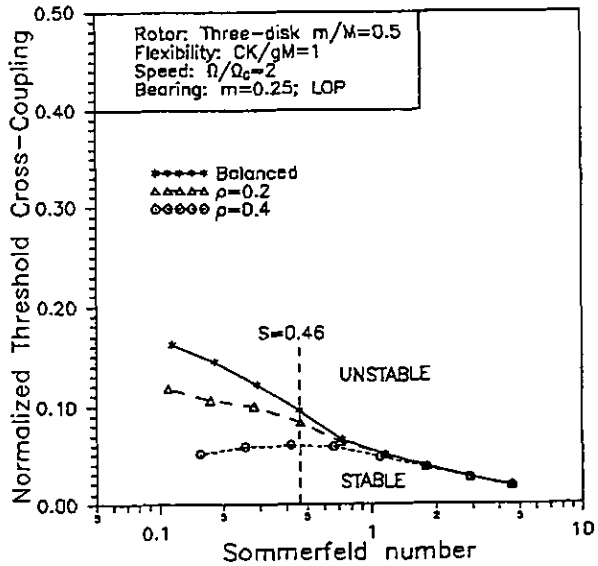



**Figure 9** Stability threshold cross-coupling curves of unbalanced rotors (CK/gM = 1) on tilting-pad journal bearings

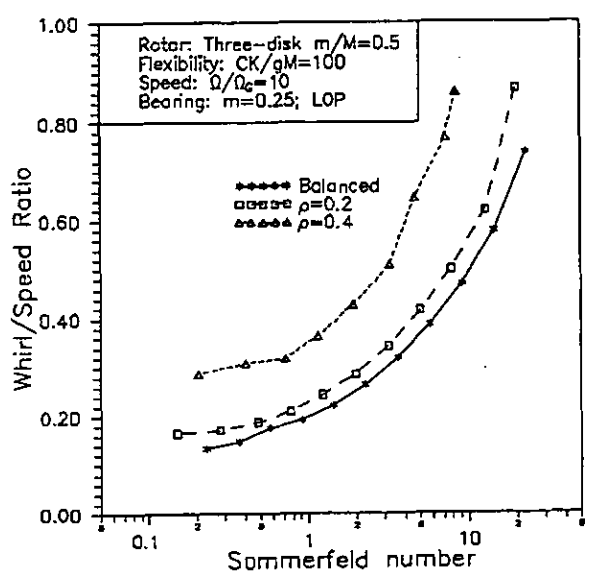
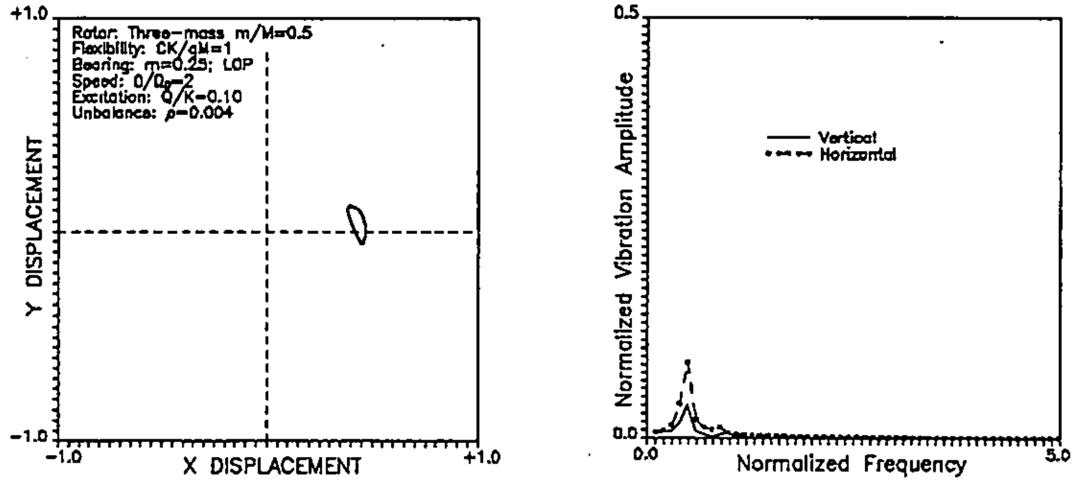


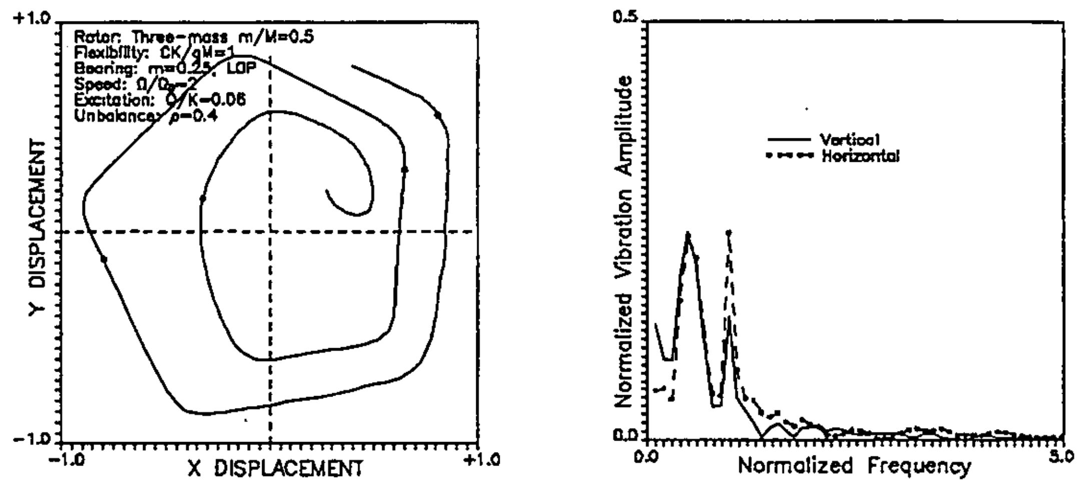

**Figure 10** Whirl/speed ratios at instability threshold of unbalanced rotors (CK/gM = 100) on tilting-pad journal bearings




(a) Response plot (Q/K=0.10)

(b) Frequency spectrum

**Figure 11** Response plot obtained by numerical integration and frequency spectra for balanced rotor (CK/gM = 1, $\rho$=0.004) on tilting-pad journal bearings




(a) Response plot (Q/K=0.06)

(b) Frequency spectrum

**Figure 12** Response plot obtained by numerical integration and frequency spectra for unbalanced rotor (CK/gM = 1, $\rho$=0.4) on tilting-pad journal bearings