Coupled 3-D Aeroelastic Stability Analysis of Bladed Disks

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Abstract: In the present work an algorithm for the coupled aeromechanical computation of 3-D compressor cascades vibrating in a traveling-wave mode is presented and applied to the determination of aerelastic stability of a transonic fan rotor. The initial vibratory modes are computed using finite-element structural analysis code. The unsteady flowfield response to blade vibration is estimated by numerical integration of the 3-D unsteady Euler equations. Coupling relations are formulated in the frequency domain using a mode-modification technique, based on modal projection. The vibratory mode is updated at the end of the aerodynamic simulation of each period, and the updated mode is used for the simulation of the next period. A number of results illustrate the method's potential.

1 INTRODUCTION

Turbomachinery flutter is analyzed using one of a number of formulations, traveling-wave, standing-wave, or influence-wave (cf. Crawley [1988]). The developments in the field have been reviewed, e.g., by Platzer [1975, 1978], Sisto [1977], Fleeter [1979], Bendiksen [1988, 1990], and in the 2 volumes of AGARDograph 298 (1987, 1988).

Many studies of turbomachinery flutter have been based on a methodology proposed by Carta [1967]. This method, which Bendiksen [1990] calls energy method, is based on the assumption that flutter occurs in one of the natural modes of vibration of the rotor and computes the energy exchange between this vibration mode and the flowfield. Any mode modification due to unsteady aerodynamic forces is neglected (but the steady-state aerodynamic forces are often included in the computation of the vibratory mode). As a consequence this methodology is essentially uncoupled, or more precisely weakly coupled, in the sense that no feedback of unsteady aerodynamic forces on mode-shape and frequency is included.

This methodology has been extensively used combined with aerodynamic models of varying complexity, mostly strip theories, based on spanwise stacking of 2-D aerodynamic models, and more recently fully 3-D methods. In the original work of Carta [1967] Theodorsen's [1935] isolated flat-plate airfoil theory was used (Bendiksen [1990] notes that "it appears that Carta's method is quite successful in estimating the relative stability of various fan designs, even if Theodorsen's isolated airfoil theory is used").


Other authors consider more realistic structural models. Kaza & Kielb [1984a] used a nonlinear elastic beam theory (Kaza & Kielb [1984b]) coupled with the flat-plate cascade theories of Smith [1973] and Adamczyk & Goldstein [1978]; this model was extended to include disk flexibility in Kaza & Kielb [1985]. Srinivasan & Fabunmi [1984] used an assumed modes method (Bisplinghoff & Ashley [1962a]), with 3 coupled bending-torsion basic modes, coupled with Smith's [1973] flat-plate cascade theory in a strip theory fashion. White & Bendiksen [1987] used an assumed modes method coupled with a strip-theory computation based on the flat-plate cascade theory of Bendiksen & Friedman [1981] to compute the aeroelastic characteristics of metal and composite blades.

Henry & Vincent [1990] used an assumed modes method (with basic modes obtained by a variety of structural methods) coupled with results from the $Z^2$-D Euler method of Gerolymos [1988a]; in this work the Euler method was used to generate tabulated force and moment results for various blade sections (intersections of steady flow streamsurfaces with the blade), i.e., to generate an aerodynamic modal basis.

The above review of the literature on coupled computation of aeroelastic eigenmodes leads to the following conclusions:

- The bulk of the literature is concerned with model parametric studies.
- Virtually all methods, including those using advanced structural models are based on 2-D aerodynamic theories, mostly of the flat-plate type, in a strip theory fashion, with the exception of Henry & Vincent who used a strip theory based on $Z^2$-D computations; however, fully 3-D computations have proved to be indispensable, especially in transonic/supersonic compressors (e.g. Epstein & al [1979], Kerrebroot [1981], Karadimas [1988]), and are believed to be so for the unsteady case as well (cf. Bendiksen [1990]).

The purpose of this paper is to develop a coupling procedure, to compute aeroelastic vibration modes, frequency and damping, using the 3-D Euler solver developed in Gerolymos [1992] as aerodynamic operator (cf. Fig. 1 for a global description of the computational procedure). This paper will be concerned with the tuned rotor problem (Lane [1956]).

