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A REDUCED ORDER APPROACH FOR THE VIBRATION OF MISTUNED BLADED DISK ASSEMBLIES

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ABSTRACT

A reduced order approach is introduced in this paper that can be used to predict the steady-state response of mistuned bladed disks. This approach takes results directly from a finite element analysis of a tuned system and, based on the assumption of rigid blade base motion, constructs a computationally efficient mistuned model with a reduced number of degrees of freedom. Based on a comparison of results predicted by different approaches it is concluded that: the reduced order model displays structural fidelity comparable to that of a finite element model of the entire bladed disk system with significantly improved computational efficiency; and under certain circumstances both the finite element model and the reduced order model predict quite different response from simple spring-mass models.

1. INTRODUCTION

The resonant amplitudes of turbine blades tend to be sensitive to minor variations in the blades' properties. It is realized that, because of the rotational periodicity of its geometry, a bladed disk usually has natural frequencies that are clustered in narrow ranges. When the natural frequencies of a system are close together, slight variations in the system's structural properties can cause large changes in its modes, and, consequently, its dynamic response. The sensitivity of a bladed disk's dynamic response to small variations in the frequencies of the blades is referred to in the literature as the *blade mistuning* problem and has been studied extensively, for examples refer to Dye and Henry (1969), Ewins (1988), Fubunmi (1980), Griffin and Hoosac (1984), or Ottarsson and Pierre (1993). It is important to understand mistuning since it can result in large blade to blade variations in the vibratory response and the high response blades can fail from high cycle fatigue.

Much of the work that has been done in mistuning utilizes spring-mass models to represent bladed disks in order to reduce the number of degrees of freedom and to make the problem computationally tractable, for examples refer to the previously cited papers. The model's parameters, such as the mass and the spring constants, are chosen in an ad hoc manner and one must question the ability of such simple models to accurately represent such complex systems. While some attempt has been made to corroborate the accuracy of spring-mass models by comparing predictions with specific test data, for example Griffin (1988), such work is relatively scarce.

Efforts have been made to develop more structurally accurate models for bladed disks by using plate elements to represent the disk and beam elements to represent the blades, for examples refer to Kaza and Kielb (1984), and Rzadkowski's two papers in (1994). While there can be blade configurations for which the beam representation may be adequate, plate, thick shell, and even solid elements are often needed to represent modern low aspect ratio blades. The finite element method could be a possible choice to accurately model a whole bladed disk, but it is recognized that the time cost and the storage space required to run these programs would be prohibitively high. For example, one could imagine using the Monte-Carlo approach of Griffin and Hoosac (1984) with detailed finite element models of the entire mistuned bladed disk. Such an approach would involve the analyses of hundreds of mistuned disks in order to determine the statistical variations in the blades' vibratory response. Clearly, such computations are currently beyond the capabilities of even supercomputers and would be hardly suitable for use as a design tool. Furthermore, because of the extremely large number of degrees of freedom involved and the closeness of the natural

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frequencies, one must question if such results would even be numerically accurate.[1]

The limitations associated with spring-mass and beam models and the direct finite element approach motivate us to consider the possibility of developing a new model for analyzing mistuned bladed disks. Our goal is to develop a methodology that will take the results directly from a finite element analysis of a tuned system and construct a computationally efficient mistuned model with a reduced number of degrees of freedom. The intent is that the approach will display structural fidelity comparable to a finite element model and computational efficiency more comparable to that of a spring-mass model.

2. APPROACH FOR REDUCING THE NUMBER OF DEGREEES OF FREEDOM

In the study of the steady-state response of complex structures, one widely used analytical approach is the *receptance method*, Bishop and Johnson (1960). The receptance method is based on the observation that the dynamic response of every substructure is determined by how it interacts with its environment at its boundaries. If the substructure interacts with its environment only at limited areas, it is convenient to express the degrees of freedom of the entire substructure in terms of the degrees of freedom of its interfaces. The benefit of this method is that when several substructures interact with each other, it is only necessary to solve for the degrees of freedom associated with the interfaces. Once the degrees of freedom of the interfaces are determined the response of all substructures and, consequently, the whole structure may be calculated. To apply the receptance method to the mistuned bladed disk, it is divided into two substructures — the disk[2] (Figure 1) and the blades (Figure 2). Modal analysis is then used to determine the substructures' behavior in terms of the degrees of freedom of the interfaces. A receptance approach was formulated for mistuned, shrouded bladed disks by Menq, Griffin, and Bielak (1986). They then specialized the formulation to the case of a tuned system and used it to solve for the vibratory response of a tuned shrouded stage in which the blade was represented using beam elements. In general, however, the direct receptance approach has two shortcomings:

