A NUMERICAL STUDY OF THE FATIGUE LIFE OF A GAS TURBINE BLADE IN TRANSIENT OPERATIONS

M. Hohlrieder and H. Irretier
Institute of Mechanics
University of Kassel
Kassel, Germany

ABSTRACT
A numerical study of the dynamical response and the life estimation of a gas turbine blade which is subjected to transient nozzle excitation is presented. The mechanical and mathematical model for the blade, the exciting unsteady aerodynamic forces and the life estimation techniques are described and the solution procedure and its realization in a computer code is discussed. For an axial gas turbine compressor blade subjected to unsteady lift and drag forces during a run-up and run-down process numerical results are presented and the relation between the damping ratios, the speed of the run-up/down and the estimated fatigue life is discussed.

NOTATIONS

$C$, $C_C$, $C_D$  'damping' matrix
$D$, $D'$, $L'$, $M'$  CORIOLIS and damping matrix
$L_E$, $I_1$  cumulative fatigue damage
$L_\Omega^{-1} \Omega$  unsteady rotor blade drag and lift force and moment
$F_\Omega^{-1} \varphi$  blade exciting forces
$F$  blade bending natural frequency
$\Omega$  static inertia forces due to rotation and acceleration
$\Delta^y$, $\Delta^z$  complex FOURIER coefficients of the blade exciting forces
$k, k^*$  stress-cycle curve parameters
$K$, $K_S$, $K^P$, $\Omega$, $K^S$, $\Omega^S$  stiffness matrix
$m$, $n$, $n_{ij}$  structural and centrifugal pre-stress stiffness matrices
$m_X$, $P_Y$, $P_Z$  supplementary stiffness matrices due to rotation and acceleration
$M$, $\eta$, $\phi$, $\varphi$  exciting torsional moment and forces
$N$  mass matrix
$N_{ij}$  rotational speed
$N_{ij}$  number of stress cycles
$N_{ij}$  number of cycles to crack initiation
$N_{ij}$  number of stress cycles of fatigue strength for finite life
$N_{ij}$  limiting value of stress cycles endured
$N_{ij}$  number of stress amplitudes
$u, v, w$  displacement vector
$\tau$  time
$\delta$  translatory displacements
$\dot{\delta}$  blade coordinates
$\sigma_{\alpha\beta}$  number of nozzles
$\theta$  blade angle of incidence
$\varphi$  stress amplitude
$\Omega$  fatigue strength
$\phi$  phase of nozzle excitation

Presented at the International Gas Turbine and Aeroengine Congress and Exposition
The Hague, Netherlands — June 13–16, 1994
\( \phi_x, \phi_y, \phi_z \quad \text{rotatory displacements} \\
\nu \quad \text{number of FOURIER-harmonic} \\
\Omega \quad \text{blade angular velocity} \\
\psi \quad \text{blade twisting angle} \\

1 INTRODUCTION

Mechanical vibrations in turbomachine stages are one of the most significant reasons for failures of blades, blade packets or bladed disk assemblies in turbines and compressors. In many types of machines these vibrations cannot be completely avoided e.g. forced vibrations in turbomachines with frequently varying speeds of rotation in a wide range of operations. Then the dynamical response of the blades must be considered to ensure a sufficient dynamic life. Numerical models dealing with this problem have been developed just in the last ten years [1]—[13]. These models allow, for various applications and with different levels of accuracy, to calculate the dynamical response and the related stresses of the blade structure, and some of them even go further and perform life estimations on the basis of appropriate theories [2], [5],[9],[10], [13].

In the present paper, a numerical model and a numerical study are described which provide the possibilities of calculating the dynamical response together with the related stresses and the estimation of the life of blades subjected to forced vibrations excited by nozzle excitation in the stage.

2 VIBRATION MODEL OF THE BLADE

Suitable for a large number of steam and gas turbine and compressor blades, the mechanical model of the blades considered in the present paper is a rotating beam which can perform bending vibration in the y-z-plane and torsional vibrations around the x-axis (Figure 1). An extended, linear beam theory is used which takes into account all important effects occurring in the vibrational behaviour of the blade: Shear deflection and rotary inertia, stagger angle at blade root, blade twist and taper, centrifugal pre-stress in the blade and supplementary inertia effects due to angular speed and acceleration of the rotating blade. Thus, the applied model is applicable to a wide spectrum of turbine and compressor blades even to those which show significant marks of shell-like blades. Of course, blades of radial impellers and three-dimensional blades of gas turbines cannot be treated with this model.

