PREFERRED PROPAGATION PATTERNS
IN SURFACE CRACKS UNDER CYCLIC LOADING

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ABSTRACT

There is a dramatic change in the shape of the semi-elliptical cracks growing due to combined cyclic bending and tensile loading of plates. The growing cracks change their shape such that they follow preferred propagation patterns (PPPs). It is observed that computation of strain energy release rate (G) in surface cracks plays an important role in determining the preferred propagation pattern (PPP). Numerical integration techniques have been employed to compute total strain energy (TSE) release rate. The variation of TSE with crack depth (a/t) for different bending ratios (R_b) is presented and discussed. A program in BASIC is written to directly simulate the propagation of crack on the screen. It is shown that for a given cyclic loading field, the PPP represents an upper limit on the aspect ratio of any surface crack growing due to this cyclic loading.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>a/c</td>
<td>aspect ratio</td>
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<tr>
<td>a/t</td>
<td>normalized crack depth</td>
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<td>G</td>
<td>strain energy release rate</td>
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<tr>
<td>K</td>
<td>stress intensity factor</td>
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<td>R_b</td>
<td>bending ratio</td>
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1.0 INTRODUCTION

Surface fatigue cracks in plates subjected to combined cyclic bending and tension loading change their shape substantially during the crack growth process. Accurate prediction of this shape variation is necessary for obtaining reliable safety margins of the cracked plates. The crack growth is controlled by the stress intensity factor (K) at the crack front. Consequently, in this paper the strain energy release rate (G) during the crack growth is calculated. As K varies along the crack front (Mahmoud, 1990), G for a crack must be obtained by integrating K over an incremental crack area. Three different techniques for calculating the same are evaluated.

This paper uses an empirical stress-intensity factor equation (Eq.1) for a surface crack (Newman and Raju, 1979), obtained from a three dimensional, finite element analysis of semi-elliptical cracks in finite elastic plates subjected to tension or bending loads, given by:

\[ K(\phi) = (S_t + HS_b) \sqrt{\frac{\pi a}{Q}} F \left( \frac{a}{c}, \frac{a}{C}, B, \phi \right) \]  

The functions of F and H are defined so that the boundary-correction factor for tension is equal to F and the boundary correction factor for bending is equal to the product of H and F.

The Eq.1 can be written (Hosseini and Mahmoud, 1985) as:

\[ K(\phi) = a \sqrt{\frac{\pi a}{Q}} \left[ M_1 + M_2 \left( \frac{a}{c} \right) \right] + M_3 \left( \frac{a}{C} \right) f(\phi) g(\phi) \]

where,

\[ M_1 = 1.13 - 0.09 \left( \frac{a}{c} \right) \]
\[ M_2 = 0.54 + 0.89 \left( \frac{a}{c} \right) \]
\[ M_3 = 0.5 - \frac{1}{0.64 + a/c} \]

\[ f(\phi) = \frac{a^2}{B} \cos^2(\phi) + \sin^2(\phi) \frac{1}{2} \]
\[ g(\phi) = 1 + 0.35 \left( \frac{a}{c} \right)^2 \left( 1 - \sin(\phi) \right)^2 \]
\[ f(\phi) = \frac{1}{2} \sec^2 \left( \frac{\pi a}{2B} \right) \left( \frac{a}{c} \right)^2 \quad \text{for} \quad \frac{c}{B} < 0.5 \]
\[ Q = 1.0 + 1.46 \left( \frac{a}{c} \right)^{1.65} \]

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Fig. 1(a) The geometry of the surface crack

Fig. 1(b) Incremental change in crack area for calculation of the average stress intensity factor

Fig. 1(c)-(d) Crack growth patterns as predicted by eq.(3) for five different initial crack configurations at (c) Re=0.0 and (b) Re=0.3.

' refer to section 3
In the following section, procedures for computing aspect ratio \((a/c)\) and strain energy release rate \((G)\) is presented.