1. The substructures' modes have to be free at the disk-blade interfaces in order to be admissible. This works reasonably well for the disk since the blades provide relatively little constraint at its rim and, consequently, only a few families of disk modes are required to represent its response. However, it is undesirable to use the free-free blade model. Because blades generally vibrate close to the clamped-free condition, a large number of the free-free modes are needed to achieve a good representation of its vibratory response.
2. A direct application of the receptance method results in a formulation with a reduced number of degrees of freedom. However, depending on the number of nodes at the blade interface, the number can still be quite large, especially if solid elements are used to model the blade's neck. Since it is undesirable to restrict the approach to blade models with only a few nodes at the disk-blade interface, the receptance method needs to be modified in order to make it even more efficient. This is done using the following simplifying assumption.

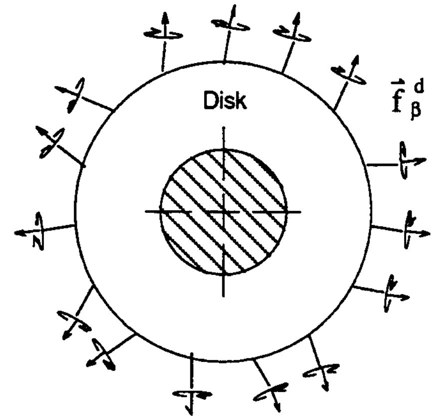


Figure 1: Disk substructure

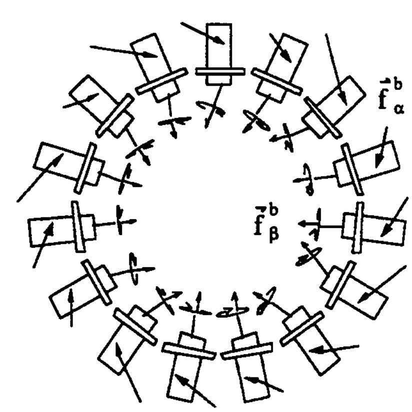


Figure 2: Blade substructure

It is assumed that the disk-blade interfaces undergo rigid body type translations and rotations. The blade vibration is then determined as a combination of blade base motion and clamped-free blade modes. As will be shown, this approach results in a formulation that has a relatively small number of degrees of freedom (six times the number of blades) and is, consequently, computationally efficient. In addition, since the blade is represented in terms of its clamped-free modes its response can be quickly characterized with only a few modes.

[1] In contrast, the reduced order model uses finite element generated modes for a single cantilevered blade and for a tuned disk. The modes of the tuned disk can be calculated using a single disk segment with cyclic symmetric boundary conditions. Disk modes calculated in this manner do not exhibit numerical instabilities since for a specified cyclic constraint their natural frequencies are well separated.

[2] Note the cross-hatching in Figure 1 indicates that in this model the disk is fully constrained at its bore.

3. MATHEMATICAL FORMULATION

3.1 Disk Equation

If the disk substructure is subjected to a harmonic excitation, its response is given by the receptance matrix **R**, i.e.,

$$\begin{Bmatrix} \bar{u}^d_\alpha \\ \bar{u}^d_\beta \end{Bmatrix} = \begin{bmatrix} \mathbf{R}^d_{\alpha,\alpha} & \mathbf{R}^d_{\alpha,\beta} \\ \mathbf{R}^d_{\beta,\alpha} & \mathbf{R}^d_{\beta,\beta} \end{bmatrix} \begin{Bmatrix} \bar{f}^d_\alpha \\ \bar{f}^d_\beta \end{Bmatrix} \tag{1}$$