2 PROBLEM STATEMENT

2.1 Assumed modes method and aerodynamic operator

In the present work, the assumed modes method is used. First (cf. section 2.2) a set of basic structural modes (in which the static deformation and stiffening due to steady aerodynamic pressures has been included) is computed. These modes are computed for various traveling-wave orders n (cf. Lane [1956]), of corresponding interblade phase-angle $\beta_n=2\pi n/N \ (n=0,1,...,N-1)$, thus generating a mechanical modal basis (mechanical/aeromechanical) which can be described by the set

$$ M = \left\{ m_{n}, m_{n} \left< M \right> \mid 0 \leq n \leq N-1 \right\} \ (1) $$

where $m$ is the blade-mode number, $M$ the number of blade-modes used for the subsequent modal synthesis, $\beta_n$ the traveling-wave order defining the interblade-phase-angle $\beta_n$, $N$ the number of blades, $Z$ the set of integer numbers, and $P$ denotes the operating point under consideration, e.g.

$$ P = \left[ \pi, \pi_T, \theta, \text{RPM}, P_0, T_0 \right]^T \ (2) $$

where $\pi, \pi_T$ is the total-to-total pressure ratio, $\pi_T$ is the steady pressure distribution for static computation and geometry modification.

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Fig. 1 Flow-diagram of the 3-D computer-program MANIAC-2 (Methodology for Aeroelastic Numerical Instability Analysis in Compressors).
mass-flow-rate, RPM the rotation velocity, \( \rho \) the inlet total pressure, and \( T_\text{in} \) the inlet total temperature (all these parameters influence the mechanical modes through centrifugal stiffening and steady aerodynamic effects). These modes will be coupled, when excited, through unsteady aerodynamic effects, to yield aeromechanical eigenmodes

\[
A = \begin{bmatrix}
15mN & 15mN \\
0 & 0 \\
\end{bmatrix}
\]

Based on the orthogonality of aeromechanical eigenmodes of different \( \beta \), (cf. Thomas [1979], Lane [1956]) the computation of the aeromechanical eigenmode \( \Phi(m, \eta, r, P) \) uses the modal basis \( \Phi(m, \eta, r, P) \)

\[
\Phi(m, \eta, r, P) = \begin{bmatrix}
15mN \\
0 \\
\end{bmatrix}
\]

(3)

for a fixed \( r \in \{0, 1, \ldots, N-1\} \). This is an important reduction to the number of degrees of freedom, since an original problem for the search of \( M \times N \) modes is reduced to \( N \) separate problems of \( M \) modes, as is usually the case in tuned rotors aeroelasticity. Then the aeromechanical mode \( \ell \) for the traveling-wave \( r \), \( \Phi(\ell, r, P) \) will be assumed to have a modeshape \( \Phi(\ell, r, P) \) given by

\[
\Phi(\ell, r, P) = \sum_{m=1}^{M} \Phi(m, \eta, r, P)
\]

(4)

where \( \Phi \) denotes the aeromechanical mode shape, \( \Phi \) the mechanical modeshape (cf. section 2-2), and \( \eta_m \) are the generalized coordinates. In order to formulate the aeroelastic problem and compute the coupling of the mechanical eigenmodes through unsteady aerodynamic effects 2 methods are encountered in the literature, either the use of Padé approximants (e.g. Edwards & al [1977], Dugundji & Bundas [1984]), or the use of an aerodynamic modal basis (e.g. Henry & Vincent [1990]). In either case, one would need to compute the aerodynamic operator, in the form of unsteady pressures on the surface of the blade, for each of the assumed modes used for the modal projection, vibrating at various frequencies, e.g. for the frequencies of each eigenmode. Hence the knowledge of an aerodynamic data base in the form

\[
P_r = \begin{bmatrix}
15mN \\
0 \\
\end{bmatrix}
\]

