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SENSITIVITY CONTROLLED RESPONSE SURFACE APPROACH FOR RELIABILITY BASED DESIGN

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ABSTRACT

The stochastic Finite-Element-Method (FEM) is a suitable tool to assess the reliability of lifetime prediction models for complex components. Due to its demands on the number of FE analysis it has been rarely used on FE models of real components under realistic operational conditions. In the following a response surface approach is suggested that minimizes the number of FE calculations. Based on a first order method the sensitivity of the failure probability with respect to the random input quantities is evaluated. Subsequently a response surface of higher order, weighting the important and unimportant input uncertainties appropriately, is used to assess the reliability of the lifetime prediction. The method is used to analyze the reliability of typical rotor disks concerning failure due to creep rupture and low cycle fatigue (LCF) during static and transient operation.

INTRODUCTION

For the design of gas turbine components the Finite-Element-Method is a common tool to assess the lifetime of such complex and highly loaded components. However, the calculated lifetime as result of the Finite-Element analysis is strongly influenced by the scatter of the input quantities. The input quantities are uncertain either due to inherent variability (material, geometry) or due to the inaccuracy of engineering knowledge and prediction (boundary conditions). The emerging of uncertainties of input quantities leads to two different problems or questions:

- If the input quantities scatter how uncertain is the predicted lifetime as consequence (uncertainty analysis)?
- Since the lifetime is uncertain as well what is the probability that some design criterion or lifetime requirement is violated (failure probability analysis)?

The need to answer these questions becomes obvious if it is taken into account that the typical lifetime models are highly nonlinear and very sensitive with respect to some input quantities. This is confirmed by the experience with lifetime data of real components, where also a huge scatter can be observed.

The probabilistic methods to answer these questions are usually quite demanding concerning the number of Finite-Element calculations. This is a key issue for the design of sophisticated components under complex loading conditions since one FE analysis may take some hours. This gives rise to the need of efficient tools that keep the number of FE calculations down to a minimum.

INTRODUCTION TO PROBABILISTIC METHODS

Monte Carlo Method

The most commonly known probabilistic method is the Monte Carlo Method. Its ease of use is opposed by the fact that the required number of FE calculations is very high. According to Bjerager (1990) the number of FE runs for a failure analysis can be estimated as $100/P_f$, where P_f is the expected failure probability. Since P_f is usually very small for technical components (10^{-4} - 10^{-8}) the Monte Carlo Method can only be used for an uncertainty analysis, where about 30 to 100 FE calculations are enough to roughly estimate the mean and the standard deviation of the predicted lifetime.

Reliability Methods

The failure of a component is usually described by the performance function $g(\underline{u})$ that follows the convention:

$$\text{Component as failed:} \quad g(\underline{u}) \leq 0 \tag{1}$$

$$\text{Component is still operable:} \quad g(\underline{u}) > 0 \tag{2}$$

The performance function depends on the vector of the uncertain input quantities denoted with $\underline{u}$. The hyper surface $g=0$ separating the failure domain from the safe domain is usually called the limit state

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surface. In terms of lifetime prediction the performance function can be expressed as

$$g(\underline{u}) = t_{predicted}(\underline{u}) - t_{required} \tag{3}$$

indicating that failure of the component can be expected if the predicted lifetime is less that the required lifetime. The failure probability is generally expressed as the multidimensional integral

$$P_f = \int\int_{g(\underline{u})\leq 0}...\int f_{\underline{u}}(\underline{u}) \cdot du_1...du_n \quad , \tag{4}$$

where $f_{\underline{u}}(\underline{u})$ is the joint probability density function of the uncertain quantities u_1 to u_n and n is the number of uncertain influence variables. Here, the integration domain is the failure domain that is described by Eq. (1). If the uncertain input quantities are standard normal distributed and statistically independent the First Order Reliability Method (FORM) provides an estimate of the failure probability according to the equation

$$P_f = \Phi(-\beta) \ , \tag{5}$$

where β is the length of the vector of the most probable failure point (MPFP). This is the point on limit state surface $g=0$ that is closest to the origin of the $\underline{u}$-space as illustrated in Figure 1. The First Order Reliability Method is based on a linear approximation of the limit state surface at the most probable failure point (Madsen et al., 1986). This is indicated in Figure 1 with the dashed line denoted with $\tilde{g}=0$.