where $\bar{u}^d$, $\bar{f}^d$, and $\mathbf{R}^d$ are the displacement, the external force, and the receptance associated with the disk. The subscript α denotes the group of nodes that do not interact with other substructures and the subscript β denotes the group of nodes that reside at the disk-blade interfaces. Since the only external forces on the disk are the interactive forces at the disk-blade interfaces

$$\bar{f}^d_\alpha = \bar{0} \tag{2}$$

Equations (1) and (2) imply that

$$\bar{u}^d_\beta = \mathbf{R}^d_{\beta,\beta}\ \bar{f}^d_\beta \tag{3}$$

To formulate the disk equation in a more reduced order form, i.e., a formulation with six degrees of freedom per interface, the following two relations are introduced

$$\bar{u}^d_\beta \approx \mathbf{Q}^d_{\beta,o}\ \bar{u}^d_o \tag{4}$$

$$\bar{f}^d_o = \Sigma^d_{o,\beta}\ \bar{f}^d_\beta \tag{5}$$

where $\bar{u}^d_o$ is a vector whose components are the six equivalent rigid body-type motions of the disk at the interfaces and $\bar{f}^d_o$ is the resultant forces on the disk at the interfaces. $\Sigma^d_{o,\beta}$ is a matrix that effectively sums the various nodal forces to get the resultant forces and moments that act on the disk. $\mathbf{Q}^d_{\beta,o}$ is the transpose of $\Sigma^d_{o,\beta}$. The detailed derivations of these matrices are provided in Yang (1994). The inverse relations of equations (4) and (5) are

$$\bar{u}^d_o = \mathbf{Q}^{d\ +}_{\beta,o}\ \bar{u}^d_\beta \tag{6}$$

$$\bar{f}^d_\beta = \Sigma^{d\ +}_{o,\beta}\ \bar{f}^d_o \tag{7}$$

where $\mathbf{Q}^{d\ +}_{\beta,o}$ and $\Sigma^{d\ +}_{o,\beta}$ are the generalized inverses of the matrices $\mathbf{Q}^d_{\beta,o}$ and $\Sigma^d_{o,\beta}$. Their expressions are

$$\mathbf{Q}^{d\ +}_{\beta,o} = \left(\mathbf{Q}^{d\ T}_{\beta,o}\,\mathbf{Q}^d_{\beta,o}\right)^{-1}\mathbf{Q}^{d\ T}_{\beta,o} \tag{8}$$

$$\Sigma^{d\ +}_{o,\beta} = \Sigma^{d\ T}_{o,\beta}\left(\Sigma^d_{o,\beta}\,\Sigma^{d\ T}_{o,\beta}\right)^{-1} \tag{9}$$

In essence, equations (6) and (7) state that $\bar{u}^d_o$ is the least squares fit of $\bar{u}^d_\beta$ and $\bar{f}^d_\beta$ is the non-self-equilibrated forces estimated from $\bar{f}^d_o$. There are many possible choices of the distributed force $\bar{f}^d_\beta$. The reason that only the non-self-equilibrated forces were used is that, because of *Saint Venant's Principle*, the self-equilibrated forces die off quickly away from the interfaces and their global effect on the system should be relatively small. Substituting equations (6) and (7) into equation (3) results in the reduced order receptance formulation for the disk

$$\bar{u}^d_o = \mathbf{R}^d_{o,o}\ \bar{f}^d_o \tag{10}$$

where

$$\mathbf{R}^d_{o,o} = \mathbf{Q}^{d\ +}_{\beta,o}\ \mathbf{R}^d_{\beta,\beta}\ \Sigma^{d\ +}_{o,\beta} \tag{11}$$