(5)

where \( p \) are the \( i \)-harmonics of pressure, \( x \) a position vector, \( \delta B \) the blade surface, and \( f_r = \{ f_1, f_2, \ldots, f_m \} \) are the frequencies of the mechanical modes \( M \) used in the assumed modes method. Hence the prediction of the coupled aeroelastic modes (and their stability or instability) for only \( 1 \) traveling-wave order would require \( M \times N \) aerodynamic computations. And since the stability of \( 1 \) operating point would require the computation of \( L \times 8 \) (an arbitrary estimation) traveling-waves, \( L \times M \times M \) computations would be needed for the construction of the aerodynamic database at each operating point. Since most aeroelastic studies in the field of turbomachinery have used simple (and computationally rapid) methods, often analytical, the construction of \( P \), and often of \( \Phi = \Phi(\ell, r, P) \), did not present a major problem. This is not the case, however, for 3-D time-marching Euler methods (and later solvers with viscous models), where the realistic representation of the steady and unsteady flowfield requires substantial computing times. A typical time-marching Euler computation of the 3-D unsteady flow in a vibrating cascade requires from \( \Delta t = \tau \) of CPU on a CRAY-XMP computer (depending on the frequency, cf. Gerolymos [1992]), say a typical \( \tau = 2 \tau \) per computation (as a mean for the first 6 frequencies of a fan rotor). This computation is done for a given vibratory mode and a given interblade-phase-angle. Hence for a typical \( M = 6 \), 36 Euler computations would be required, resulting to unrealistic computing-times, since this operation should be done \( L \times 8 \) times at each operating point \( P \), at various operating points. The purpose of the present work is to circumvent this problem by developing a coupling method using only 1 aerodynamic computation for each aeroelastic mode, thus reducing the computing time per mode by a factor \( M \).

To this purpose a method of mode-modification during the time-marching iterations was developed. This procedure, that will be described in detail in section 3, is schematically described in Fig. 2. At the end of the aerodynamic simulation of each period of the aeromechanical mode \( \Phi(\ell, r, P) \), the aeromechanical mode is reactualized by the contributions of the modes of Eq. 5, computed on the basis of the unsteady pressures generated by \( \Phi(\ell, r, P) \) itself (\( \Phi(\ell, r, P) \), \( x \in \delta B \)), instead of an equivalent linear superposition of unsteady pressures generated by each

![Fig. 2 Procedure for coupled aeromechanical computation.](image-url)
assumed mode and interpolated to the frequency of interest. The frequency is now an unknown determined by the coupling procedure, \( f = |B(E,r,P)| \).

### 2-2 Basic structural modes

The mechanical modes are computed using the theory for rotationally periodic structures developed by Thomas [1974, 1979] and its extension to substructuring coupled with wave propagation developed by Henry [1980], Henry & Ferraris [1984]. The modes used in section 4 ‘Results’ were computed by Mascarilli & coworkers [1990] using the finite element computer program SAMCEF (SAMTech [B]) in which the aforementioned analysis method has been implemented by Lombard & al [1986]. In the following are summarized the basic hypotheses of the structural model:

- computation of undamped natural modes.
- inclusion of structural deformation and stiffening induced by centrifugal forces and steady aerodynamic pressures (e.g. from a 3-D Euler steady-state solution).
- modes in traveling-wave formulation for a basic blade+disk sector, with appropriate phase-shifted boundary conditions at disk and shroud interfaces.
- shroud interface modelled by appropriate boundary conditions (Mascarilli [1990]).

The resulting eigenmodes (cf. Thomas [1979]) are complex, because of the phase-shifted boundary conditions, this being the only difference with classical undamped systems. The resulting generalized eigenvalue problem is hermitian (cf. Wilkinson [1965]). A number of properties of the mechanical modes, \([M(r,P)]=\mathbf{e}^{\text{NDOF}x\text{NDOF}}\) and \([K(r,P)]=\mathbf{e}^{\text{NDOF}x\text{NDOF}}\), dependent on interblade-phase-angle and operating point, are hermitian. This problem can be solved by decomposition to 2 real symmetric generalized eigenvalue problems (cf. Wilkinson [1965]). A number of properties of the solutions are of interest here, viz.