2.0 NUMERICAL FORMULATION

The shape variation of semi-elliptical cracks is usually presented as a plot of the aspect ratio \((a/c)\) against the normalized crack depth \((a/t)\), see (i) set of Figure-1. Now, assuming that the material resistance due to cracking is constant along the crack front and using the Newman-Raju (1981) expression for \(K\) together with the Paris law, it is possible to show that the aspect ratio is the solution of differential eq. (3) (Hosseini and Mahmoud, 1985):

\[
\frac{da}{dt} = \left( 0.9 \frac{a}{c} \right)^{1.3} \left( 1.2 + 0.35 \frac{(a/t)^2}{(1-R_s+H_1R_0) / (1-R_s+H_2R_0)} \right) \cdot \left( H_1 \cdot 0.34 \frac{a}{c} \right) \cdot \left( 1 - 0.11 \frac{a}{c} \right)
\]

where

\[
H_1 = 1 - 0.34 \frac{a}{c} - 0.11 \left( \frac{a}{c} \right) \frac{a}{c}
\]

\[
H_2 = 1 + G_1 \frac{a}{c} + G_2 \left( \frac{a}{c} \right)^2
\]

\[
G_1 = -1.22 - 0.12 \left( \frac{a}{c} \right)
\]

\[
G_2 = 0.55 - 0.05 \left( \frac{a}{c} \right)^{0.75} + 0.47 \left( \frac{a}{c} \right)^{1.5}
\]

The bending ratio \(R_b = S_b/(S_b+S_t)\); \(S_b\) and \(S_t\) are the bending and tension components, respectively.

Based on thorough examination by Hosseini and Mahmoud (1985) of statistical parameters, the value \(m=3\) is recommended as a value for many materials. Examples of the crack shape variation during the crack growth under cyclic tension and bending are shown in Figure-1. The curves are obtained by numerical integration of eq.(3). For a crack initial dimensions \(a_i\) and \(c_i\), an increment \(da=400\) is selected and the increment \(dc\) is then calculated from eq.(3). The new crack dimensions become \(a_i+da\) and \(c_i+dc\). These values are then used in eq.(3) to obtain a second set of values, i.e. \(a_1\) and \(c_1\). This procedure is repeated until \(a_i/t\) becomes 1.0. Obviously, the predictions of this equation depend on the step size \(da\). The results presented in this paper are calculated using \(da=400\).

The strain energy release rate \(G\) is calculated for unit stress applied remotely on the crack by integrating \(K\) over an increment of a crack area \((dA)\) as shown in Figure-1(b). In this case:

\[
G = \frac{\int K^2 \phi \ dA}{\Delta A} \quad (5)
\]

where \(dA\) is a small element of \((\Delta A)\).

The incremental area \((\Delta A)\) is obtained by increasing \(a\) and \(c\) by \(da\) and \(dc\) respectively. Cruse and Besuner (1975) calculated \(G\) by assuming \((\Delta A)\) such that both \(a\) and \(c\) increase by an equal amount, i.e. \(da=dc\).

Obviously, other assumptions may also be valid. Therefore, three different methods of achieving \(\Delta A\) are discussed below:

Case-1 Assuming that \(a\) and \(c\) increase by the same amount, similar to Cruse and Besuner (1975), i.e. \(da=dc=400\).

Case-2 Assuming that the shape of the crack after the increment \(dA\) is similar to the shape of the crack before the increment, i.e. \((a+da)/(c+dc)=a/c\).

Case-3 Considering the real increases of the crack are due to fatigue, i.e. \(da=t/400\) and \(dc\) is calculated from eq.(3). It is observed that the crack growth patterns as predicted by Eq. (3), match very well with the experimental data reported by Kawahara and Kurihara (1977) (refer Fig. 2).

In performing the numerical integration of eq.(5), \(d(\Delta A)\) was taken such that \(d\theta=5^\circ\). This value was found after successive refinement, to be small enough to give accurate estimates of \(G\).

