UNBALANCE RESPONSE OF ROTORS CONSIDERING THE DISTRIBUTED BEARING STIFFNESS AND DAMPING

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ABSTRACT

Usually while modelling rotor-bearing systems the bearings are treated as point supports. In the present paper, using the finite element technique, the unbalance response of rotors is studied by considering distributed bearing stiffness and damping. The bearing stiffness and damping terms are derived by the principle of virtual work. Unbalance responses of rotors with bearing distributed effects are compared with the model using point supports and for different supports viz., cylindrical journal bearings, tilting pad journal bearings, offset and three lobe journal bearings.

NOMENCLATURE

B bearing length
[B] bearing element stiffness matrix
[C] damping matrix
C bearing location
C, c bearing damping coefficients
C c bearing damping coefficients per unit length
C c bearing radial clearance
D bearing diameter
F x, y bearing forces in x, y directions
[K] stiffness matrix
K k, k bearing stiffness coefficients
k k bearing stiffness coefficients per unit length
L length of the rotor
l element length
[M] mass matrix
{Q} external force vector
{q} nodal displacement vector
s, L axial displacement along the shaft element
u, v shaft displacements in x, y directions
Δu, Δv virtual displacements in x, y directions
Δw virtual work
ω angular velocity of shaft

1. INTRODUCTION

Fluid film bearings, commonly used in heavy rotating machines, play a significant role on the dynamic behaviour of the rotor. The stiffness and damping properties of the oil film significantly alter the critical speeds and unbalance response of the rotor. The classical way of representing the effects of a bearing when making a mathematical model of a rotating shaft, is to treat the shaft as though it were pinned at the bearing location. If flexible bearings are used the rigid constraint is relaxed and the bearing effect is represented by two springs acting in orthogonal directions.

Assuming distributed mass and elasticity of the rotor, Rieger (1971) analysed the unbalance response of flexible rotor supported on hydrodynamic bearings. Joshi and Dange (1976) extended the distributed mass model of Eshleman and Eubanks (1967). Huang and Huang (1967) included other parameters such as coupling flexibilities, external loads and long and short bearing supports. Modal analysis of rotor-bearing systems (Craggs, 1987; Genta and Bona, 1990) has also been used for the study of dynamic response. However, in all these papers, the bearings are treated as point supports having stiffness and damping. When fairly long bearings are used, the distributed effects of bearings are to be considered. Craggs (1993) observed that the significant moments at each bearing locations resulted in a large deviation from the computer simulation, when using an experimental work where fairly long bearings are used. He proposed a model for distributed bearing stiffness effects. The present paper extends the work of Craggs (1993) to find the unbalance responses of
rotors for different bearing supports Viz., cylindrical journal bearings, tilting pad journal bearings, offset and three lobe journal bearings.

2. SYSTEM EQUATION OF MOTION

The equation of motion of the rotor bearing system is of the form

\[ [\mathbf{M}] \{\ddot{\mathbf{q}}\} + [\mathbf{C}] \{\dot{\mathbf{q}}\} + [\mathbf{K}] \{\mathbf{q}\} = \{\mathbf{Q}\} \]  

(1)

where the masses matrix includes the rotary and translational mass matrices of the shaft and rigid disc mass and diametral moments of inertia. The damping matrix includes the gyroscopic moments and bearing damping. The stiffness matrix considers the shaft and bearing stiffnesses. The details of the individual matrices of eq. (1) are given in the (Nelson and McVaugh, 1976).

For computational purposes, eq. (1) is written in the first order state vector form and can be solved for the eigenvalues. The unbalance response of the system is solved by assuming a harmonic solution (Nelson and McVaugh, 1976).

3. FORMULATION WITH DISTRIBUTED BEARING FORCES

3.1 Stiffness Matrix:

The bearing stiffness is assumed to be uniformly distributed over the length. Fig. la. shows the shaft finite element under the influence of a distributed bearing force. For a length of shaft ds, the forces \( F_x \) and \( F_y \) acting at the bearing sections are given as

\[
\begin{bmatrix}
F_x \\
F_y
\end{bmatrix} = \begin{bmatrix}
k_{xx} & k_{xy} \\
k_{yx} & k_{yy}
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix}
\text{ds}
\]

(2)

where \( k_{xx} \), \( k_{xy} \), etc., are stiffnesses per unit length and \( u \) and \( v \) are the shaft displacements in the \( x \) and \( y \) directions.