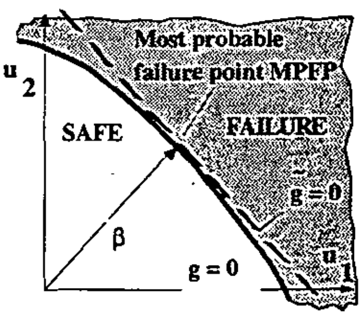

Figure 1: Illustration of the most probable failure point location

It is important to mention that the most probable failure point has to be found iteratively. Several search algorithms are available to find the MPFP (Wu et al., 1987, Abdo and Rackwitz, 1990), but it is beyond the scope of this paper to discuss this in detail. It is important to mention that all first order methods need the gradient of the performance function with respect to the input quantities.

A great advantage of the First Order Reliability Method is its capability to deliver also probability sensitivity factors once the most probable failure point has been found. The probabilistic sensitivities are given by

$$\alpha_i = \frac{\partial\beta}{\partial u_i} = \left.\frac{u_i}{\|\underline{u}\|}\right|_{MPFP} = \left.\frac{u_i}{\beta}\right|_{MPFP} \quad ,i = 1,...,n \ . \tag{6}$$

The explanations given above are valid if all uncertainties follow a Gaussian distribution and are statistically independent. Without going into detail there are several transformations available to transform a set of arbitrarily distributed and correlated uncertainties into a set of normal distributed and uncorrelated random variables, e.g. the Nataf model or the Rosenblatt transformation (Ditlevsen, 1981, Liu and Der Kiureghian, 1986). Due to its advantageous properties the Nataf model was used for the work presented here.

Response Surface Methods

Due to its wide range of applicability the response surface method is a very useful tool for reliability analyses. In contrast to the linear First Order Method a failure relevant response of the component, e.g. the predicted lifetime, is approximated using a second order polynomial of the uncertain input quantities. Such an approximation is given the following equation:

$$\tilde{t}_{predicted} = c_0 + c_1 \cdot u_1 + c_2 \cdot u_2 + c_{11} \cdot u_1^2 + c_{12} \cdot u_1 \cdot u_2 + c_{22} \cdot u_2^2 + ... \tag{7}$$

The coefficients c_0, c_i and c_{ij}, with $i=1,...,n$ and $j=1,..,i$, have to be determined using a least squares fit based on a limited number of FE calculations. Once these coefficients are determined the approximated response from Eq. (7) can be used in a Monte Carlo simulation. In contrast to a Finite-Element calculation the evaluation of a polynomial expression takes only a negligible time. The different response surface approaches known from literature mostly differ in the type of the so called experimental design plan, i.e. in the strategy to locate the points of the FE calculations in the $\underline{u}$-space of the uncertainties. Typical examples of experimental designs are the factorial design or the central composite design (Faravelli, 1989), to name only two.

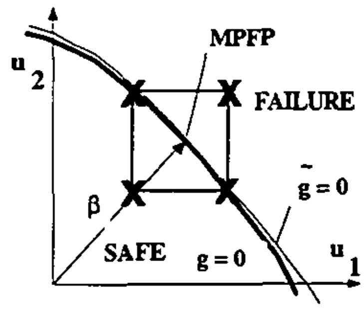

Figure 2: Location of FE calculations for response surface method

Decisive for the accuracy of the probabilistic analysis is the center location of the design plan. Ideally, the design plan should be built up in the region of the $\underline{u}$-space that contributes most to failure, i.e. the region around the most probable failure point. This is illustrated in Figure 2 for a factorial design where the location of the FE calculations are marked with crosses. Therefore, a combination of the First Order Reliability Method and the Response Surface Method seams to be a logical consequence for two different reasons:

- Using FORM the center location of the experimental design can be identified.
- Since the Response Surface Approach is based on a higher order approximation it is a very accurate tool.