Figure-3 shows the results of the three methods of calculating \(G\) for the two different cracks. From the figure, \(G\) calculated from case-3 (real crack growth) is generally greater than the results of the other two cases. However, the difference is less than 4% except when \(a/t\) approaches 1.0. The figure also shows that for initially oblong cracks \([a/c=0.2\) to 0.4] the \(G\) variation using case-3 has more curvature than the other two cases [Fig. 3(a)]. For cracks with the higher initial aspect ratio, the three curves are similar. Based on the above, it was decided to use case-3 in the remainder of this paper because it uses the real crack shapes. Also, it is observed that trend of \(G\) variation in all three cases is similar.

3.0 VARIATION OF THE TOTAL STRAIN ENERGY RELEASE RATE WITH \(R_b\)

It is obvious from Eq.(3) that crack growth is sensitive to bending ratio \(R_b\). The variation of total strain energy release rate \((TSE)\) with crack depth \((a/t)\) is presented for different bending ratios \((R_b)\). The total strain energy release rate \((TSE)\) in this paper is defined as the amount of energy released over an area \(\Delta A\) i.e., \((G\Delta A)\).
The results are plotted for five different initial crack configurations:

i. \((a/c)_0 = 1.0\) and \((a/t)_0 = 0.01\) (PPP)

ii. \((a/c)_0 = 0.6\) and \((a/t)_0 = 0.05\)

iii. \((a/c)_0 = 0.2\) and \((a/t)_0 = 0.1\)

iv. \((a/c)_0 = 1.0\) and \((a/t)_0 = 0.4\)

v. \((a/c)_0 = 0.4\) and \((a/t)_0 = 0.4\)

Fig. 3 (a) Comparison of three methods of calculating strain energy release rate \((G)\) for a crack with (a) initial \(a/t=0.01\) and \(a/c=1.01\) (b) initial \(a/t=0.1\) and \(a/c=0.2\).

Fig. 3 (b)

Fig. 4 (a) Strain energy release rate \((G)\) Vs. normalized crack depth \((a/t)\) for five different initial crack configurations at (a) \(R_a=0.0\) and (b) \(R_a=0.3\)

Fig. 4 (b)

4.0 DISCUSSION AND CONCLUSIONS

It has been observed by Mahmoud (1992), that the PPP of the surface cracks in a plate under combined cyclic tension and bending is the growth pattern of a shallow surface crack with aspect ratio 1.0. This preferred pattern is such that \(a/c\) decreases continuously as the crack grows deeper. Other cracks grow such that they converge to this preferred pattern. The convergence is extremely strong for cracks above the preferred pattern. For cracks below the preferred pattern, they grow faster at first in the depth direction, thereby increasing \(a/c\), then converge to the preferred pattern by growing in the surface direction. It is seen that the prediction of crack growth patterns computed by numerical integration of differential Eq. (3) match very well with the experimental data (Kawahara and Kurihara (1977) and Aamodt and Klem (1976)).
The quantitative value of PPP should lead to improved life prediction of these surface cracks. The variation of aspect ratio \((a/c)\) and strain energy release rate \((G)\) with normalized crack depth \((a/t)\) for different values of \(R_b\) would be shown at the time of presentation. Also, computer simulated crack propagation could be viewed at the time of presentation.

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REFERENCES


Cruse, T. and Besuner, P., 1975, "Residual Life Prediction for Surface Cracks in Complex Structural Details", J Aircraft 12, 369-375


The variation of the strain energy release rate \((G)\) is calculated for surface fatigue cracks propagating under combined cyclic tension and bending loading. Three methods of calculating strain energy release rate \((G)\) are presented. It was observed that the variation of \((G)\) during fatigue growth of a crack qualitatively resembles the shape variation of the crack, i.e. \(G\) varies such that its values converge to the \((G)\) values for the PPP (refer to Fig. 4). This trend is also true for the variation of \(G\) as a function of the aspect ratio. Thus the \(G\) variation trends explain the crack growth shape variation, because \(G\) of a crack converges to a preferred \(G\) variation, the crack aspect ratio changes such that it converges to a PPP.

The comparison of TSE curves (see Fig(5)) for different \(R_b\) shows that the total strain energy release rate for the preferred propagation pattern (PPP) is the minimum of all the other initial crack configurations. This justifies the convergence of the other cracks with different initial configuration into the preferred propagation pattern (PPP).