The free-standing blade has root clamping conditions described by linear springs relating the shear forces and the bending and torsional moments at the blade root with the corresponding displacements and rotations. Coupling effects due to the elasticity of the disk between adjacent blades or for the complete bladed disk assembly are not included in the model.

The mathematical model of the blade can be described by its governing differential equations, in the following designated as continuous model, or by the principle of virtual work resulting in a finite element formulation, designated as discrete model. Both models, completed by the adequate boundary conditions, are used in the numerical model.

The continuous model of the blade is governed by differential equations [3], [4], [7], which can be arranged in the form

\[ M \ddot{q} + C \dot{q} + K q = f \]  

where \( q = q(x,t) \) is the displacement vector

\[ q = [u, v, w, \phi_x, \phi_y, \phi_z]^T \]  

of the longitudinal displacement \( u \), the transversal bending displacements \( v \) and \( w \), the associated angles of cross section \( \phi_x \) and \( \phi_y \), and the torsional angle \( \phi_z \). The matrices \( M, C \) and \( K \) and the vector \( f \) are continuous functions of \( x \) which, as e.g. \( K \), may also contain derivatives with respect to \( x \). \( M \) are the diagonal mass matrix, \( C \) the 'damping' matrix which consists by

\[ C = C_D + C_C \]  

of the real damping matrix \( C_D \) and the skew-symmetric CORIOLIS-matrix \( C_C = C_C(\Omega) \), and \( K \) the stiffness matrix which is of the form

\[ K = K_S + K_O + K^s + K^\Omega \]  

where \( K_S \) is the symmetric structural stiffness matrix of differential operators due to the longitudinal, bending, shear and torsional stiffness of the blade, \( K_O \) is the matrix of the centrifugal prestress \( \sigma^s \), \( K^s \) is the symmetric supplementary stiffness matrix and \( K^\Omega \) is the skew-symmetric supplementary stiffness matrix both due to the inertia forces in the rotating or accelerated reference system, for the deformed configuration of the blade. The right hand side of equation (1) is

\[ f = f_E + f_O + f^s + f^\Omega \]  

where \( f_E = f_E(x,t) \) are the exciting forces including static components distributed over the blade length, and \( f_O \) and \( f^s \), \( f^\Omega \) are the static inertia forces caused by the rotating and accelerated reference system, respectively.
The discrete model of the blade is of similar form than equation (1) and reads

\[ M \ddot{g} + C \dot{g} + K g = f \]  

(6)

where now the displacement vector \( g = g(t) \) with

\[ g = [\ldots u_i \ y_i \ v_i \ w_i \ \varphi_i \ \psi_i \ \varphi_i \ y_i \ z_i \ldots]^T \]  

(7)

contain all the discrete displacements at all nodes \( i \). The matrices \( M, C, K \) and the vector \( f \) are given in [14]. They have the same structure as in the continuous model, however, now due to the applied discrete model they consist of discrete real numbers instead of continuous functions of \( x \) as in the continuous model.

The equations of motion (1) or (6) are completed by the appropriate boundary condition of the blade. Here, as mentioned above, linear elastic conditions may be considered at the blade root to include the effect of local disk elasticity.

The excitation of the blade is modelled by time-dependent external forces and moment distributed over the blade length which are related to the unsteady aerodynamic drag, lift and moment on the blade profile (Figure 2). These unsteady aerodynamic forces are periodic or sweep-periodic functions of time, depending on the considered steady or transient case of constant or varying speed of the blade, respectively. So the typical and important nozzle excitation, including partial admission, can be considered while a flutter analysis is not included in the model.

The mathematical description of the vector \( f_E \) of the exciting forces distributed along the blade reads for the continuous model

\[ f_E = [0 \ p_y \ p_z \ m_x \ 0 \ 0]^T \]  

(8)

where \( p_y = p_y(x,t) \) and \( p_z = p_z(x,t) \) are the exciting forces with respect to the transversal directions \( y \) and \( z \), and \( m_x = m_x(x,t) \) is the exciting torsional moment, respectively (Figure 1).