- the eigenvalues are real, i.e. the modes are undamped.
- the modal mass and stiffness are real, i.e. defining
  \[
  \mu(m,r,P) = (\varphi(m,r,P))^* [M(r,P)] (\varphi(m,r,P)) |_{\text{1st} \leq m \leq M} \\
  k(m,r,P) = (\varphi(m,r,P))^* [K(r,P)] (\varphi(m,r,P)) |_{\text{1st} \leq r \leq N-1} \tag{7}
  \]

where \(\mu(m,r,P)\) is the modal mass, and \(k(m,r,P)\) is the modal stiffness, \((\cdot)^*\) denotes the transpose complex conjugate of a vector and \((\varphi(m,r,P))e^{\text{NDOF}}\) the modeshape vector of the mode \(M(m,r,P)\), it follows that

\[
\mu(m,r,P) \in \mathbb{R} ; k(m,r,P) \in \mathbb{R} ; 0 \leq r \leq N-1 \tag{8}
\]

Furthermore Thomas [1979] shows that the traveling-wave modes appear in counter-rotating pairs (waves \(r\) and \(N-r\), with identical frequency \((\omega(M(r,P))=\omega(M(N-r,P)))\) and whose modeshapes are complex conjugates (i.e. for a sector with NDOF degrees of freedom \(\varphi(m,N-r,P))=\varphi(m,N-r,P))e^{\text{NDOF}}\). Of importance for the coupling procedure used is that modal mass and stiffness are real and that modeshapes are orthogonal with respect to the mass and stiffness matrices, i.e.

\[
(\varphi(m,r,P))^* [M(r,P)] (\varphi(m,r,P)) = \delta_{m,0}\mu(m,r,P) |_{\text{1st} \leq m \leq M} \\
(\varphi(m,r,P))^* [K(r,P)] (\varphi(m,r,P)) = \delta_{m,0}k(m,r,P) |_{\text{0st} \leq r \leq N-1} \tag{9}
\]

where \(\delta_{m,0}\) is Kronecker's \(\delta\).

### 3 AEROELASTIC COUPLING

Consider the computation of the coupled mode \(\Xi(\ell,r,P)\). As described in section 2-1 this mode is computed using an assumed modes method based on modes from \(M\). Since in the following the traveling-wave order \(r\), and the operating point \(P\) are fixed the explicit functional dependence on \((r,P)\) of various properties will be dropped to simplify notation. Let \(\lambda\epsilon \mathbb{C}\) be the eigenvalue associated with the aero-mechanical eigenmode \(\Xi_{\ell}\Xi(\ell,r,P)\)

\[
\lambda = -\zeta + i\omega \tag{10}
\]

where \(i\) is the imaginary unit, \(\omega\) is the vibration frequency, and \(\zeta\) is the damping factor. This definition is slightly different from the classic SDOF oscillator definition (e.g. Meirovitch [1975]), where the undamped natural frequency is used to define \(\zeta\). The modeshape \(\Phi_{\ell}\) of the aeroelastic eigenmode \(\Xi_{\ell}\) is assumed to be represented by a finite number of mechanical modes \(M\)

\[
\mathbf{r}_e \left[ \Phi_{\ell}(e^{\lambda t}) \right] - \mathbf{r}_e \left[ \sum_{m=1}^{M} \eta_m e^{\lambda t} (\varphi_m) \right] \tag{11}
\]

where \((\varphi_m)=\varphi(m,r,P))e^{\text{NDOF}}\) are the modeshapes of modes \(M(m,r,P)e_{\mathbb{C}}\), \(\eta_m e_{\mathbb{C}}\) are generalized coordinates in the modal space \(M\). Using Eq. 11 the aeroelastic equations of motion become (cf. Crawley [1988])

\[
\lambda^2 [M] \left( \sum_{m=1}^{M} \eta_m e^{\lambda t} (\varphi_m) \right) + [K] \left( \sum_{m=1}^{M} \eta_m e^{\lambda t} (\varphi_m) \right) = \\
= ( \lambda ) \left( \eta_1, \eta_2, \ldots, \eta_M; \lambda \right) e^{\lambda t} \tag{13}
\]

