Vibration of a Rigid Shaft Supported by Radial Ball Bearings with Several Defects

J. M. FRANCO, N. AKTURK and R. GOHAR

Imperial College of Science, Technology and Medicine
Department of Mechanical Engineering, Tribology Sec.
London SW7 2BX, U.K.

ABSTRACT.

In this paper a vibration analysis of a rigid shaft-ball bearing system has been carried out, in order to approximate the nonlinear load-deformation characteristics of the ball/race contacts, an approximation by means of the Taylor series is studied. The response of an imperfect model system in both time and frequency domains is analyzed. Factors affecting the vibration characteristics of the system such as rotating unbalance, the varying diameter of the balls and the surface waviness are investigated. The vibration analysis of the system, including various external excitations, is carried out for each set of conditions and for more than one defect simultaneously.

It is shown that the mathematical model of the total ball load-deflection relationship has a linear part that depends on the initial deflection, a small time dependent component, the load-deflection coefficient; and a number of terms contributing to nonlinearity of the contact stiffness.

NOMENCLATURE

Symbol Description Units

M mass of the rotor Kg
m number of lobes or waves
N number of balls
r rotor speed/mode order rpm
Q ball/race contact load N
t time s
x, y, z deflection in these directions m
α ball/race contact angle rad
θ angular position of balls rad
ω angular velocity rad/s
δ contact deflection m
ϕ angular position around the bearing rad
γ angular position around the raceway rad

Subscripts

b ball
c cage
e equivalent
i i\textsuperscript{th} ball, inner
n natural frequency
0 Corresponds to the initial conditions, outer
g groove
p related to wave amplitude, pitch diameter
x, y, z coordinate axes
w related to waviness amplitude
1, 2, 3, 4 related to dependent variables, estimations

INTRODUCTION

Rotating machines subjected to unexpected problems lead to an increase in operation and maintenance costs as well as to the possibility of fatal failure. Knowledge of the dynamic behaviour of the machine components is essential in order to predict a malfunction or to understand in depth the machine performance.
To investigate the performance or to diagnose the malfunctions of rotating machines, several variables have been studied. However, the best evidence is given by mechanical vibrations. Each type of turbomachinery has its own specific vibration characteristics. For all ball bearing supported machinery these characteristics are altered by external loads by unbalance and misalignment of the rotor, and by other factors related to the condition of the ball bearings themselves, such as manufacturing bearing seating to the required level of precision and surface waviness.

Since the vibrations generated by ball bearings are complex and difficult to analyze, analytical models have been employed to obtain evidence is given by mechanical vibrations. To investigate the performance or to diagnose the study of the relation between excitation and response has been carried out both theoretically and experimentally by various researchers.

Models and their experimental verification have been investigated to identify the frequency vibrational signature either at low or high frequencies. In modelling a shaft supported by ball bearings, the ball to raceway contact characteristic is one of the governing parameters. Although it is known that this relationship is nonlinear, it has been assumed for a long time that a linear curve can represent in the actual curve in dynamical problems to avoid the difficulties caused by the nonlinearity. Today, with the help of computers, in dynamic as well as static problems, the relationship is modelled as nonlinear. In the modern studies, three main approaches can be seen to represent the nonlinear ball to race contact relationship. In the first, the nonlinear relationship is assumed to be linear over a narrow range of frequency, resulting in a piece-wise characteristic curve that represents the actual curve [1,2]. This is convenient for studying continuous systems since for each range of frequency the contact characteristic will be linear and hence the superposition technique holds. The second method employs the Hertzian contact relationship. This technique is a closer representation of the actual nonlinear relationship and is widely employed by many researchers [3,4,5]. Finally, the third approach uses the Taylor series in order to obtain an approximate curve fit to the Hertzian contact [6,7,8]. If at least the first two terms of the Taylor series are used the nonlinear curve can be linearised over a certain frequency range.

Because of the nonlinearity of the elastic force at points of contact for each ball, the vibration produced by the shaft-ball bearing system is complex. Therefore a numerical method is used to solve the simultaneous nonlinear, first order differential equations. Even so, by means of the Taylor's series expansion of Eq. (4) gives an approximation to the total ball load which is expressed by:

\[ F_i = K \delta_i \left( \delta_i \right) \frac{3}{2} \left( \frac{\delta_i(t)}{\delta_o} \right)^{3/2} \]  