They are related to the unsteady aerodynamic drag \( D^r \), lift \( L^r \) and moment \( M^r \) (Figure 2) by

\[ p_y = L^r \sin(\psi - \delta) + D^r \cos(\psi - \delta) \]
\[ p_z = L^r \cos(\psi - \delta) - D^r \sin(\psi - \delta) \]
\[ m_x = - M^r \]  

(9)

where \( \psi \) is the angle of twist and \( \delta \) is the angle of incidence between the flow velocity \( v_r \) and the chord line \( \eta_r \) of the blade profile which is assumed to be parallel to the principal axis of the cross section (Figure 2). When nozzle excitation is considered the unsteady aerodynamic forces are periodic functions of time with the fundamental nozzle passing (circular) frequency \( z \Omega \) where \( z \) is the number of nozzles and \( \Omega \) the angular speed of the blade. Consequently, the exciting force vector (8) by introducing equation (9) is of the form of the complex FOURIER-series

\[ f_E = \sum_{v=-\infty}^{\infty} \hat{F}_E^v e^{jv\varphi} \]  

(10)

where \( \hat{F}_E^v \) are the complex FOURIER-coefficients and

\[ \varphi = z \int_0^t \Omega dt \]  

(11)

is the fundamental phase of the periodic nozzle excitation.

In the particular steady case (\( \Omega = \text{const} \)) it reduces to \( \varphi = z \Omega t \) and \( f_E \) is really periodic. If in a transient case \( \Omega = \Omega(t) \neq \text{const} \)
const, \( f \) can be considered as a sweep-periodic function with increasing or decreasing exciting frequencies.

Practically, it is still an unsolved problem to estimate the unsteady aerodynamic forces \( L, D \) and \( M \) leading to the FOURIER-coefficients in equation (10) both in a sufficiently accurate and general manner and, simultaneously, with a reasonable computing effort. In the blade model presented here, a technique and a computer code [15], [16] on the basis of a plane, compressible, subsonic flow up to \( M \leq 0.8 \) for thin airfoil blades is used. The theory and analysis considers the potential interaction i.e. the interference of the steady circulation around the stator blades on the rotating rotor blades in the stage which generally is the dominating effect for the unsteady aerodynamic forces and, additionally, the vortex wake interaction i.e. the effect of vortex shedding from the stator on the rotor blades. Both are effects from a frictionless flow through the stage which may be completed by the generally small viscous wake interaction i.e. effects of flow friction.

When the exciting forces are calculated for various cross-sections of the blade they are either introduced directly into the force vector if the discrete model is used or interpolated linearly to yield the continuously distributed exciting forces required in the force vector for the continuous model.

The damping of the blade or blade group is introduced into the model by assuming a viscous modal damping described by corresponding damping ratios associated with the modes of the system. In view of the very complicated nature of damping in blade systems this is without doubt a very rough assumption. However, considering the unsolved problem of sufficiently accurate and general damping estimations for real blade constructions including the clamping conditions, the surrounding medium, etc., this assumption seems to be adequate for the basic studies for which the model presented here was developed.

The angular speed of the blade or the blade packet may be constant, called the steady case, or may vary linearly with time, called the transient case. Both, positive and negative angular accelerations for the transient cases of run-up and run-down, respectively, can be considered. Thus, operations can be simulated for which the angular velocity of the blade starts from an initial value e.g. from standstill, runs up to nominal speed, remains constant for a certain time, and then runs down again to the initial speed. During the period of varying angular speed, the change of the natural frequency due to the altering centrifugal effects is included in the considerations while the modes are assumed to remain constant. Also, the change of the exciting frequency spectrum of the nozzle excitation when the blade or blade packet accelerates is taken into account. Basically, also the change of the damping ratios and of the amplitudes and phases of the exciting unsteady aerodynamic forces with the rotational speed can be considered in the simulation process.

3 LIFE ESTIMATION MODEL OF THE BLADE

During a simulated range of operation, the vibrations of the blade result in dynamic stresses which show a transient character versus time with significant maxima near to the resonances between the blade natural frequencies and the harmonics of excitations. This dynamic stress is then classified by a stress-level classification technique resulting in a stress-level distribution. By comparing the stress-level distribution with the stress-cycle curve of the blade material the cumulative fatigue damage of the dynamic stresses is calculated using an appropriate hypothesis which results in a life estimation of the blade for the considered simulation range of operation.