The coordinates \( q_1, q_2, ..., q_8 \) are the time dependent end point displacements (translations and rotations) of the finite shaft element and are indicated in Fig. la. The finite shaft element equation of motion is developed by specifying spatial shape functions and then treating the shaft element as an integration of an infinite set of differential discs (Nelson and McVaugh, 1976).

The translation of a typical point internal to the element is chosen to follow the relation,

\[
\begin{bmatrix}
u \\
v
\end{bmatrix} = \begin{bmatrix}
\psi_1(s) \\
\psi_2(s) \\
\psi_3(s) \\
\psi_4(s)
\end{bmatrix}
\begin{bmatrix}
u_0 \\
v_0
\end{bmatrix}
\]  

(3)

The spatial constraint matrix is given by

\[
[k_0] = \begin{bmatrix}
\psi_1 & \psi_2 & \psi_3 & \psi_4 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \psi_1 & \psi_2 & \psi_3 & \psi_4
\end{bmatrix}
\]

(4)

From the eq. (3) the displacements are

\[
\begin{bmatrix}
u \\
v
\end{bmatrix} = (\psi)^T \begin{bmatrix}
u_0 \\
v_0
\end{bmatrix} (\psi)
\]

(6)

where \( \psi_0, \psi_4 \) are the nodal displacements as in \( \{q^0\} \) and the vector \( (\psi) \) contains the Hermitian polynomials of eq. (5).

As explained in Craggs (1993), the bearing stiffness matrix may be derived by allowing small virtual displacements \( \delta u, \delta v \) in the equilibrium position and integrating the virtual work over the length \( \ell \) of the element.

The total virtual work \( \delta W \) is given as

\[ \delta W = \ell \int_0^\ell \begin{bmatrix}
\delta u \\
\delta v
\end{bmatrix}^T \begin{bmatrix}
F_x \\
F_y
\end{bmatrix} \text{ds} \]  

(7)

Substituting for \( u, v \) from eq. (6) and using eq. (2)

\[ \delta W = [\delta u, \delta v] \begin{bmatrix}
\psi_0 \\
\psi_4
\end{bmatrix} \begin{bmatrix}
u_0 \\
v_0
\end{bmatrix}^T \begin{bmatrix}
\psi_0 & \psi_4
\end{bmatrix} \begin{bmatrix}
u_0 \\
v_0
\end{bmatrix} \begin{bmatrix}
\psi_0 \\
\psi_4
\end{bmatrix} \text{ds} \]  

(8)

where

\[ [B] = \ell \int_0^\ell \begin{bmatrix}
\{\psi\} & 0 \\
0 & \{\psi\}
\end{bmatrix} \begin{bmatrix}
k_{xx} & k_{xy} \\
k_{yx} & k_{yy}
\end{bmatrix} \begin{bmatrix}
\{\psi\}^T \\
\{\psi\}
\end{bmatrix} \text{ds} \]  

(9)
On simplifying this the distributed bearing stiffness matrix can be written as,

\[
[B] = \begin{bmatrix}
  k_{xx} & k_{xy} \\
  k_{yx} & k_{yy}
\end{bmatrix}
\]

where

\[
[A] = \frac{1}{420} \begin{bmatrix}
  156 & 4t^2 \\
  22t & 13t^2 & 156 \\
  -13t & -3t^2 & -22t & 4t^2
\end{bmatrix}
\]

3.2 Damping :

Assuming synchronous vibration response, the terms \(k_{xx}, k_{xy}\), etc. can be considered as complex to include the damping as \((k_{xx} + i \omega c_{xx})\), etc. The damping matrix for the distributed bearing effects can be derived in a similar way as the stiffness matrix which is explained in section 3.1. In the present analysis the linearised bearing coefficients are assumed. However, for accurate results including the non-linear effects, the solution has to be obtained for fluid film forces by integrating over the pressure profile.

4. RESULTS & DISCUSSIONS

Numerical results have been obtained from a simple rotor model, shown in Fig.1b. The rotor with a central disc and with the following data has been considered for the analysis to compare the results of distributed bearing model with those of point support model.

Length of the rotor : 1.8 m
Diameter of the shaft (d) : 0.06 m
Disc : Mass = 22 kg, Polar and diametral inertia = 0.1687, 0.0842 km².
Unbalance eccentricity = 0.001 mm.