The drawback of the method is that the number of FE analysis still can be quite large. In case of the factorial design the experimental design plan consists of 2^n FE calculations, where n is the number of uncertainties. For the central composite design plan this number is even higher. Therefore, the number of uncertainties is usually restricted to about 8 or 9. However, this is very often an inadmissible constraint to the probabilistic model.

SENSITIVITY CONTROLLED RESPONSE SURFACE

To reduce the number of FE calculations necessary for the experimental design plan a sensitivity controlled response surface approach is suggested. As mentioned above the most probable failure point has to be found first to locate the center of the experimental design plan.

Therefore, the sensitivities α_i from Eq. (6) can be used to identify those uncertainties that really have an important influence on the failure probability. Since sensitivities with a similar value but opposite sign also have a similar impact on failure the sensitivities from Eq. (6) have to be transformed to non-negative and normalized measures.

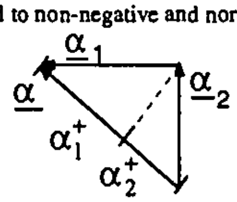

Figure 3: Sensitivities and non-negative importance measures

From Eq. (6) it is obvious that the vector $\underline{\alpha}$ of all sensitivities is coaxial to the vector of the most probable failure point and has a unit length. Therefore, as non-negative importance measure the quantities

$$\alpha_i^+ = \frac{\underline{\alpha}_i \cdot \underline{\alpha}}{\|\underline{\alpha}\|} = \alpha_i^2 \quad , i = 1,\ldots,n \qquad (8)$$

are suggested, with $\underline{\alpha}_i=(0,\ldots,\alpha_i,\ldots,0)^T$. The geometrical interpretation of the importance measures α_i^+ is illustrated in Figure 3. Using these importance measures the reliability analysis is performed according to the following procedure:

1. Find the most probable failure point using the First Order Reliability Method. Here all uncertainties are taken into account.
2. For the constant coefficient c_0 and the linear coefficients c_i of all uncertainties the value of the performance function g and the gradient of the performance function both at the most probable failure point can be used.
3. Determine the importance measures from Eq. (8).
4. Select the uncertainties according to their importance measure in descending order until the importance measures have accumulated the value of 0.9, i.e.

$$\sum_{i=1}^{n_{imp}} \alpha_i^+ \geq 0.9 \qquad (9)$$

where n_{imp} is the number of selected most important uncertainties. The value of 0.9 is only a suggestion that seems to be reasonable.

5. Since only the quadratic terms in Eq. (7) remain to be determined it is sufficient to built up a fractional factorial experimental design plan only for the uncertainties identified as the most important ones. The number of experiments, i.e. FE calculations, and the number of coefficients to be determined are given in Table 1. Here, the number of calculations is given by 2^{n-p}, where p is the replication fraction of the factorial design. In Table 1 the replication fraction p was increased systematically to avoid the number of FE calculations getting excessively high. For further details about experimental design plans reference is given to the works of Myers (1979) and Faravelli (1989).

Using this procedure the uncertainties having a lower impact on the failure probability are not ignored. They are taken into account using a linear approximation that is available from the First Order Reliability Analysis in step 1.) at no extra cost.

In this sense the uncertainty analysis to analyze the scatter of the lifetime as mentioned in the introduction is inherently included in this procedure. For an uncertainty analysis the experimental design plan should be centered around the expectation point, which is usually the starting point of the search for the most probable failure point. Therefore, an uncertainty analysis is automatically achieved with the procedure described above if the iterative search for the most probable failure point is stopped after the very first iteration. Therefore, in the following only the problem of calculating the failure probability is discussed.