Besides the concepts of fracture mechanics, there exist basically three concepts of determination of an effective dynamic stress or strain for the life estimation. The nominal stress concept considers the nominal stress and takes into account notch effects by using stress-cycle curves for a corresponding notch specimen. This concept is limited to elastic stresses and strain below the yield point and the necessary stress-cycle curve often are not available. The notch stress concept considers the real stress at the root of the notch e.g. from an elastic finite or boundary element solution or from the elastic nominal stress increased by appropriate notch factors and uses then the stress-cycle curves of an unnotched specimen. This concept can also be applied for stresses beyond the yield point by reducing the elastic plastic stress and strain to an elastic comparative stress by NEUBER's rule [17]. Finally, the notch root concept starts from the real stress and strain determined from an elastic-plastic finite or boundary element method and uses strain-cycle curves for the life estimation.

In the present model, the notch stress concept is used based on the dynamic displacement field from the solution of the equations of motion (1) or (6) for the continuous or discrete model, respectively, from which the corresponding linear-elastic nominal stresses \( \sigma^n \) are calculated with help of the constitutive equations. This calculation is performed for the contour of the blade root cross-section because generally this is the part of the blade with the highest static and dynamic stress level. Considering the stress patterns versus time for different points on the blade root contour those points are chosen for which a life estimation procedure should be performed. The criterion for this choice can be the highest average or cumulation of the dynamic...
stress or the highest stress peaks during the range of operation.

As already pointed out, the calculated dynamic stress is the linear-elastic nominal stress $\sigma^{en}$ in the blade root cross section. If necessary, this nominal stress must be converted into the higher real stress $\sigma^{er}$ by an appropriate stress concentration or fatigue notch factor if notch effects must be taken into account. It might happen that either only $\sigma^{er}$ or both $\sigma^{er}$ and $\sigma^{en}$ are larger than the yield stress of the blade material. In this case, additional consideration are required which allow the calculation of an elastic comparative stress $\sigma^{ec}$ on the basis of the elastic-plastic stress-strain relation of the material following NEUBER's rule. Details considering this technique in application on turbine blades are discussed in [10]. So far, they are not included in the present model which is described here. In any case, the result of these procedures is a certain dynamic elastic stress value $\sigma = \sigma(t)$ which is now considered further for the intended life estimation of the blade.

On principle, this calculated dynamic stress is a transient function versus time (Figure 3) and a classification procedure is required to convert this function into a stress level distribution. There exist various techniques dealing with this problem [18] - [20] divided into one-parameter methods as peak counting, level crossing counting or range counting and two-parameter methods as range-mean counting, peak-through counting or the rainflow-cycle counting method. Here the rainflow-cycle counting technique and a modification of the level crossing counting technique are realized in the life estimation procedure. The first one counts the closed loops of the elastic or elastic-plastic stress-strain curves during the considered range of operation (Figure 4) and records the number of loops of different classes of mean stress values $\bar{\sigma}^m$ and ranges of stress amplitudes $\sigma^i$. The second counting technique in its classical form counts the number of crossings of the stress-time curve over the boundaries of certain classes of stress levels (Figure 5) while the mean stress value is taken as constant for the whole range of operation. In the present procedure such modification is used which devides also the complete stress-time curve in classes of mean stress values $\bar{\sigma}^m$ and for each class the number $p^i$ of level crossings are recorded. Thus, processes with increasing or decreasing
mean stresses as they occur in the blade vibrations during run-up or run-down, respectively, can be evaluated not only with respect to the vibratory stresses but also to the related actual mean stress.