where \((\varphi_m)=\varphi(m,r,P))e^{\text{NDOF}}\) is the finite-element representation vector of the i-harmonics of pressure forces applied on each point of the blade surface (its elements being 0 at points of the structure that are not on the 'wetted' surface). Note that although the aerodynamic code used in the present work is nonlinear, only the i-harmonics of the aerodynamic forces are considered in the coupling procedure, which is linearized. Instead of searching an explicit relation of \((\varphi)\) with the generalized coordinates themselves and the frequency \(\lambda\), its value is directly computed by the Euler solver following the diagramme of Fig. 2. The aeroelastic computation of the mode \(\Xi_{\ell}\) starts with the simulation of 1 vibration period of the mode \(\Xi_{\ell}\Xi_{r}\) with \(\lambda_1\). At the end of this period a first estimation (note that the aerodynamic simulation of \(\Xi_{\ell}\) is not continued until periodic convergence of the unsteady aerodynamic forces) of \((\varphi_0, \ldots, 0, 1, \ldots, 0; \omega)\), with \(\eta_m = \delta_{m,0}\) \((m=1, \ldots, M)\) is obtained. These aerodynamic forces will not be orthogonal to
the other mechanical modeshapes, and will excite them. With the procedure that will be described in the following, the generalized coordinates of Eq. 11 will be reactualized, and a new vibration period will be simulated by the Euler code, with the reactualized modeshape and frequency as input, which will furnish a new frequency and modeshape, and this procedure is continued until convergence.

In order to determine the coupling relations consider the projection of the equations of motion (Eq. 12) on the modal basis by premultiplying with \( \{ \psi_m \} \): 

\[
\{ \phi_n \}^* \lambda^2 \{ M \} \sum_{m=1}^{M} \eta_m e^{\lambda t} \{ \phi_m \} + \{ \phi_n \}^* \{ K \} \{ \eta_m \} e^{\lambda t} = \{ \phi_n \}^* \{ f \} \quad (13)
\]

defining

\[
\{ f \} = \{ \chi \}^* \{ \eta_n, \eta_2, \ldots, \eta_m, \lambda \}
\]

(14)

and rearranging the terms, after simplifying by \( e^{\lambda t} \) yields

\[
\sum_{m=1}^{M} \left( \lambda^2 \{ \phi_n \}^* \{ M \} \{ \phi_m \} + \{ \phi_n \}^* \{ K \} \{ \phi_m \} \right) \eta_m = \{ \chi \}^* \{ \eta_n, \eta_2, \ldots, \eta_m, \lambda \}
\]

(15)

and using the orthogonality relations of Eq. 9 the following equations of motion are obtained (returning to the index \( n \) instead of \( m \))

\[
\lambda^2 \eta_n + \eta_n = \sum_{m=1}^{M} \left( \lambda^2 \{ \phi_n \}^* \{ M \} \{ \phi_m \} + \{ \phi_n \}^* \{ K \} \{ \phi_m \} \right) \eta_m = \{ \chi \}^* \{ \eta_n, \eta_2, \ldots, \eta_m, \lambda \}
\]

(16)

The coupling relations are described as an iterative procedure with iteration count \( n \) the period simulated by the Euler code. The iterations are necessary for the computation of the generalized aerodynamic forces \( f_m \). Since the coupling relations are linear, it was chosen to invariably set \( \eta_1=1 \), thus interpreting \( \eta_m \) for \( n=1 \) as perturbations of the corresponding mechanical mode.

This iterative procedure has not presented problems in the applications. On inspection of step 2 of the iterative procedure (Eq. 19) it is seen that because the steady stiffness \( k_t \) (structural + centrifugal + steady pressure field) is an order-of-magnitude higher than the unsteady aerodynamic stiffness \( f_m \) (for supersonic fan applications) the frequency perturbation is small. The stability of the second relation of step 2 of Eq. 19 requires that the eigenfrequencies be well separated; keeping in mind that the modal coordinates used are for a fixed interblade-phase-angle \( \beta \), common for all the modes, this requirement is usually satisfied if the frequencies closer than 60Hz are not encountered in the first 6 blade-modes of the shrouded Fan C examined in section 'Results'.

The aerodynamic damping \( \delta \) (logarithmic decrement) is computed for \( \zeta = \text{Re}(\alpha \omega) \) by the well-known approximate formula (Carta [1967])

\[
\delta = 2\pi \sqrt{1 - \zeta^2}
\]

(20)

A note concerning the computation and physical significance of \( f_m \) is in order here. The vector \( \{ f \} = \{ \eta_n, \eta_2, \ldots, \eta_m, \lambda \} \) is the vector of the \( l \)-harmonics of pressure forces applied at each point on the blade surface. Consequently if \( \eta_m(x) \) is the complex displacement amplitude of the mode \( m \) at the point \( x \) on the blade surface

\[
f_m = \{ \phi_m \}^* \{ f \} = \sum_{x \in \partial \Omega} \left( -\eta_m(x) \cdot \hat{n}(x) \right) p(x) dS
\]