No. of important uncertainties n_{imp}	No. of quadratic coefficients	Fraction of design plan p	No. of experiments (FE runs)
1	1	0 (full factorial)	2
2	3	"	4
3	6	"	8
4	10	"	16
5	15	1 (1/2-replicate)	16
6	21	"	32
7	28	2 (1/4-replicate)	32
8	36	"	64
9	45	3 (1/8-replicate)	64
10	55	4 (1/16-replicate)	64

Table 1: Experimental design for a quadratic response surface

EXAMPLES: RELIABILITY ANALYSIS OF A TURBINE DISK

General Remarks

Apart from dynamic aspects creep at high temperatures and low cycle fatigue (LCF) due to thermally induced transient stresses are the most damaging mechanisms that limit the lifetime of gas turbine components. To demonstrate the probabilistic assessment of these effects and to outline the benefit of a sensitivity oriented approach as suggested here a simple Finite Element model of a turbine disk was used. The Finite Element mesh and the major influence quantities are sketched in Figure 4.

The FE calculations have been performed using the commercial Finite-Element program ANSYS. For the probabilistic analyses illustrated in the following all probabilistic methods above have been seamless integrated into the FE code. For better user-friendliness the preprocessing of the probabilistic model and the postprocessing of the probabilistic results can be done interactively using a menu technique. In addition, all required FE calculations are performed automatically. For the response surface part of the analyses the factorial response surface tool available in the FE code has been used.

The geometry parameters regarded as random input parameters are the outer radius R_{out}, the inner radii R_1 and R_2 and the width of the foot section W. The material of the disk is a 12% chromium steel with the uncertain material properties Young's modulus E, density ρ, creep rupture strength R_{mtnT} and the cyclic strain amplitude to crack initiation ε_{aIMT}. Most of these parameters are functions of the metal temperature.

The lifetime limiting parameters R_{mltT} and ε_{alNT} additionally depend on the service duration (operation time t or number of cycles N).

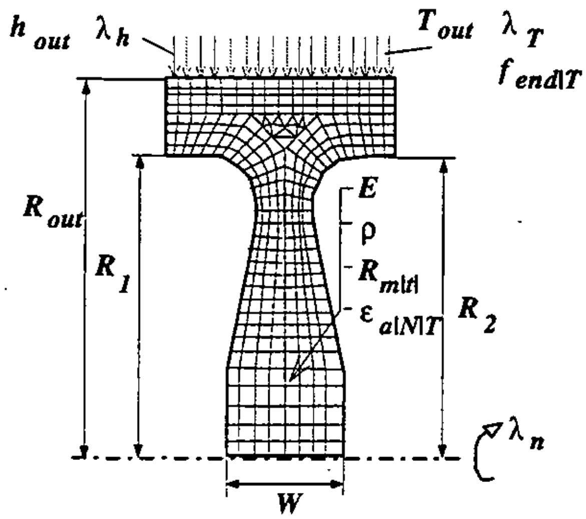

Figure 4: Sketch of FE model of turbine disk

The thermal boundary conditions in steady state are described by the heat transfer coefficient h_{out} and the bulk temperature T_{out}. The thermal transients behavior is described by the parameters λ_h, λ_T and T_{end} that are explained in more detail below. During transient operation the rotational speed is strongly influenced by dissipating mechanisms, e.g. aerodynamic and mechanical friction, that are very difficult to predict. Therefore, the parameter λ_n describing the transient rotational speed is regarded as uncertain input. This parameter is also explained in more detail below.

For the input quantities that are uncertain due to the inaccuracy of the engineering assessment or a lack of knowledge a triangular distribution (TRIA) was used. The geometry is controlled by the manufacturing tolerances. However, it is not known in advance which values geometrical extensions have within these tolerances. Therefore, a uniform distribution (UNIF) was used for geometrical parameters. For parameters where a statistical database was available the appropriate normal distribution (N) or logarithmic-normal distribution (LN) was applied.

The lifetime, i.e. time until creep rupture occurs or the number of cycles until crack initiation is obtained, was determined with ABB own programs. Due the open structure of FE code it was possible to execute these programs from within the FE code. It is beyond the scope of this paper to describe these ABB specific lifetime evaluation codes in more detail. They are based on textbook knowledge and on proprietary know-how.