Corresponding to the number of classes of mean stresses, both classification procedures result in a certain number of stress level distributions (Figure 6) from which follow the number $p_{nj}$ of stress amplitudes $a_{ij}$ for each class $j$ of mean stress $\sigma_{mj}$. These stress amplitude distributions are now compared with stress-cycle curves for the corresponding mean stress values (Figure 7). In general, for this procedure these curves are approximated by the classical WOEHLER-line (index $j$ is omitted in the following equations)

$$N = \begin{cases} \frac{\sigma^{\infty}}{\sigma} k & \text{for } \sigma \geq \sigma^{\infty} \quad (12) \\ \infty & \text{for } \sigma < \sigma^{\infty} \end{cases}$$

between the stress amplitudes $\hat{\sigma}$ and the number $N$ of cycles to crack initiation where $\sigma^{\infty}$ and $N^{\infty}$ are the fatigue strength and the limiting value of stress cycles endured, respectively, and $k$ is an exponent depending on the material, its processing and other circumstances. A life estimation for a stochastic dynamic stress based on this classical WOEHLER-line yields the well-known PALMGREN-MINER damage hypothesis [21], [22], for which follows that stress amplitudes below the fatigue strength do not contribute to the estimated fatigue damage. This assumption, however, is known to be not sufficient for the damage process of stochastic stresses in many cases and modified hypothesis of cumulative fatigue damage were developed considering also the damage contribution of these stresses. In particular, the hypothesis of HAIBACH [19] and that one described by CORTEN-DOLAN [23] appear to be better approximations

$$N = \begin{cases} \frac{\sigma^{\infty}}{\sigma} k & \text{for } \sigma \geq \sigma^{\infty} \\ \infty & \text{for } \sigma < \sigma^{\infty} \end{cases}$$

for the stress-cycle curves in view of an accurate life estimation. The first one is described by

$$N = \begin{cases} \frac{\sigma^{\infty}}{\sigma} k & \text{for } \sigma \geq \sigma^{\infty} \\ \infty & \text{for } \sigma < \sigma^{\infty} \end{cases}$$

which corresponds to a continuation of the decreasing curve of stress amplitudes versus number of cycles also beyond the limiting value of stress cycles endured by a straight line with approximately the half of slope than above this point (Figure 7). The second one uses the approximation

$$N = N_{a} \left( \frac{\sigma}{\sigma_{a}} \right)^{k_{a}}$$

Fig. 7: Stress-cycle curves for cumulative fatigue damage

Fig. 8: Influence of mean stress on the fatigue strength

Fig. 9: Axial compressor blade
for the whole range if stress amplitudes where a modified value of \( k^* = (0.8...0.9)k \) is taken as exponent which yields a stress-cycle curve with a slope more than the classical WOEHLER-line but continued into the range of fatigue strength (Figure 7).

It is known that in the equations (12) to (13) in particular the fatigue strength \( \sigma_{\infty} \) depends on the mean stress \( \sigma_m \). Different approximations were made considering this effect from which the most important ones were described by HAIGH, GOODMAN, GERBER, BAGCI and ZENNER [24]. They are plotted in Figure 8. All these approximations are included in the developed program and can be optionally chosen for the life estimation procedure.

The final step in this procedure is the application of the hypothesis of cumulative fatigue damage which compares the numbers \( n_{ij} \) of cycles on a certain level of the stress level distribution with the numbers \( N_{ij} \) of fatigue strength for finite life for the same stress amplitude level (Figure 6). The cumulative fatigue damage is then defined by

\[
D = \sum_{j=1}^{m} \sum_{i=1}^{m_j} \frac{n_{ij}}{N_{ij}}
\]

which gives the percentage of life which is used up by one of the repeating ranges of blade operations considered in the simulation. Depending on the stress-cycle curve which is used to calculate the number \( N_{ij} \) of fatigue strength for finite life (Figure 6), the three hypothesis of PALMGREN-MINER, HAIBACH and CORTEN-DOLAN come into consideration. As already pointed out, the main difference between these hypothesis is the evaluation of the stress amplitudes below the fatigue strength for the cumulative fatigue damage. In the developed program the three hypothesis are realized, and the final result is the estimated fatigue life of the blade considered in the simulation process.

On the basis of the theories and algorithms described above a computer package named TUBSIM (Turbine Blade Simulation) was developed at the Institute of Mechanics of the University of Kassel in the last five years for the dynamic analysis and life estimation for beam-like turbine blades [12], [24], [25].

In the following, a basic example shows the possibilities and applications of the developed simulation program for an axial compressor blade subjected to a nozzle excitation during operations with varying rotational speeds.

### 4 EXAMPLE AND NUMERICAL RESULTS

The blade considered in the following basic study is an axial compressor blade of a gas turbine with a typical shallow-shell-like geometry (Figure 9).