By applying standard modal analysis, the disk receptance $\mathbf{R}^d_{\beta,\beta}$ can be written as

$$\mathbf{R}^d_{\beta,\beta} = \sum_j \frac{\bar{\phi}^d_{j,\beta}\ \bar{\phi}^{d\ T}_{j,\beta}}{m^d_j\left(\omega^{d\ 2}_j - \Omega^2 + 2i\Omega\omega^d_j\zeta^d_j\right)} \tag{12}$$

where $\bar{\phi}^d_j$ is the j[th] disk mode, and m^d_j, ζ^d_j, and ω^d_j are the modal mass, modal damping ratio, and natural frequency of the j[th] disk mode. Combining equations (11) and (12), the reduced order disk receptance can be written as

$$\mathbf{R}^d_{o,o} = \sum_j \frac{\bar{\phi}^d_{j,o}\ \bar{\phi}^{d\ T}_{j,o}}{m^d_j\left(\omega^{d\ 2}_j - \Omega^2 + 2i\Omega\omega^d_j\zeta^d_j\right)} \tag{13}$$

where $\bar{\phi}^d_{j,o}$, the j[th] reduced order disk modes, is written as

$$\bar{\phi}^d_{j,o} = \mathbf{Q}^{d\ +}_{\beta,o}\ \bar{\phi}^d_{j,\beta} \tag{14}$$

3.2 Blade Equation

Consider a single blade whose base is constrained to move in a plane undergoing small, harmonic translations and rotations. Let $\bar{u}^b_o$ be the six-degree-of-freedom displacement vector of the blade base and, again, denote the group of nodes that do not interact with the disk by the subscript α and the group of nodes that reside at the interface by β. The receptance equation for the blade can be written as

$$\bar{u}^b_\alpha - \mathbf{Q}^b_{\alpha,o}\,\bar{u}^b_o = \mathbf{R}^b_{\alpha,\alpha}\left(\bar{f}^b_\alpha + \Omega^2\,\mathbf{M}^b_{\alpha,\alpha}\,\mathbf{Q}^b_{\alpha,o}\,\bar{u}^b_o\right) \tag{15}$$

$$\bar{u}^b_\beta = \mathbf{Q}^b_{\beta,o}\ \bar{u}^b_o \tag{16}$$

where Ω is the excitation frequency. $\bar{u}^b$, $\bar{f}^b$, and $\mathbf{R}^b$ are the displacement, the external force, and the receptance associated with the blade. $\mathbf{M}^b_{\alpha,\alpha}$ is the blade mass matrix associated with the group-α nodes. $\mathbf{Q}^b_{\alpha,o}$ and $\mathbf{Q}^b_{\beta,o}$ are geometric functions such that $\mathbf{Q}^b_{\alpha,o}\,\bar{u}^b_o$ and $\mathbf{Q}^b_{\beta,o}\,\bar{u}^b_o$ describe the motion of the entire blade provided that the blade follows the motion of its base and does not deform. The term $\Omega^2\mathbf{M}^b_{\alpha,\alpha}\mathbf{Q}^b_{\alpha,o}\bar{u}^b_o$ in the right-hand side of equation (15) describes the inertial force introduced by the blade base motion. Rearranging equation (15), the motion of the group-α nodes $\bar{u}^b_\alpha$ can be expressed in terms of the external excitation force $\bar{f}^b_\alpha$ and the blade base motion $\bar{u}^b_o$, i.e.,

$$\bar{u}^b_\alpha = \mathbf{R}^b_{\alpha,\alpha}\;\bar{f}^b_\alpha + \left(\Omega^2\,\mathbf{R}^b_{\alpha,\alpha}\,\mathbf{M}^b_{\alpha,\alpha} + \mathbf{I}\right)\mathbf{Q}^b_{\alpha,o}\,\bar{u}^b_o \qquad (17)$$

From modal analysis, the receptance $\mathbf{R}^b_{\alpha,\alpha}$ can be written as

$$\mathbf{R}^b_{\alpha,\alpha} = \sum_j \frac{\bar{\phi}^b_{j,\alpha}\;\bar{\phi}^{b\,T}_{j,\alpha}}{m^b_j\left({\omega^b_j}^2 - \Omega^2 + 2i\Omega\omega^b_j\zeta^b_j\right)} \qquad (18)$$

where $\bar{\phi}^b_j$ is the j[th] mode for a blade clamped at its base, and m^b_j, ζ^b_j, and ω^b_j are the modal mass, modal damping ratio, and natural frequency of the j[th] blade mode.

