The Torsional Stability of a Compressor Cascade

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ABSTRACT

This paper includes new experimental data on the torsional stability of a compressor or fan type of blade row. This data is linked in with published material to form a three dimensional figure describing the stability of such rows. The data is then discussed by considering relative values of influence coefficients and some of the trends in the stability figure are explained. New data is also included on the relative magnitudes and phases of the influence coefficients.

Nomenclature

c Blade chord
Cm Non-dimensional moment coefficient
Im(Cm) Imaginary component of moment coefficient
Re(Cm) Real component of moment coefficient
Cm(n) Moment coefficient acting on blade n when only blade 0 vibrates
M Mach number
N Number of blades in row
S Blade spacing
w Cascade approach velocity
\beta Inter-blade phase angle in radians
\phi Phase angle between angular displacement of a blade and its self induced moment
\omega Reduced frequency (\omega/v)

INTRODUCTION

The torsional stability of axial flow compressors and fans continues to be of interest to gas turbine manufacturers. Throughout the 1970's increasingly sophisticated computational methods were developed to predict the unsteady forces acting on cascaded blades, Whitehead (1982) and Verdon and McCune (1975) are prominent examples. The assumptions always being that the flow is two dimensional and inviscid; a strip theory could then be used to integrate the loading along the blade span.

Experimental validation of these theories has not been so forthcoming, largely due to the difficulty of setting up blade to blade repeatability for the steady and unsteady flow. It is fair to say that to date none of the codes have been fully validated in the open literature.

The usual approach to presenting both predictive and experimental results is to show how the magnitude of the unsteady moment varies with Mach number and inter-blade phase angle. The moment is usually split into its real and imaginary components to show the phase relationship with the displacement of the blade. The number of parameters may, however, be reduced by simply noting whether a cascade state is stable or not. That is to ignore the magnitude of the moment. An aerodynamically unstable cascade is one with negative aerodynamic damping. A picture may then be drawn to show trends in stability which may not have been apparent from plots of the magnitude of the moments.

This approach has the further advantage that it is stability that is of primary importance to the designer. A safe design will generally be as far into the stable domain as possible. Also, in the new experimental data presented, here problems were encountered with obtaining repeatable values for the magnitudes of the moments whereas the stability of the cascade was clear. Chi and Srinivasan (1985) reported similar variations in a stall flutter test on a rotating fan.

The new experimental results are also presented with theoretical and experimental stability information from the open literature in an attempt to display the dominant trends that occur in Mach number, frequency parameter and phase angle space. Some of these trends are then explored using the Fourier decomposition of the influence coefficients and then the limitations of this are discussed.

The approach is to consider only torsional oscillations about the mid-chord position for compressor type cascades in a two dimensional flow.
EXPERIMENTAL DATA

Davies and Whitehead (1984) and Davies and Bryanston-Cross (1985) describe an experiment to measure the unsteady moments on torsionally vibrated bi-convex aerofoils over a wide range of Mach numbers and inter-blade phase angles in an annular cascade. Table 1 and Figure 1 shows the important characteristics of the cascade.

Not all of the data from these experiments was presented at the time because of the lack of blade to blade repeatability of the unsteady moments. Typical plots showing the extent of this are presented in Figure (2) along with theoretical predictions from the two dimensional Finite Element program Finsup, described by Whitehead (1982). Further reflection on this data, however, showed there was considerable information here if it was approached from the perspective of stability rather than magnitude. The magnitude of the moments is considerably scattered but the stability information is often clear.

It is important to note that the cascade was periodic in amplitude, stagger angle, inter-blade phase angle, frequency, space chord ratio and approach Mach number for Mach number less that 0.9, Davies and Bryanston-Cross (1985). Moreover, the strain gauges measuring the moment and amplitude were regularly recalibrated statically and dynamically between experiments. For instance, the lower plot of Figure 1 at M = 0.65 shows data taken from nine different blades in a periodic cascade, as defined above. The scatter in magnitude under these conditions becomes of interest in its own right.