This tapered and twisted blade of a length of \( L=228.7 \) mm is fixed on a disk with a radius of \( R=316.8 \) mm; the average width and profile thickness are 50 mm and 6 mm, respectively. The further detailed geometric data of the blade are given in [25].

The material stress data of the blade made of steel X 10 Cr 13 as the tensile strength \( \sigma_b \), the yield stress \( \sigma_y \), and the fatigue strengths \( \sigma_{\infty} \) and \( \sigma_{\infty} \) under reversed and pulsating stress, respectively, are given in Table 1 for an operating temperature of 300°C.

The stress-cycle curve parameters \( N_{\infty}, k \) and \( k^* \) in the equations (14) to (16) are presented in Table 2, and the

\[
\begin{array}{cccc}
\sigma_b & \sigma_y & \sigma_{\infty} & \sigma_{\infty} \\
610 & 495 & 275 & 407 \\
\end{array}
\]

Table 1: Material stress data [N/mm²]

\[
\begin{array}{ccc}
N_{\infty} & k & k^* \\
3.10^7 & 10 & 10 \\
\end{array}
\]

Table 2: Stress-cycle curve parameters

\[
\begin{array}{ccc}
E [N/mm²] & v & \rho [kg/dm³] \\
0.21.10^6 & 0.3 & 7.85 \\
\end{array}
\]

Table 3: Material data

\[
\begin{array}{ccc}
\text{Case} & T_{up}=T_{down}[s] & \omega [Hz/s] \\
SLOW & 50 & 2 \\
FAST & 25 & 4 \\
\end{array}
\]

Table 4: 'Slow' and 'fast' run-up/down and corresponding rotational frequency sweep
YOUNG's modulus $E$, POISSON's ratio $v$ and the material density $\rho$ are given in Table 3.

The CAMPBELL-diagram for the first three bending natural frequencies shows the increase of these natural frequencies due to the centrifugal stiffening effect from the values $f_1 = 118$ Hz, $f_2 = 473$ Hz and $f_3 = 1048$ Hz at standstill up to a rotational speed of 6000 rpm (Figure 10). The two harmonics of excitation $v = 1$ and $v = 2$ which are considered in the study that follows are also indicated in the CAMPBELL-diagram (Figure 10). In the present study an operation is simulated where the blade runs up from standstill to a rotational speed of 6000 rpm within a time $T_{up}$ and runs down again to standstill within the time $T_{down} = T_{up}$ (Figure 11). Two cases of time periods for run-up and down and, consequently, of acceleration and deceleration during this process are considered marked as the cases SLOW and FAST respectively (Table 4 and Figure 11).

![Fig. 10: CAMPBELL-diagram for the first three bending natural frequencies](image)

![Fig. 11: Rotational speed during run-up, steady state and run-down (load block)](image)

### Table 5: Mean values and exciting amplitudes of unsteady lift and drag

<table>
<thead>
<tr>
<th>$v$</th>
<th>$L_v^f$ [N/mm]</th>
<th>$D_v^f$ [N/mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0</td>
<td>0.1</td>
</tr>
<tr>
<td>1</td>
<td>0.05</td>
<td>0.005</td>
</tr>
<tr>
<td>2</td>
<td>0.025</td>
<td>0.0025</td>
</tr>
</tbody>
</table>

### Table 6: 'Low' and 'high' modal damping ratios of the three blade bending modes

<table>
<thead>
<tr>
<th>case</th>
<th>$\xi_1$</th>
<th>$\xi_2$</th>
<th>$\xi_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>0.00075</td>
<td>0.00094</td>
<td>0.0011</td>
</tr>
<tr>
<td>High</td>
<td>0.0015</td>
<td>0.0019</td>
<td>0.0023</td>
</tr>
</tbody>
</table>

During the simulated process the blade is subjected to the mean values of lift and drag forces $L^f_o$ and $D^f_o$ and, additionally, to two harmonics of excitation $v = 1$ and 2 in a stage of $z = 20$ nozzles of each of the unsteady lift and drag forces $L_v^f = 2[L_o^f]$ and $D_v^f = 2[D_o^f]$. The assumed values of both the steady and unsteady forces are presented in Table 5 from which follows that all the drag values are taken to be...
10% of the corresponding lift values and that the first exciting amplitude is 5% and the second one 2.5% of the mean value of lift and drag, respectively.