Creep Rupture Reliability Analysis

Probabilistic model. For a creep rupture analysis only the static loadings during steady state have to be taken into account. The uncertainties influencing the steady state temperatures and stresses are listed in Table 2. All input quantities are regarded as statistically independent for this example. According to Eq. (10) the random behavior of the creep rupture strength R_{mltT} is described by applying a random and temperature independent coefficient C_R to the mean values of R_{mltT}.

$$R_{mltT}\Big|_{random} = C_R \cdot R_{mltT}\Big|_{mean} \qquad (10)$$

Similarly, random coefficients have been introduced to account for the uncertain behavior of all other random input quantities accordingly.

Uncertainty coefficients for	- Distribution
Bulk temperature T_{out}	TRIA(0.963, 1.0, 1.085)
Outer radius R_{out}	UNIF(0.9991, 1.0009)
Inner radius R_1	UNIF(0.9985, 1.0015)
Inner radius R_2	UNIF(0.9985, 1.0015)
Foot width W	UNIF(0.9978, 1.0022)
Density ρ	N(1.0, 0.0065)
Creep rupture R_{mltT}	N(1.0, 0.0775)

Table 2: Random input quantities for the creep rupture analysis

Results. The probabilistic importance measures evaluated at the most probable failure point are illustrated in Figure 5. Obviously only the random coefficient of the bulk temperature T_{out} and the creep rupture strength R_{mltT} are important input quantities according to the definition in Eq. (9). Consequently, a quadratic response surface was built up around the most probable failure point with all uncertainties entering as linear terms and only T_{out} and R_{mltT} as quadratic terms.

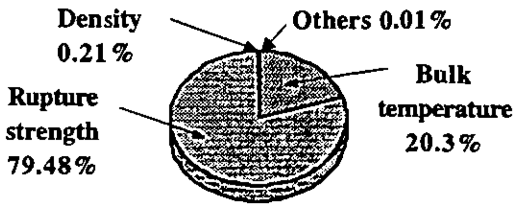

Figure 5: Probabilistic importance measures for rupture lifetime

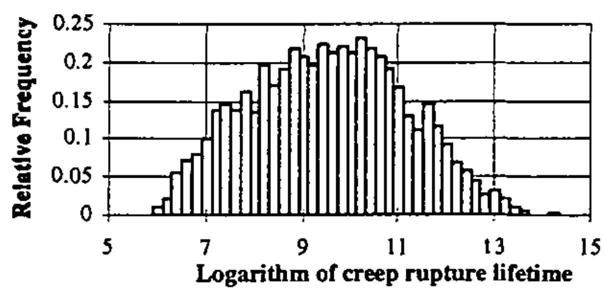

Figure 6: Histogram of creep rupture lifetime

As indicated in Table 1 only 4 additional Finite-Element calculations are necessary to determine the 3 coefficients of the quadratic terms. Therefore, 28 FE analyses have been saved as compared to a response surface including all 7 uncertainties described by linear and quadratic terms. The probability histogram as obtained from 6000 Monte Carlo Simulations on the response surface is shown in Figure 6 and the corresponding cumulative distribution function is given in Figure 7. For the creep rupture lifetime a logarithmic scale was used.

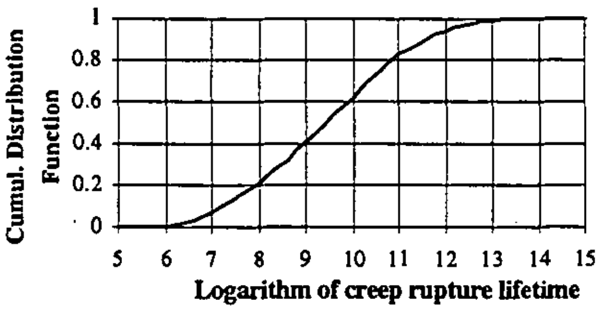

Figure 7: Distribution Function of Creep Rupture Lifetime

Low Cycle Fatigue Reliability Analysis

Probabilistic model. Low cycle fatigue is dominated by the cyclic loading of the disk with temperatures and the resulting thermal stresses. Therefore, a transient analysis has to be performed to assess the damaging effect of starting up and shutting off the machine. The uncertainties taken into account in this analysis are listed in Table 3. Similar to the previous example random coefficients are used to describe the random behavior of the relevant input quantities.