Using the Taylor's series expansion of Eq. (4) about a point \( \delta_i(t) \) and the substitution of this in Eq. (3) gives an approximation to the total ball load which is expressed by:

\[ F_i = K \delta_i \frac{3}{2} \left[ \frac{3 \delta_i(t)}{2 \delta_o} \right] + 3 \left( \frac{\delta_i(t)}{\delta_o} \right)^2 - \frac{3}{48} \left( \frac{\delta_i(t)}{\delta_o} \right)^3 + \frac{3}{128} \left( \frac{\delta_i(t)}{\delta_o} \right)^4 + \text{higher terms} \]  

GENERAL CONSIDERATIONS

For slow speed of rotations and/or large magnitude of applied loads, the ball bearings have been analyzed without taking into account dynamic effects. From the kinematics of ball bearing components, the cage speed is given by [5]:

\[ \omega_c = \frac{1}{2} \left[ \omega \left( 1 - \frac{d}{d_p} \cos(a) \right) + \omega \left( 1 + \frac{d}{d_p} \cos(a) \right) \right] \]  

Note that, as in this paper a radial ball bearing is studied, the contact angle is equal to zero (\( a = 0 \)).

The total proportionality constant can be obtained from the inner and outer ball/raceway contact proportionality constants as follows [5]:

\[ K_e = \sqrt[3/2]{ \frac{1}{1/(K_r)^{3/2} + 1/(K_o)^{3/2}} } \]  

AN APPROXIMATION TO THE LOAD-DEFLECTION RELATIONSHIP

According to Hertzian theory, the total elastic force at the points of contact of the \( i^a \) ball with the inner and outer raceways is:

\[ F_i = K \delta_i \]  

Due to the nonlinearity of Eq. (3) an approximation by means of the Taylor’s series has been employed in this paper.

Using the Taylor’s series expansion of Eq. (4) about a point \( \delta_i(t) \) and the substitution of this in Eq. (3) gives an approximation to the total ball load which is expressed by:

\[ F_i = K \delta_i \frac{3}{2} \left[ \frac{3 \delta_i(t)}{2 \delta_o} \right] + 3 \left( \frac{\delta_i(t)}{\delta_o} \right)^2 - \frac{3}{48} \left( \frac{\delta_i(t)}{\delta_o} \right)^3 + \frac{3}{128} \left( \frac{\delta_i(t)}{\delta_o} \right)^4 + \text{higher terms} \]  

In the work presented here, an analysis of the response of the system has been made using an imperfect model. It is assumed that off-sized ball and surface waviness (distributed defects) as well as rotating unbalance are present. The examples quoted include the analysis of both externally damped and undamped rotor-bearing systems.
EQUATIONS OF MOTION OF THE ROTOR-BEARING SYSTEM

\[ \delta_1(t) = \delta_2 + x \cos \theta_i + y \sin \theta_i \] (6)

where \( \theta = \omega t + i \phi \) is the angle between the lines of action of the radial load (direction of displacement of the moving ring) and the radius passing through the centre of the ith ball; \( \omega t \) is the turning angle of the cage; \( \phi \) is the angular distance between two adjacent balls; \( \phi = 2\pi/N \), where \( N \) is the number of balls (see Fig. 1).

The total elastic forces in the X and Y directions are given by:

\[ F_X = \sum_{i=0}^{N-1} F_i \cos \theta_i \] (7)

\[ F_Y = \sum_{i=0}^{N-1} F_i \sin \theta_i \] (8)

Approximating the load-deflection relationship and limiting ourselves to the first four terms, Eq. (7) becomes:

\[ F_X = K_0 \sum_{i=0}^{N-1} \cos \theta_i + \frac{3}{2} K_0 \sum_{i=0}^{N-1} \delta(t) \cos \theta_i \\
+ \frac{3}{8} K_0 \sum_{i=0}^{N-1} \delta(t)^2 \cos \theta_i \\
- \frac{3}{4} K_0 \sum_{i=0}^{N-1} \delta(t)^3 \cos \theta_i \] (9)

A similar equation can be written in the y direction. However, in this paper, for brevity, only the equations in x direction have been derived.

The work presented here assumes a rigid rotor supported on two radial ball bearings which vibrates in two degree-of-freedom (i.e. the x and y directions) as shown in Fig. 2.