The resultant force on the blade base, $\bar{f}^b_o$, is needed in order to assemble the entire system. To determine $\bar{f}^b_o$ consider the blade as an object which can move and deform in space. The forces acting on the blade are the excitation force $\bar{f}^b_\alpha$ on the airfoil and the interactive force $\bar{f}^b_o$ at the interface. Since linear and angular momentum have to be conserved, one can deduce that

$$\Sigma^b_o\left(-\Omega^2\,\mathbf{M}^b\,\bar{u}^b\right) = \Sigma^b_o\,\bar{f}^b = \Sigma^b_{o,\alpha}\,\bar{f}^b_\alpha + \bar{f}^b_o \qquad (19)$$

where Σ^b_o is a geometric function which calculates the resultant force and moment about the blade base O. Substituting equations (16) and (17) into equation (19), $\bar{f}^b_o$ can be expressed in terms of the blade base motion $\bar{u}^b_o$ and the external excitation $\bar{f}^b_\alpha$, i.e.,

$$\bar{f}^b_o = \mathbf{Z}^b_{o,o}\,\bar{u}^b_o + \mathbf{H}^b_{o,\alpha}\,\bar{f}^b_\alpha \qquad (20)$$

where

$$\begin{aligned}\mathbf{Z}^b_{o,o} &= -\Omega^2\,\Sigma^b_o\,\mathbf{M}^b\,\mathbf{Q}^b_o \\ &\quad -\Omega^4\,\Sigma^b_{o,\alpha}\,\mathbf{M}^b_{\alpha,\alpha}\,\mathbf{R}^b_{\alpha,\alpha}\,\mathbf{M}^b_{\alpha,\alpha}\,\mathbf{Q}^b_{\alpha,o}\end{aligned} \qquad (21)$$

$$\mathbf{H}^b_{o,\alpha} = -\Omega^2\,\Sigma^b_{o,\alpha}\,\mathbf{M}^b_{\alpha,\alpha}\,\mathbf{R}^b_{\alpha,\alpha} - \Sigma^b_{o,\alpha} \qquad (22)$$

Equation (20) describes the relation between the interactive force $\bar{f}^b_o$, the blade base motion $\bar{u}^b_o$, and the external excitation $\bar{f}^b_\alpha$ for a single blade. Clearly, similar equations apply for all blades. In fact, equations (20), (21) and (22) describe the motions of all of the blades provided the vectors represent the collections of the corresponding vectors of individual blades and the matrices become block diagonal matrices with the corresponding matrices of individual blades at the diagonals.

3.3 Assembling the Substructures

Equations (10) and (20) describe the relations between the interactive forces and the motion of the interfaces for the disk and the blades respectively. Since there are four unknown vectors, $\bar{u}^d_o$, $\bar{f}^d_o$, $\bar{u}^b_o$, and $\bar{f}^b_o$ to be determined, there needs to be two additional constraints. The first is Newton's law of interactive forces, i.e.,

$$\bar{f}^d_o = -\bar{f}^b_o \qquad (23)$$

The second one is Hooke's law which provides a constitutive relation at the interfaces, i.e.,

$$\bar{f}^d_o = \mathbf{K}_{o,o}\left(\bar{u}^d_o - \bar{u}^b_o\right) \qquad (24)$$

where $\mathbf{K}_{o,o}$ is the stiffness matrix associated with the attachment at the disk-blade interfaces.[3] By solving equations (10), (20), (23), and (24) simultaneously, the four unknowns can be expressed in terms of the external excitation force $\bar{f}^b_\alpha$, i.e.,

$$\bar{u}^b_o = -\left(\mathbf{R}^d_{o,o} + \mathbf{K}^{-1}_{o,o}\right)\left[\mathbf{I} + \mathbf{Z}^b_{o,o}\left(\mathbf{R}^d_{o,o} + \mathbf{K}^{-1}_{o,o}\right)\right]^{-1}\mathbf{H}^b_{o,\alpha}\,\bar{f}^b_\alpha \qquad (25)$$