Table 1 Steady and unsteady cascade data

<table>
<thead>
<tr>
<th>Overall Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tip diameter</td>
</tr>
<tr>
<td>Hub diameter</td>
</tr>
<tr>
<td>Hub/Tip ratio</td>
</tr>
<tr>
<td>Axial height</td>
</tr>
<tr>
<td>Inlet Guide Vanes</td>
</tr>
<tr>
<td>Number of blades</td>
</tr>
<tr>
<td>Spacing at mean diameter</td>
</tr>
<tr>
<td>True chord</td>
</tr>
<tr>
<td>Axial chord</td>
</tr>
<tr>
<td>Stagger angle</td>
</tr>
<tr>
<td>Air outlet angle</td>
</tr>
<tr>
<td>Filtrating Cascade</td>
</tr>
<tr>
<td>Number of blades</td>
</tr>
<tr>
<td>Spacing at mean diameter</td>
</tr>
<tr>
<td>Chord</td>
</tr>
<tr>
<td>Space/chord ratio</td>
</tr>
<tr>
<td>Stagger angle</td>
</tr>
<tr>
<td>Thickness</td>
</tr>
<tr>
<td>Type double circular arc</td>
</tr>
<tr>
<td>Reynolds number</td>
</tr>
<tr>
<td>Blade material</td>
</tr>
<tr>
<td>Unsteady Data</td>
</tr>
<tr>
<td>Frequency</td>
</tr>
<tr>
<td>Frequency parameter</td>
</tr>
<tr>
<td>Torsional axis - mid chord amplitude of vibration</td>
</tr>
<tr>
<td>Interblade phase angle</td>
</tr>
</tbody>
</table>

**Figure 2** Annular Cascade Imaginary Moment Coefficient versus Mach Number and Comparison with Program Finsup.

The torsional stability of a cascade is dependent upon the sign the imaginary component of the aerodynamic moment, a positive value indicating instability. The lower plot in Figure (2) shows that the cascade is stable at an inter-blade phase angle (B) of 225° for all Mach numbers considered. The upper plot for 22.5° is more problematic. Over some Mach number ranges the data spans the neutral stability axis and therefore it is not clear whether the cascade is stable or not. However, there are ranges where the stability information is clear. From M = 0.7 to 0.89 there are 24 data points indicating stability and 1 indicating instability. This is taken to be sufficient evidence of stability over this range. Similarly from 0.9 to 1.27 there are 21 data points indicating instability and only one representing stability, the instability of the cascade is therefore assumed in this region. All of the experimental data of Davies (1985) has been processed in this manner to produce the stability plot of Figure 3. Here, data from Figures similar to Figure (2) for B = 0, 22°, 45, 90, 135, 180, 225 and 270 degrees have been condensed into a single...
plot showing clearly defined regions of stability and instability. Regions of ambiguity are marked as "No data". The theoretical predictions from program Finsup are in accord with this figure except in very limited regions around the lines of neutral stability where the picture is bound to be unclear.

**Figure 3** Stability Plot for the Annular Cascade

### COLLATION OF DATA

The approach of the previous section has been extended to the experimental and theoretical information available in the open literature. The stability information from this literature has been plotted in a three-dimensional space with phase angle ($\beta$) Mach number ($M$) and frequency parameter ($\omega'$) making up the three axis. Figures (4) and (5) show this data taken and Table (2) summarises the sources of information and symbols used. The form the data takes can be better understood by referring to Figures (6) and (7); here a volume has been drawn around the regions of instability and stability to show them more clearly. These volumes are conservative in that the unstable region has been extended so that flat planes may be used to make the figure.

The striking feature of instability polyhedron is its uniformity. Despite the variation in the theoretical approach and experimental method the message is clear. The instability region is always in the range of $\beta = 0 \rightarrow 180^\circ$ and this range decreases in magnitude with increasing frequency parameter.

The instability region may be described by the following inequalities:

- $\beta = 0.116 M + 1.366 \omega' - 3.367 < 0$
- $\beta = 568.69 M - 666.0 \omega' + 30 > 0$
- $0 < M < 1.35$
- $\omega' > 0.05$
- $2 \pi > \beta > 0$

These inequalities must be true for any point falling within the unstable region, hence the torsional stability of any two-dimensional blade row may be assessed. A contour plot of the instability region is also included, Figure 8. The right hand face and the face facing the reader are both bounded by stability points. No information is included for Mach numbers greater than 1.35. The region of instability is therefore generally enclosed by a defined region of stability.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>Circled Points Refer to Experimental Data.</td>
</tr>
<tr>
<td>0</td>
<td>Table 2 Sources of Collated Data.</td>
</tr>
</tbody>
</table>

The only evidence that contradicts this picture is that due to Verdes (1977). Here resonances serve to destabilise the cascade over a range of negative inter-blade phase angles at high Mach numbers. The exact theory of Adamczyk and Goldstein (1978), however, contradicts this view when applied to the same cascade. The implication being that the resonances arise as a result of the integral formulation of the problem. This, and the absence of experimental data showing resonance phenomena, gives sufficient justification for the omission of resonance instabilities in the three dimensional plots. Kurosaka (1975) may also be referred to in support of this omission.