The two harmonics of excitation as indicated in the CAMPBELL-diagram (Figure 10) by the two straight lines show from the points of intersection with the natural frequency curves that during each run-up and run-down during one block of operation (Figure 11) the blade passes through six resonances resulting in high levels of dynamic response superimposed to the static stress due to the mean lift and drag and to the centrifugal force field caused by the rotation.

For the modal damping of the three bending modes two cases are considered (Table 6) which cover a realistic range of the very small damping values of free standing axial compressor blades. The two cases of damping, designated as LOW and HIGH damping, respectively, differ from each other by a factor two; both have a slight increase with the mode number of 25% and 50% for the second and third mode, respectively, with respect to the fundamental mode.

First, Figure 12 shows for the two cases SLOW and FAST of run-up and -down (Table 4 and Figure 11), each of them considered with the two cases LOW and HIGH of damping (Table 6), the four different dynamic response curves versus time in form of the resulting static and vibratory normal stress \( \sigma_{xx} \) at the blade root at the leading edge of the blade root contour. This point proved to be that one with the highest dynamic stress and, consequently, it should be considered for the intended life estimation.

Fig. 12: Dynamic stress response during one load block for various speeds of run-up/down and levels of damping
All the curves show that the mean stress which is the sum of the static bending stress of the mean values of the lift and drag forces and the normal stress due to the centrifugal force field increases during the run-up phase of the operation up to its final constant value while it decreases during the run-down period correspondingly. This mean stress field is superimposed by more or less strong dynamic stress responses shortly after the resonance points (Figure 12). The maxima of these transient resonance responses are primarily determined by the intensity of excitation of the two considered harmonics (Table 5) and by the mode number because the exciting intensity of a mode rapidly decreases with the mode number when uniformly distributed forces along the blade as considered here are responsible for the excitation of the modes. Besides these two aspects, the maximum transient resonance responses depend on the modal damping ratio of the mode and the rotational frequency sweep rate during the run-up or run-down period. Estimation formulas considering this effect were given by Iretier and Leul (1991). They prove that the influence of the damping on the response maximum in a transient state is much less than in the steady state, in particular for the low modes. On the other hand, the influence of the acceleration or deceleration on the response maximum during the transition through resonance is lower the higher the damping is and vice versa. Also is known that the transient response peaks during run-down are almost slightly higher than during run-up provided the resonance peaks are well separated from each other so that each peak can be considered as for a single-degree-of-freedom system. Otherwise, the phase relations between the different response harmonics caused by the various harmonics of excitation and the considered natural frequencies may result in response levels during run-up and run-down which differ from this rule. All these facts are visible in the four diagrams shown in Figure 12.

Next, the cumulative fatigue damage and the estimated fatigue life is considered for the four combination of SLOW and FAST run-up/down and LOW and HIGH damping. For all the following results the mean stress hypothesis given by HAIGH is used as it is that one just between the two extreme hypothesis of GOODMAN and BAGCI (Figure 8). It is important to point out here that the influence of the mean stress hypothesis on the estimated life can be of considerable magnitude as it was shown in [13]. Here, the hypothesis of HAIGH is applied as a compromise among the various other mean stress hypothesis.

First, the influence of the classification procedure and of the stress-cycle curve used for the life estimation is considered. Table 7 shows for the case of SLOW run-up and LOW damping the estimated fatigue life as number of load blocks as one is shown in Figure 11. The results prove that

<table>
<thead>
<tr>
<th>stress-cycle curve</th>
<th>Cortan-Dolan</th>
<th>Haibach</th>
<th>Palmgren-Miner</th>
</tr>
</thead>
<tbody>
<tr>
<td>level crossing</td>
<td>1.66</td>
<td>1.69</td>
<td>1.73</td>
</tr>
<tr>
<td>rainflow-cycle</td>
<td>1.48</td>
<td>1.51</td>
<td>1.53</td>
</tr>
</tbody>
</table>