(21)

where \( \hat{n} \) is the outgoing normal to the blade surface \( \partial \Omega \), and \( p \) is the \( l \)-harmonics of pressure on the blade surface, and \( \hat{n} \) means a numerical approximation of the integral. The physical significance of \( f_m \) is made clear by observing that

\[
\frac{3m}{f_m} - \frac{3m}{\int_{\Omega} \left( \int_{\Omega} \left( -\eta_m(x) \cdot \hat{n}(x) \right) p(x) dS \right) dS} = \int_{\Omega} \left( \int_{\Omega} \left( -\eta_m(x) \cdot \hat{n}(x) \right) p(x) dS \right) dS
\]

(22)

i.e. \( \pi^2 m f_m \) is the energy virtually accumulated by the blade during one period of vibration at frequency \( \omega \) in the mode \( m \) due to the composite unsteady pressure field (response to the resultant aeromechanical vibration mode \( \ddot{\theta}_t \)).
4 RESULTS

4.1 Configuration and Steady Flow

4.1.1 Configuration: Results of the coupled aeromechanical analysis are given for Fan C at a point on the 107% speed line. Fan C has 38 blades and its design rotation speed is 29,000 RPM. Fan C has a part-span shroud, which was not included in the mechanical model, where it has a great influence on vibration modes, as explained in the work of Carta [1967]. Although the shock-waves system of the rotor is influenced by the part-span shroud, with important effects on efficiency (cf. Derrien [1986]), it is expected that reasonably accurate results may be obtained for the steady aerodynamics of the vibrating cascade. The inclusion of the shroud into the aerodynamic model will be the subject of a future study (cf. Gerolymos [1992]).

4.1.2 Steady Flow: All the computations were run on an 80×15×15 grid. Although this grid is relatively coarse it is believed, in view of the parametric studies on the influence of the computational grid on results, described in Gerolymos [1992], that reasonably accurate unsteady results may be obtained.

Steady flow results are shown in Fig. 3, where the Mach number level, on the surface of the blade, is plotted. Axialwise Mach number distribution, at various spanwise stations is plotted in Fig. 4. It is seen that there is a strong shock-waves system on the suction-surface. The operating point computed was at 107% speed and a relatively low pressure-ratio. Therefore, the shock-waves on the suction-surface and near the tip are very close to the trailing-edge. The shock-wave is well downstream into the interblade channel on the pressure-surface as well. The shock-system penetrates to the hub (cf. Fig. 4 at 54% and 23% span), on the suction-surface, although this effect is probably exaggerated by the coarseness of the grid in the neighborhood of the hub.

4.2 Unsteady Results and Aeroelastic Coupling

4.2.1 Mechanical Modes: Computations of the coupled aerelastic eigenmodes were run for the ±4-order traveling-waves (the term -4-order is used in the following for the (N-4)-order traveling-wave). The frequencies of the mechanical modes used for the modal projection in the assumed modes method are given in Tab. 1 (note that the frequencies are the same for the ±4-order traveling-waves, because undamped basic mechanical modes are considered).

The corresponding modeshapes are depicted in Fig. 5 for the ±4-order traveling-wave (the corresponding -4-order traveling-wave mechanical modeshapes are qualitatively similar). The presence of the shroud complicates the modes, introducing important bending/torsion coupling. Note that modes 3 and 4 are quite similar, and their frequencies are close (a difference of 60Hz). It is expected that non-negligible coupling between these 2 modes will occur through unsteady aerodynamic forces. Modes 5 and 6 are higher modes. Especially mode 5 exhibits important amplitudes beneath the shroud. These 6 modes were used in the assumed modes method, for computing the first 4 coupled aeromechanical modes. Some anomalies at the ~70% span neighborhood mark the position of the shroud (not shown).

4.2.2 Computation of Coupled Aeromechanical Modes: Computations for the coupled aeromechanical eigenmodes will be presented for the ±4-order traveling-waves. It is clear that this is not sufficient for examining the stability...
FAN C MECHANICAL MODESHAPEs

Fig. 5 Mechanical modes \( \Phi(m,4) \) for Fan C at \( \text{rpm} \) speed on the operating line, used in the assumed modes method.