For numerical solutions, the initial conditions and step size play a crucial role. The step size should be small enough to achieve an adequate accuracy; but it should be as large as possible to avoid an excessive number of evaluations of the derivative that is directly related to computing time. Therefore a compromise should be made between them. At time \( t=0 \) the following assumptions are made:

a) The arbitrary displacements in the X and Y directions are:
\[ x = 1.6 \mu m \quad y = 0.0001 \mu m \]
b) The velocities in the x and y directions are zero:
\[ \frac{dx}{dt} = 0 \quad \frac{dy}{dt} = 0 \]

The step size is assumed to be:
\[ h = 0.0001 \text{ sec.} \]
The response of the rotor-bearing system was carried out for the ball bearing with the specifications below:

Ball Diameter = 11.9062 mm
Inner race radius = 20.0470 mm
Outer race radius = 31.9530 mm
Inner race groove radius = 6.0130 mm
Outer race groove radius = 6.3100 mm
Number of Balls = 8
Load-deflection coefficient = 9.95 x 10^9 N/m

Rotor Mass = 16 kg

DYNAMIC ANALYSIS OF AN IMPERFECT MODEL

Ball bearings are not perfect; they have several defects related to the design and manufacture of bearing components as well as the assembly of the bearings. Moreover in rotating machines there are many factors altering the balance of the rotor, which is one of the most common cause of problems. Some of these problems are discussed below:

Off-sized Ball Model
If within a ball bearing all the balls are perfectly spherical but one or more have different diameters, a situation arises in which the magnitudes of all ball to race contact forces are not equal. In a perfect model, the initial elastic deformation in the radial direction of any rolling body, caused by the interference fitting, has a unique value. However, if one ball is, for example, larger than the others, the initial elastic deformation of the ball is proportional to the increment in diameter. Therefore, constant ball load-and hence spring force-is produced by the difference in diameter of the ball with respect to the rest of the balls in the set. Hence:

$$\delta_0 = \delta_e - \delta_p$$  \hspace{1cm} (11)

The excitation due to unequal diameter of a ball has the cage set frequency, and this is in agreement with that obtained from Eq. (3). The magnitude of the cage set frequency depends on the elastic properties of each ball in the set as well as the unequal diameter of the off-sized ball or balls.

Since the response in time domain is also important, Fig. 3 shows the response of the system under the conditions presented above. It can be seen that the most dominant vibration amplitude is due to the excitation of the cage set frequency.

From the computed results, the generation of the vibration frequencies mentioned above is based on the ball geometry and the speed at which the bearing is rotating.

Rotating Unbalance
The most common source of rotor excitation, in rotating machines, is mass unbalance, where the local mass centre is offset by an eccentricity (e) from the axis of rotation. Thus the contributions of the external forces produced by this excitation (in both x and y directions) are given by:

$$F_x(t) = Mw^2 e(\cos(\omega t))$$  \hspace{1cm} (12)
$$F_y(t) = Mw^2 e(\sin(\omega t))$$  \hspace{1cm} (13)

Rotating unbalance generates a vibration with a frequency equal to that of the rotation of the rotor, while its magnitude varies with the square of the rotational speed.

Waviness
Experimental determination of the individual and combined effects of geometrical errors in the raceways of rings and rolling elements on the level of vibrations in ball bearings has been carried out by various researchers [3,6,7,9,10]. As a result of the analysis of experimental investigations it has been concluded that the waviness of the outer ring has less effect than of the inner ring. It has been shown that raceway waviness has the maximum influence on the level of vibrations of a bearing.

Waviness on the inner raceway can be seen as a sinusoidal deformation with a number of lobes (m). Thus the nonlinear spring forces will change according to the ball-raceway contact compliance and the amplitude of the waviness. This means that a simple harmonic excitation takes place in the direction of rolling and normal to the direction of rolling.

Taking in account the inner ring rotation, the waviness amplitude ($\delta_w$) is given by

$$\delta_w = \delta_p \sin(m \gamma)$$  \hspace{1cm} (14)

where $\delta_p$ is the amplitude of the wave, m is the number of waves per circumference, ($\gamma$) is measured with respect to a point on the inner ring, and $\gamma = \phi + (\omega_i - \omega_e)t$. 

![Fig. 3 Calculated vertical vibration of a bearing having one off-sized ball (1 µm larger), n = 7500 rpm, N = 8](image_url)
The analysis of the imperfect model having a wavy inner race has been carried out for an undamped system. The vibration produced by the eight-ball undamped system with eight waves is shown in Fig. 4.

The spectral analysis of the complex response of the system having wavy inner race shows that this sort of imperfection can be observed in the spectrum of the response of the system which is characterized by sideband frequencies due to the interaction between the various constituent components of the bearing.

The complex spectrum shows excitation of frequency components at \( [n (f - f_c) + f_c] \), \( [n (f - f_c) - f_c] \).

This effect is due to the nonlinear load-deflection characteristics, having the effect of the number of terms in the approximation presented in this paper.

Fig. 4 Calculated vertical vibration of a bearing having wavy inner race \( (N = 8, n = 7500 \text{ rpm, } e = 1 \mu m) \)

MODEL WITH SEVERAL DEFECTS

The response of a imperfect model which includes rotating unbalance, an off-sized ball and a wavy inner race surface is analyzed simultaneously.