Table 7: Estimated fatigue life (values x 10^4) as number of load blocks

<table>
<thead>
<tr>
<th>run-up/down damping</th>
<th>damage of one whole load block</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>run-up only</td>
</tr>
<tr>
<td>SLOW LOW</td>
<td>6.55</td>
</tr>
<tr>
<td></td>
<td>1.36(21%)</td>
</tr>
<tr>
<td>SLOW HIGH</td>
<td>2.68</td>
</tr>
<tr>
<td></td>
<td>0.77(29%)</td>
</tr>
<tr>
<td>FAST HIGH</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>0.35(78%)</td>
</tr>
<tr>
<td>FAST HIGH</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>0.05(50%)</td>
</tr>
</tbody>
</table>

Table 8: Cumulative fatigue damage (values x 10^5) for various speeds of run-up/downs and levels of damping

The rainflow-cycle technique is the safer one of the two classification procedures. However, the difference is never more than 13%. The differences between the three stress-cycle curves used for the cumulative fatigue damage calculation even do not exceed 4%. Consequently, it follows from both facts that is sufficient to apply only one combination of the methods indicated in Table 7 to study now the influence of the speed of the run-up/down and the
level of damping on the fatigue life of the considered compressor blade.

For each of the four combinations of SLOW and FAST run-up/down on the one hand and of LOW and HIGH damping on the other, Table 8 shows the cumulative fatigue damage for one whole load block (Figure 11) and, additionally, only for the run-up and for the run-down phase of the load block.

The rainflow-cycle classification procedure and the PALMGREN-MINER-hypothesis were used for the calculations. As expected from the dynamic stress response curves considered above in Figure 12 the highest damage occurs in the case of SLOW run-up/down and LOW damping.

Doubling the level of damping reduces the damage by a factor of 2.44, however, the contrary case, where the damping remains on the low level and the speed of run-up/down is doubled, results in a decrease of the damage by a factor of 14.56 because on this low level of damping the influence of the speed of the run-up and run-down process is dominant for the transient vibratory response of the blade and exceeds considerably the influence of the damping. This fact is also visible from the comparison of the cases SLOW / HIGH and FAST / HIGH where the reduction of damage is of a factor 26.8 whereas in the cases FAST / LOW and FAST / HIGH the reduction of damage by a factor of only 4.5 is much less.

Besides the total damage of one whole load block (Figure 11), Table 8 shows also the values and percentage of damage for the run-up and the run-down phase separated from each other. For the SLOW run-up/down where the six resonances are all relatively well separated from each other the damage during the run-down is much higher than during the run-up because of the higher transient resonance response levels. However, in the FAST run-up/down case the tendency changes because now the close resonances in combination with an extremely slow amplitude decay due to the very low damping level result in total dynamic stress levels which are of the same magnitude or even higher during the run-up in comparison to the run-down phase. This fact was already observed in the dynamic stress plots in Figure 12.

Finally, Table 9 shows the estimated life of the blade again as number of load blocks for the four combinations of speed of run-up/down and damping. Once more the results emphasize the very important influence of the speed of the run-up/down on the expected fatigue life which is of much more significance than the level of damping.

### Table 9: Estimated fatigue life (values x 10⁴) as number of load blocks

<table>
<thead>
<tr>
<th>run-up/down</th>
<th>damping</th>
<th>fatigue life</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLOW</td>
<td>LOW</td>
<td>1.53</td>
</tr>
<tr>
<td>SLOW</td>
<td>HIGH</td>
<td>3.73</td>
</tr>
<tr>
<td>FAST</td>
<td>LOW</td>
<td>22.22</td>
</tr>
<tr>
<td>FAST</td>
<td>HIGH</td>
<td>101.01</td>
</tr>
</tbody>
</table>

5 CONCLUSIONS
A numerical model was presented for the simulation of forced vibrations and the estimation of fatigue life of turbine blades subjected to stationary and transient nozzle excitation. The presented results show that all important effects for this process are included in the program. This allows sufficient classifications of the correlation of the dominating parameters of the problem like the acceleration during run-up and run-down operations, the natural frequencies of the blades, the related damping ratios, the unsteady blade forces etc. and the fatigue life of blades under conditions of steady and transient nozzle excitation.

6 ACKNOWLEDGEMENT
The work and results presented here were partly supported by the Deutsche Forschungsgemeinschaft, Bonn, Germany, and the author acknowledges with thanks the financial help to develop the program and to carry out some of the presented studies.

7 REFERENCES


[22] MINER, M.A.: Cumulative Damage in Fatigue, J. Applied Mechanics, 12, 1961, pp. 159-164