Uncertainty coefficients for	Distribution
Bulk temperature T_{out}	TRIA(0.963, 1.0, 1.085)
Transient param. λ_T	LN(0.995, 0.103)
Transient param. f_{endT}	TRIA(0.765, 1.0, 1.275)
Heat transf. coeff. h_{out}	TRIA(0.793, 1.0, 1.207)
Transient param. λ_h	LN(0.992, 0.127)
Outer radius R_{out}	UNIF(0.9991, 1.0009)
Inner radius R_1	UNIF(0.9985, 1.0015)
Inner radius R_2	UNIF(0.9985, 1.0015)
Foot width W	UNIF(0.9978, 1.0022)
Transient param. λ_n	LN(0.997, 0.076)
Young's modulus E	N(1.0, 0.0435)
Density ρ	N(1.0, 0.0065)
LCF strain ε_{aIMT}	N(1.0, 0.0656)

Table 3: Random input quantities model for LCF analysis

For the transient analysis usually a scaling factor is used to describe the boundary conditions as a function of time, e.g. the time dependency of the heat transfer coefficient follows the equation:

$$h(t) = f_h(t) \cdot h_{ss} . \tag{11}$$

Similar equations exist to describe also the other boundary conditions as function of time. An illustration of the transient scaling factor f is given in Figure 8. The peak values of the thermal stresses are introduced during the shut-down when very sharp gradients with respect to time occur. Therefore, the start-up phase is described by a simple ramp function. The important shut-down phase was parameterized using an exponential function that is capable to cover the steep time gradients at the beginning of the shut-down correctly. Hence, during shut-down the transient scaling factor f is described by

$$f(t) = f_{end} + (f_{ss} - f_{end}) \cdot \exp(-\lambda \cdot t) \tag{12}$$

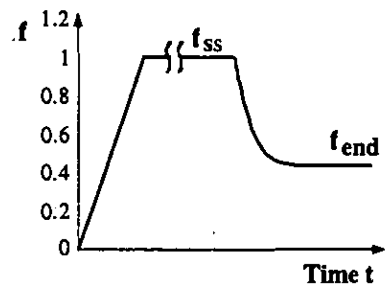

Figure 8: Sketch of transient scaling factors

The transient scaling factors for heat transfer coefficients, bulk temperatures and the rotational speed are evaluated with thermodynamic models and programs. Without going into detail, it should be mentioned that these thermodynamic models and programs have been used to obtain the distributions for the shut-down rate parameter of the heat transfer coefficient λ_h, the bulk temperature λ_T and the rotational speed λ_n by means of Monte Carlo Simulations. Due to the enormous heat capacity of the entire system the temperature at the end of the shut-down is not equal to the ambient temperature. Since this is very difficult to evaluate the value of the scaling factor at the end of the shut-down f_{endT} was included in the probabilistic model.

The shut-down parameters λ_h, λ_T and λ_n are not independent from each other. The heat transfer coefficient is a temperature dependent quantity and it is also influenced by the hotgas mass flow. The hotgas mass flow itself is determined by the rotational speed of the rotor. In contrast to that there is no interaction between the hotgas temperature and the rotational speed. Using the thermodynamic models mentioned above the correlations between λ_h, λ_T and λ_n have been evaluated and the results are given in form of the correlation coefficient matrix in Table 4.

	λ_h	λ_T	λ_n
λ_h	1.000	0.797	0.586
λ_T	0.797	1.000	-0.018
λ_n	0.586	-0.018	1.000

Table 4: Correlation matrix for transient parameters

The negligibly low correlation coefficient between λ_T and λ_n confirms the expectation of no interaction between these parameters.