The primary use for the magnitudes of the acting moments is in considering a strip theory approach for assessing the overall stability of a fan. A typical case for consideration might be a part span shrouded fan with coupled or uncoupled torsional instability outboard of the shroud. Here the frequency and inter-blade phase angle will be constant along the span but the Mach number will vary. The points to be considered along the span would then fall into a plain perpendicular to the $\beta$ axis in $M$, $\beta$, $\omega'$ space, Figures (6) and (7). A conservative design for stability would then be to ensure that this plane did not cut into the instability polyhedron. Less conservative designs which may be unstable at certain span sections could not be assessed by the above method because of the lack of magnitude data.
Figure 4 Points making up Unstable Region
See Table 2 for Data Sources.

Figure 5 Points making up Stable Region.
See Table 2 for Data Sources

Figure 6 Approximated Region of Stability from Fig 5.

Figure 7 Approximated Region of Instability from Figure 4.
A further feature of the three dimensional plots is that despite differences in blade contour and stagger between the cascades considered by the authors the results are still well ordered, all the instability information falls into a uniform polyhedron. This feature of the data would also appear to apply to the three dimensional rotating fan investigated by Mikiolajczak et al (1975), here two fans with differing blade sections were investigated for stability and found to be very similar.

![Figure 8](image)

**Figure 8** Contour Plot Stability Region from the Collated Data of Table 2.

**ANALYSIS**

A Fourier summation approach will now be used to explain some of the salient features of the instability plot.

Consider a cascade of blades where one blade is vibrated and all others are held stationary. The moment on all of the blades is measured. For a constant inter-blade phase angle \( \beta \) the Argand diagram of Figure (9) is true when \( C_m(0) \) is in phase with the motion of blade zero and \( C_m(n) \) is phased relative to motion of blade \( n \) when it vibrates with phase angle \( \beta \).

Then for all blades vibrating:

\[
\text{Re}(C_m(\beta)) = \sum_{n=0}^{N-1} C_m(n) \cos n\beta \quad - (1)
\]

\[
\text{Im}(C_m(\beta)) = \sum_{n=0}^{N-1} C_m(n) \sin n\beta \quad - (2)
\]

or:

\[
C_m(\beta) = \sum_{n=0}^{N-1} C_m(n) e^{-in\beta} \quad - (3)
\]

For such a Fourier series:

\[
\tilde{C}_m(n) = \frac{1}{N} \sum_{\beta=0}^{2\pi} C_m(\beta) e^{in\beta} \quad - (4)
\]

See Kreysig (1979).

![Figure 9](image)

**Figure 9** Argand Diagram Showing the Relationship between the Real and Imaginary Moment Components and the Influence Coefficients.

Showing that a single blade vibration theory or experiment will yield all \( C_m(\beta) \) and a multiblade vibration test will yield all \( C_m(n) \). The validity of this approach was demonstrated by Davies and Whitehead (1984) for low frequency parameters. The extent of its applicability is important; Fleeter and Heyniak (1987), for instance, use the method explicitly for assessing the possibility of detuning a cascade, and Szechanyi et al (1984) use it to process experimental data. In both these contexts the relative value of the influence coefficients are also of importance. Both of these points will now be addressed.

The criteria for torsional stability is

\[
\text{Im}(C_m(\beta)) < 0 \quad - (5)
\]

Or using equation (2):

\[
\sum_{n=0}^{N-1} C_m(n) \sin (n\beta) < 0 \quad - (6)
\]

Next it is assumed that pressure waves take a negligible amount of time to pass from blade to blade, therefore all \( C_m(n) \) are real and positive. The inter-blade phase angle would play no part in a single-blade vibration experiment therefore all \( C_m(n) \) are also independent of \( \beta \). They will, however, depend upon \( M \) and \( \omega' \).

At \( \beta = 0 \)

\[
\text{Im}(C_m(\beta)) = 0 \quad - (7)
\]

and

\[
\frac{d\text{Im}(C_m(\beta))}{d\beta} = \sum_{n=0}^{N-1} nC_m(n) \quad - (8)
\]

is finite. Therefore \( \beta = 0 \) is not a turning point and \( \text{Im}(C_m(\beta)) \) will take positive and negative values. Therefore according to this analysis a completely stable cascade is not possible for all \( \beta \). This is in accord with the low frequency parameter data of Figures (6) and (7).

The experiments of Davies and Whitehead (1984) showed that for single blade excitation the effect of vibrating blade zero could only be measured on blades 0, 1, 2, and -1, where blades are numbered in the direction of swirl. The data in that reference was only presented for Mach numbers 0.86 and 0.77. Further experiments carried out at the time showed that this was also true at higher Mach numbers. Although the higher Mach number data could not be quantified as moments due to a calibrating oversight, the extent of the influence was measured. It is shown in Figure (10) as a moment strain gauge voltage plotted against Mach.
number. Szecenyi et al (1984) showed similar results at a higher reduced frequency.