Fig. 5 shows the vertical vibration of a bearing having an off-sized ball (1 μm larger than the others), a rotor centre of mass with eccentricity \( e = 1 \mu m \), and a wavy inner race having \( \xi_r = 1 \mu m \), \( m = 8 \). The frequency spectrum is shown in Fig. 6. The results were determined at rotor speed value of 10000 rpm.

Table 1 shows the dominant frequencies in the spectrum of vertical vibration of a bearing having a wavy inner race.

The response of the system in the time domain shows a main vibration component having a frequency value of 575 Hz and carrier which corresponds to various modulating effects.

The interaction between the various defects generates a complex motion including the frequencies identified earlier.

The component at a frequency as same as cage speed (64 Hz) is the most dominant in magnitude. The ball pass frequency (512 Hz) component is also dominant in magnitude.

The spectrum also shows excitation of frequency components at \( [n (f - f_c)] \), \( [n (f - f_c) + f_c] \) and \( [n (f - f_c) - f_c] \). This effect is also due to the nonlinear load-deflection characteristics, being effected by the number of terms in the approximation used for contact deflection. In the lower speeds a decrease in magnitude for these sideband frequencies was observed.

Considering the case with an off-sized ball which can be considered as a sort of waviness,
Table 1. Dominant frequencies in the spectrum of vertical vibration of a bearing having a wavy inner race.

<table>
<thead>
<tr>
<th>Peak</th>
<th>Frequency (Hz)</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>64</td>
<td>$f_c$</td>
</tr>
<tr>
<td>b</td>
<td>167</td>
<td>f</td>
</tr>
<tr>
<td>c</td>
<td>256</td>
<td>4f,</td>
</tr>
<tr>
<td>d</td>
<td>309</td>
<td>3(f - $f_s$)</td>
</tr>
<tr>
<td>e</td>
<td>412</td>
<td>4(f - $f_s$), $f_s$ - f</td>
</tr>
<tr>
<td>f</td>
<td>512</td>
<td>8f, 5(f - $f_s$), $f_s$ - f</td>
</tr>
<tr>
<td>g</td>
<td>575</td>
<td>$f_s$, 9f, 5(f - $f_s$) + $f_c$</td>
</tr>
<tr>
<td>h</td>
<td>620</td>
<td>6(f - $f_s$)</td>
</tr>
<tr>
<td>i</td>
<td>658</td>
<td>4f, 7(f - $f_s$) - $f_s$</td>
</tr>
<tr>
<td>j</td>
<td>760</td>
<td>8(f - $f_s$) - f, 10f, 9(f - $f_s$) - f, 5f - $f_c$</td>
</tr>
<tr>
<td>k</td>
<td>824</td>
<td>8(f - $f_s$) + $f_c$</td>
</tr>
<tr>
<td>l</td>
<td>890</td>
<td></td>
</tr>
</tbody>
</table>

From the computed results, the response of the system shows that frequency components of higher orders at $\{n(f - f_s)\}$, $\{n(f - f_s) \pm f\}$ and $\{n(f - f_s) \pm f\}$ are excited due to the increment in the number of waves on the inner race. These sideband frequencies indicate modulating effects.

CONCLUSIONS

1. Vibration characteristics of radial deep groove ball bearings which support a rigid horizontal shaft have been carried out while the system is subjected to the following external excitations: off-sized ball, rotating unbalance and surface waviness.

2. The mathematical model shows similarities to mechanical systems subjected to parametric excitation, which can be reduced to Mathieu's equation.

3. When more than one off-sized ball was considered, cage rotation frequency sidebands from the principal or dominant frequency were observed.

4. Under unbalance, the base frequency as well as the frequency of the excitation force was the dominant component in the frequency spectrum. The ball pass frequency also appears but is small in magnitude when compared to those mentioned above.

5. When surface waviness was included in the system, the dynamic behaviour was complex and an excitation of significant frequencies was observed. This behaviour is due to the nonlinear load-deflection characteristics, which are a result of the effect of a number of terms in the approximation in the work carried out. At high shaft speeds an increase in magnitude for these sideband frequencies was observed. This shows that at higher speeds wavy surfaces cause severe vibration.

6. The interaction of the effect of off-sized balls, which can be assumed to be a kind of waviness with a wavy inner race, also produces sideband frequencies.

Fig. 7 Calculated frequency spectrum for vertical vibration of a bearing having wavy inner race, rotating unbalance and an off-sized ball (N= 8, $n = 7500$ rpm, $e = 0.3$ µm, 4th ball 1 µm larger than the others).
REFERENCES