Tab. I Frequencies of 14-order traveling-wave modes used in the assumed modes method.

<table>
<thead>
<tr>
<th>Mode ( r )</th>
<th>Frequency ( \text{Hz} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>235</td>
</tr>
<tr>
<td>2</td>
<td>408</td>
</tr>
<tr>
<td>3</td>
<td>626</td>
</tr>
<tr>
<td>4</td>
<td>688</td>
</tr>
<tr>
<td>5</td>
<td>805</td>
</tr>
<tr>
<td>6</td>
<td>901</td>
</tr>
</tbody>
</table>

4.2.3 Stability and Aeroelastic Coupling:

Results concerning frequency modification, damping \( \zeta \) and logarithmic decrement \( \delta \) (computed from Eq. 20) for the coupled aeromechanical modes computed in section 4.2.2 are given in Tab. 2. It is seen that in general frequency modifications are very small. Indeed they are negligible, except for modes \( \tilde{A}(4,+4) \) and \( \tilde{A}(4,-4) \) that show a -2% frequency modification due to aerelastic coupling. Concerning damping it is seen that as expected from inspection of \( \tilde{p} \) peaks mode \( \tilde{A}(4,+4) \) is particularly stable. Modes \( \tilde{A}(4,4) \) are both unstable. All computed results for mode \( \tilde{A}(4,-4) \) show instability. Examination of Fig. 5 shows that the results is indeed caused by traveling-wave frequencies close to the rotating stall frequency, due to insufficient near-hub resolution of the computational grid.

Analysis of mechanical modes coupling is presented in Tab. 3, where the values of \( |\phi_m| \) for the various aeromechanical modes is given, representing the participation of each mechanical mode \( \Phi \) to each aeromechanical mode \( \tilde{A} \). This participation is normed to 1 for the mechanical mode corresponding to the aeromechanical one \( \Phi(4,4) \) for \( \tilde{A}(4,4) \), and as a consequence participation of other mechanical modes is given as a fraction of this mode. It is seen that generally coupling is quite weak, of the order of -5% for most cases, with the exception of mode \( \tilde{A}(4,+4) \) which has a 13% participation of mode \( \Phi(3,+4) \), a phenomenon justified by the proximity of frequency of the two modes. However the coupling effect is dependent on the aerelastic mass ratio (cf. Bendiksen [1990]) and therefore the limited number of results presented here, for a particular fan blade, and a particular operating point are insufficient to substantiate any conclusion.

4.3 Comparison of Coupled and Uncoupled Computations

4.3.1 Mean-Accumulated-Power \( p_a \): A detailed comparison of \( p_a \) distributions vs. axial distance for coupled and uncoupled computations for modes \( \Phi(4,4) \) and \( \tilde{A}(4,4) \) is presented in Fig. 8. This is interesting because \( \tilde{A}(4,4) \) is characterized by important coupling of mechanical modes 4 and 3. It is seen that in there is very little difference between coupled and uncoupled computations, with the exception of the \( p_a \) peaks at the shock-wave foots and, in the neighborhood of the trailing-edge.

4.3.2 Damping and Stability: Although the differences between coupled and uncoupled computations in Fig. 8 seem very small, they have an effect on aerodynamic damping. This is illustrated in Fig. 9a, where the convergence of logarithmic decrement \( \delta \), for coupled \( \tilde{A}(4,4) \) and uncoupled computations is presented, as a function of the number of
### Tab. 2 Frequency modification, damping and logarithmic decrement for the +4-order traveling wave coupled aeroelastic modes.8

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency</th>
<th>Frequency</th>
<th>$\zeta$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>234.8 Hz</td>
<td>235.4 Hz</td>
<td>1.70%</td>
<td>10.6%</td>
</tr>
<tr>
<td>2</td>
<td>408.3 Hz</td>
<td>406.6 Hz</td>
<td>-0.43%</td>
<td>-2.7%</td>
</tr>
<tr>
<td>3</td>
<td>626.1 Hz</td>
<td>615.8 Hz</td>
<td>0.17%</td>
<td>1.0%</td>
</tr>
<tr>
<td>4</td>
<td>688.1 Hz</td>
<td>679.4 Hz</td>
<td>0.07%</td>
<td>0.4%</td>
</tr>
</tbody>
</table>