Only three terms need to be considered therefore in inequality (6).

\[ Cm(1) \sin \beta + Cm(2) \sin (2\beta) - Cm(-1) \sin \beta > 0 \quad -(9) \]

\[ \sin \beta (Cm(1) - Cm(-1) + 2Cm(2) \cos \beta) > 0 \quad -(10) \]

The results of Figure (8) show that instability only exists in the range \( \beta = 0 \pm \pi \).

\[ Cm(1) - Cm(-1) + 2Cm(2) \cos \beta > 0 \quad -(11) \]

For this to be true for \( \beta = 0 \pm \pi \)

\[ Cm(1) > Cm(-1) \quad -(12) \]

This criterion has been derived for the case of an unstable blade row, therefore each blade in the row is being driven. An assumption leading to equation (3), the basis of the analysis, is that a single cascaded blade, vibrating on its own, will have no out of phase moment acting on it. Therefore any instability will be the result of the motion of near neighbours. The criterion shows that in the unstable region the magnitude of the driving moment on the suction side is greater than that on the pressure side, due to the motion of these near neighbours.

Closer study of Figure (8) shows that the instability region stops short of \( \beta = \pi \) and the degree of this foreshortening increases with increasing frequency parameter. Figure (11) shows how the terms in the inequality (11) vary with \( \beta \), the implication being that blade rows would be expected to be more stable around \( \beta = \pi \) than around \( \beta = 0 \), which is in accord with the evidence of Figure (8).

It may also be noted that the inequality indicates that neutral stability may be obtained for \( Cm(n) = Cm(-n) \) which is the condition for a symmetrical or unstaggered cascade. The effect of stagger is therefore to stabilise in the region \( \beta = 180^\circ \rightarrow 360^\circ \) and destabilise in the region \( \beta = 0 \rightarrow 180^\circ \).

Almost all the theories and experiments are aimed at understanding a two dimensional inviscid flow with constant inter-blade phase angle. Such flows do not exist in engines and any variation from this ideal may serve to destabilise a blade row. Rows are therefore not only designed to be stable but also to be as stable as possible. Figure (7) gives a conservative estimate of the stability range of \( \beta = 180^\circ \rightarrow 360^\circ \) for all frequency parameters. All the terms in inequality \( \text{Im}(Cm(\beta)) \) are negative in this range, therefore increasing the magnitude of any or all of the influence coefficients will increase the margin of stability. How to do this is not clear, but it does require further investigation.

DEVIATIONS FROM THE SIMPLE FOURIER DECOMPOSITION

There are two major assumptions leading to the simple Fourier analysis. The first is that the time taken for a wave to pass between blades will be an insignificant proportion of the period of oscillation. The order of magnitude of this ratio is given by

\[ t_r = \left( \frac{\pi}{c} \right) \sqrt{\frac{\omega}{2\pi}} \quad -(13) \]
Although $0$ is small $C_m(0)$ is much greater than all other $C_m(n)$, Davies and Whitehead (1984), therefore this effect may become significant. The effect is to always stabilise the blade-row at all Mach numbers and inter-blade phase angles as long as $0$ remains negative for all frequency parameters. This may explain the tendency shown in Figure (6) for blade rows to become completely stable at high frequency parameters.

It is important to note in this context that the simple Fourier analysis implies that a completely stable cascade is not possible for all phase angles. Therefore the high frequency parameter stable cascades are not in accord with this analysis and hence it should not be applied under these conditions.

**CONCLUSIONS**

* The aerodynamically governed vibration of a compressor type cascade with a mid-chord torsional axis in a two dimensional flow is discussed in terms of the stability of the row, rather than the magnitude of the aerodynamic moments.

* Data is presented showing an annular cascade of compressor type blades to be generally unstable in the region $\phi = 0 \rightarrow 180^\circ$ over a wide range of Mach numbers and at a low frequency parameter.

* Experimental data and theoretical predictions from the open literature are presented in a three dimensional stability plot which is well defined in $\phi$, $\omega'$, $M$ space. Stability is shown to be largely independent of cascade geometry.

* A criteria is derived which allows the torsional stability of a two dimensional cascade to be assessed.

* The Fourier decomposition of influence coefficients implies that a torsionally stable blade row is not possible for all interblade phase angles.

* The Fourier method is also used to show that at instability the magnitude of the moment acting on the suction surface is greater than that acting on the pressure surface and that this criterion is independent of Mach number.

* The simple Fourier Analysis is shown to be inappropriate at high reduced frequencies and a suggestion is made as to why this is so.

**ACKNOWLEDGEMENT**

The author makes grateful acknowledgement to Dr. D.S. Whitehead for the results of program Finsup and to Rolls Royce Ltd. for supporting the experimental work.

**REFERENCES**