$$\bar{f}^b_o = \left[\mathbf{I} + \mathbf{Z}^b_{o,o}\left(\mathbf{R}^d_{o,o} + \mathbf{K}^{-1}_{o,o}\right)\right]^{-1}\mathbf{H}^b_{o,\alpha}\,\bar{f}^b_\alpha \qquad (26)$$

$$\bar{u}^d_o = -\mathbf{R}^d_{o,o}\left[\mathbf{I} + \mathbf{Z}^b_{o,o}\left(\mathbf{R}^d_{o,o} + \mathbf{K}^{-1}_{o,o}\right)\right]^{-1}\mathbf{H}^b_{o,\alpha}\,\bar{f}^b_\alpha \qquad (27)$$

$$\bar{f}^d_o = -\left[\mathbf{I} + \mathbf{Z}^b_{o,o}\left(\mathbf{R}^d_{o,o} + \mathbf{K}^{-1}_{o,o}\right)\right]^{-1}\mathbf{H}^b_{o,\alpha}\,\bar{f}^b_\alpha \qquad (28)$$

The vibration of the blades can then be easily derived by substituting equation (25) into equation (17).

3.4 Remarks on the Mathematical Approach

There are several advantages of using the reduced order formulation (equations (17) and (25)) to solve the blade mistuning problem:

1. The coefficient matrices $\mathbf{R}^d_{o,o}$, $\mathbf{Z}^b_{o,o}$, and $\mathbf{H}^b_{o,\alpha}$ are calculated from the substructures' modes. Since the frequency range is usually limited to a narrow range near a particular engine order crossing, only a few dominant modes need to be included in the calculation and, consequently, the computational cost is low.

[3] If the attachment is infinitely stiff, then (24) implies that the displacements in the blade and disk are equal, and, consequently, (24) becomes a continuity requirement.

2. For blade mistuning problems, the disk is treated as a cyclic symmetric structure. As a result, its modes can be calculated efficiently by applying cyclic symmetric boundary conditions on a disk segment corresponding to a single blade.

3. The most computationally intensive part of this approach lies in computing the inverse of the matrix $\left[\mathbf{I}+\mathbf{Z}^{b}_{o,o}\left(\mathbf{R}^{d}_{o,o}+\mathbf{K}^{-1}_{o,o}\right)\right]$ in equation (25). This is a square matrix with a dimension equal to six times the number of the blades. In many problems the rigidity of the disk is such that only three degrees of freedom are needed to describe the blade's motion at its base, two rotations and translation normal to the disk. In either case the number of degrees of freedom are orders of magnitude smaller than for a finite element model for an entire mistuned bladed disk.

4. The reduced order method is especially computationally efficient when running Monte Carlo simulations of blade mistuning since the disk's modes and the nominal blade's modes only need to be calculated once. The coefficient matrices associated with the blades $\left(\mathbf{Z}^{b}_{o,o}\text{ and }\mathbf{H}^{b}_{o,\alpha}\right)$ are calculated by changing the cantilever blade frequencies in the receptance calculation for each blade. On the other hand, a direct simulation using finite element models would require completely new calculations for each mistuned stage.

4. COMPARISON OF RESULTS

A computer code named LMCC (Linear Mistuning Computer Code) has been developed based on the reduced order approach. A test problem was developed that was sufficiently simple that a finite element solution of the complete bladed disk could be determined, Figure 3. The test problem is analyzed using: 1) modal summation based on a direct finite element analysis, 2) LMCC, and 3) the computer code BLDVIB which calculates the response of a mass-spring model of the bladed disk, Figure 4.