Results. As illustrated in Figure 9, at the most probable failure point the random coefficient of the strain amplitudes to crack initiation ε_{aIMT} clearly dominates the failure probability. According to Table 1 only two additional calculations have been necessary to obtain a response surface with all uncertainties taken into account by linear terms and for ε_{aIMT} as the only quadratic term. If all 13 uncertainties would have been taken into account in the quadratic approach 128 FE analyses would have been necessary. Also for the LCF lifetime 6000 Monte Carlo Simulations have been performed on the response surface. The probabilistic results for LCF lifetime in logarithmic scale are given in form of a histogram plot in Figure 10 and in Figure 11 the corresponding cumulative distribution function is shown.

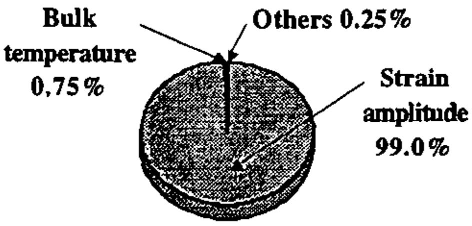

Figure 9: Probabilistic importance measures for LCF lifetime

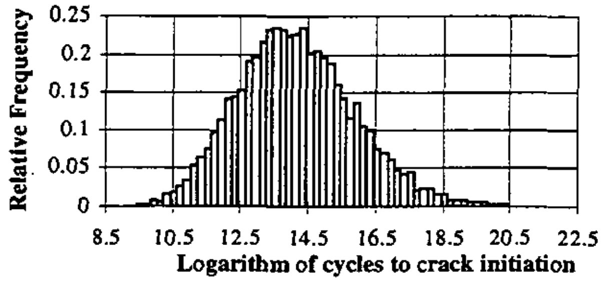

Figure 10: Histogram of LCF lifetime

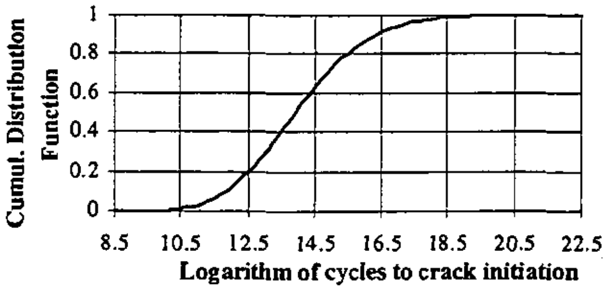

Figure 11: Distribution Function of LCF lifetime

Comments on the Results

For both examples given above the results turned nut to be quite trivial since only one or two uncertainties have the greatest impact on failure for the analyzed component. On one hand this coincides with the experience gained also from other application examples that by far not all uncertainties really matter in the end. This experience is in fact the justification to prefer a sensitivity controlled Response Surface Method. On the other hand the results given above must not lead to the conclusion that the uncertainties having a low impact on failure for the examples above can be generally kept out of consideration. First, the analyzer cannot a priori quantify which input quantities will be the important ones in the end. Secondly, the same type of uncertainties may have a completely different impact in case of a different problem or component. For instance, for cast components like turbine blades the uncertainty associated with the geometry is much larger than for machined parts like in the examples above. It is well known, that especially the location of the cooling channels has a significant influence on the mechanical integrity of the blades.

SUMMARY

In the present contribution the Stochastic Finite-Element-Method was used for the reliability analysis of a turbine disk. The different types of Stochastic Finite-Element-Methods have been explained briefly. Subsequently, the Response Surface approach in general and especially the sensitivity controlled Response Surface Method suggested here have been discussed in detail. The proposed method is aimed to reduce the required number of Finite-Element calculations. It has been applied to the probabilistic assessment of a gas turbine disk. Two different lifetime limiting effects have been addressed, namely creep rupture and low cycle fatigue. The benefit of the sensitivity controlled Response Surface Method to reduce the numerical efforts for a probabilistic analysis has been outlined at hand of these examples.

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