The rotor is supported by two flexible bearings 0.3 m from each end and with \(K_{xx} = K_{yy} = 10^7\) N/m, \(C_{xx} = C_{yy} = 10^3\) Ns/m and \(B/D = 3\).

From the Fig. 2 it can be noticed that the deviation of critical speed is approximately 4%. But for the same bearings the unbalance peak response at the disc (Fig. 3) location with bearing distributed effects is much lower (36%) compared to that of with point model. This is due to the distributed effects of damping. Hence, response calculations are essential for the realistic study of the dynamic behaviour of the rotors.
The deviations in critical speeds are very large when using point and distributed bearing models. As observed by Craggs (1993), these deviations occur at very high bearing stiffness for small values of B/D (say B/D < 1) where as at lower bearing stiffness for high B/D values. Consider the first mode in Fig. 2, as the bearing stiffness increases, there is very little difference between the point and distributed bearing results until a stiffness of 10^7 N/m is obtained. With further increase in stiffness, the critical speeds for the point support remain constant, corresponding to those that would be calculated under pinned-pinned or simply supported conditions. However, the moment effect induced with the distributed bearing is of significance and the model gives much higher critical speeds, these are asymptotic to clamped values for a beam of span (L-2C-B), so the longer bearings have higher speeds. At higher values of stiffness, then, the critical speed calculated with distributed model can almost double that of the point supported model. For a bearing stiffness of 10^7 N/m, and B/D = 3, there is obvious clamping over the length of the bearing in the first mode and the critical speed is doubled. However, with the second and third modes, the shaft flexing is much stiffer and the clamping effect will not be as great as for the first mode, but there is still a larger change in the critical speeds due to the moment effect. Hence, for the rotor dynamic analysis, the critical speed maps with the two models are to be plotted, essentially to know above which bearing stiffness the distributed effects are significant.

A typical industrial rotor (Electrical motor rotor - Fig. 4) supported on two cylindrical journal bearings, B/D = 0.8, c = 74 µm, is also considered. Fig. 5 shows the variation of unbalance response with speed, at the centre of rotor for the two models. Since the bearings are not long enough, the difference between the two models is not appreciable. Bearings with B/D > 1 can be treated as long bearings and the distributed bearing effects are to be included.

A comparative study for the distributed and point bearing models in the case of different type of fluid film bearing supports commonly used Viz., cylindrical journal bearings, tilting pad journal bearings, offset and three lobe journal bearings is also made. The data for the rotor supported by different fluid film bearings, except for the bearings, is same as that mentioned earlier. The stiffness and damping coefficients used in the analysis are obtained from Lund and Thomsen [9] and Glencoe et al [10]. B/D = 1., bearing radial clearance of 100 µm, and unbalance eccentricity of 0.005 mm are considered.

Figs. 6(a-d) show the variation of unbalance response with speed for the disc. It can be noticed in all the cases the peak values reduced drastically with the inclusion of the distributed effects since the bearings are quite long. In the case of offset and cylindrical bearings the reductions are much higher compared to the other bearings. But in the case of tilting pad bearings there is a shift of peak response also towards the higher speed with the distributed effects. This may be due to the absence of bearing cross coupled stiffness and damping terms.

For practical length of bearings (say B/D < 1), it may be noticed that the distributed stiffness does not have much effect on the critical speeds, but the distributed damping has greater influence on the amplitude of the response. The individual effect of
stiffness and damping has been studied. The amplitude reduction with stiffness alone distributed is much lesser compared to that of when damping alone is distributed. As explained in the paper by Craggs (1993), for short bearings (B/D < 1), the moment effects are less and the difference between critical speeds obtained using the distributed and point support model is high only when the bearing stiffnesses are very high. In the Figs. 5, 6 bearings stiffnesses are not high and hence the distributed effect on the critical speed is not there. The distributed effect of damping has however, reduced the amplitude of response. In the Fig. 3, since the B/D is very high (B/D = 3), even with the stiffness of 10^7 N/m, there is considerable increase in the critical speed and with damping also being distributed, there is reduction in the amplitude.

The finite element model of the rotor-bearing system is presented with the distributed bearing effects proposed by Craggs.

The unbalance response studies are carried out on different types of rotors and with different types of bearing supports. It may be concluded that the distributed bearing effects are to be considered for bearings with B/D > 1. For bearings with B/D < 1, the distributed effects are to be considered when the bearing stiffnesses are high.

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REFERENCES