### Tab. 3 Aeroelastic coupling of mechanical modes through unsteady aerodynamic effects for the +4-order traveling-wave coupled aeroelastic modes.9

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\zeta(1,+4)$</th>
<th>$\zeta(2,+4)$</th>
<th>$\zeta(3,+4)$</th>
<th>$\zeta(4,+4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>0.049</td>
<td>0.010</td>
<td>0.009</td>
</tr>
<tr>
<td>2</td>
<td>0.014</td>
<td>1.000</td>
<td>0.009</td>
<td>0.006</td>
</tr>
<tr>
<td>3</td>
<td>0.010</td>
<td>0.035</td>
<td>1.000</td>
<td>0.130</td>
</tr>
<tr>
<td>4</td>
<td>0.025</td>
<td>0.023</td>
<td>0.089</td>
<td>1.000</td>
</tr>
</tbody>
</table>

### -4-order traveling-wave

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency</th>
<th>Frequency</th>
<th>$\zeta$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>234.8 Hz</td>
<td>233.9 Hz</td>
<td>0.15%</td>
<td>0.9%</td>
</tr>
<tr>
<td>2</td>
<td>408.3 Hz</td>
<td>407.1 Hz</td>
<td>-0.04%</td>
<td>-0.2%</td>
</tr>
<tr>
<td>3</td>
<td>626.1 Hz</td>
<td>623.5 Hz</td>
<td>0.10%</td>
<td>0.6%</td>
</tr>
<tr>
<td>4</td>
<td>688.1 Hz</td>
<td>686.4 Hz</td>
<td>-0.07%</td>
<td>-0.4%</td>
</tr>
</tbody>
</table>

### Mean-Accumulated-Power $p$ (W/m²)

**Fig. 6** Mean-accumulated-power on Fan C blade at 107% speed, for the 4 first coupled aeroelastic modes for +4 traveling-wave.

**Fig. 7** Mean-accumulated-power on Fan C blade at 107% speed, for the 4 first coupled aeroelastic modes for -4 traveling-wave.
Fig. 8 Comparison of mean-accumulated-power on Fan C blade for coupled and uncoupled computations (modes \( M(4,4) \) and \( E(4,4) \)).

periods simulated by the Euler solver. It is seen that convergence of \( \delta \) for both coupled and uncoupled computations is quite rapid, the level of damping being obtained already after the simulation of 3+4 periods. In Fig. 9b is presented the convergence of the frequency of the coupled mode. It is clear that already after simulating 1 vibration period the frequency of the coupled aeroelastic mode is obtained. What is remarkable is that the effects of aeroelastic coupling on damping are important. There is a \(-30\%\) difference in damping between the coupled and the uncoupled computations, the effect of aeroelastic coupling being destabilizing. It is clear that small differences in \( p \) distributions, such as those of Fig. 8, can induce substantial changes in damping. This is due to the fact, that aerodynamic damping is itself a small quantity, sensible to modeshape changes. It is again remarked that the number of computations run is not sufficient to draw general conclusions about aeroelastic coupling.

5 CONCLUSIONS

In the present work a methodology of mode modification was described, which is used in conjunction with an unsteady 3-D Euler solver for determining the aeroelastic eigenmodes of tuned rotors. The advantage of the methodology proposed is that it induces practically no computational overhead when compared to the aerodynamic operator computing time requirements. Example results, illustrating the potential of the method were presented. A limited number of studies on the importance of aeroelastic coupling showed that the effects of coupling are relatively small on unsteady flow response, but can have a non-negligible effect on damping. It is reminded that the aerodynamic computations were performed with a rather coarse mesh. While this might lead to some scepticism regarding the exact values of results, it does not negate the importance and applicability of the coupling procedure.

6 DISCUSSION

It is believed that the work presented in this paper offers interesting perspectives for the study of turbomachinery aeroelasticity, since it is the first attempt to include realistic 3-D aerodynamic computations in a coupled aeroelastic stability analysis. There certainly are several points which require further research, viz., methodology validation, evaluation of 3-D effects, evaluation of the influence of the shroud on the flowfield (steady and unsteady), a systematic study of coupling effects. Further developments should include fully nonlinear computations in the time domain. Such a methodology, in 2-D is currently being developed by Bendiksen [1990].

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