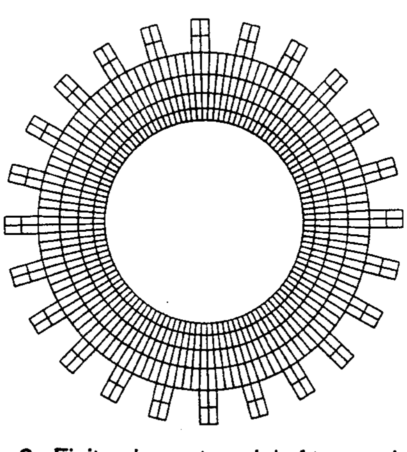


Figure 3: Finite element model of test problem

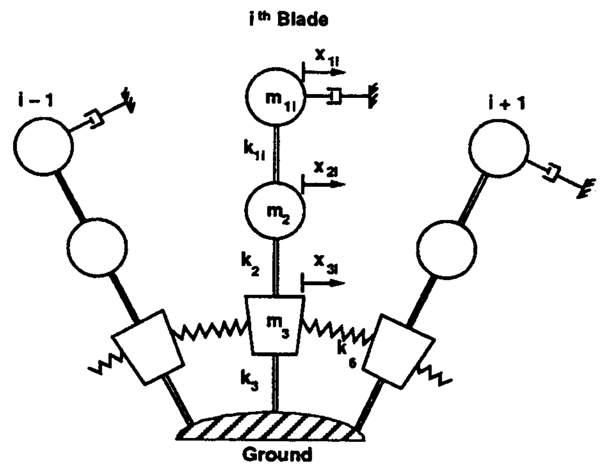


Figure 4: Spring-mass model used by Hoosac and Griffin

The development of BLDVIB is documented in Griffin and Hoosac (1984). The vibratory response of more than one hundred different resonant crossings have been examined of which the examples presented in this section are representative.

The geometry of the test problem was chosen so that the following requirements were satisfied:

1. The disk and the nominal blade have natural frequencies that are representative of a turbo-pump. Both the blades and disk are assumed to have material properties associated with a super nickel alloy.

2. The test problem represents a realistic three dimensional structure with low aspect ratio, plate-like blades. A test problem consisting of beam-type blades would not be appropriate since a goal is to check the validity of the simplifying constraint that the bases of the blades move as rigid bodies. The "blades" are 2.54 cm high by 1.27 cm wide and are 0.381 cm thick. The "disk" has inner and outer radii of 7.62 cm and 12.7 cm, and is 2.03 cm thick. The disk is constrained at its inner radius.

3. The entire system does not have too many degrees of freedom. As a result the entire bladed disk can be directly analyzed by finite elements without expending too much computational time or having numerical stability problems.

There are still several parameters of the reduced order model that need to be determined before the algorithm can be implemented. The attachment stiffness matrix $\mathbf{K}_{o,o}$ is set to infinity to represent the simplified geometry of the test problem, i.e., the blade and disk nodes at the interface are identical. The other parameters that need to be determined are the damping ratios of the substructures' modes. It is assumed that both the blades and the disk have the same damping ratio. Its value is chosen so that the resonant response predicted by LMCC for the tuned system is the same as that predicted by the finite element method.

4.1 Tuned Response

First a comparison will be made between LMCC and the finite element method based on the natural frequencies of a tuned bladed disk. LMCC does not readily compute natural frequencies. However, if the system is lightly damped they can be inferred by the system's resonant frequencies since they can be calculated using LMCC's forced response algorithm. Figure 5 shows a comparison of natural frequencies. The integer "j" refers to the engine order of the excitation which excites the j^{th} nodal diameter mode. The mode shapes can be described in terms of how the strain energy is primarily distributed in the system. For example, for j equal to one, the lowest frequency mode is essentially a first cantilevered bending mode, the second mode is a disk mode, and the third mode is essentially a blade torsional mode. The mode shapes are described in more detail in Yang (1994).

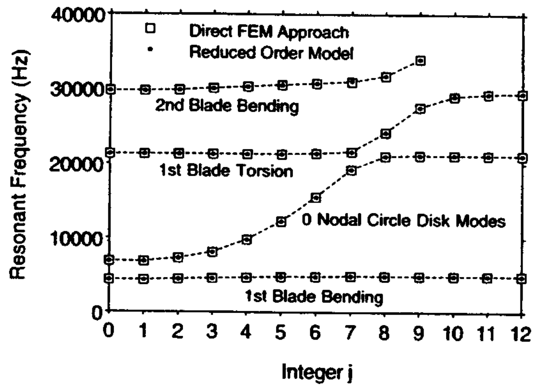


Figure 5: Tuned system frequencies

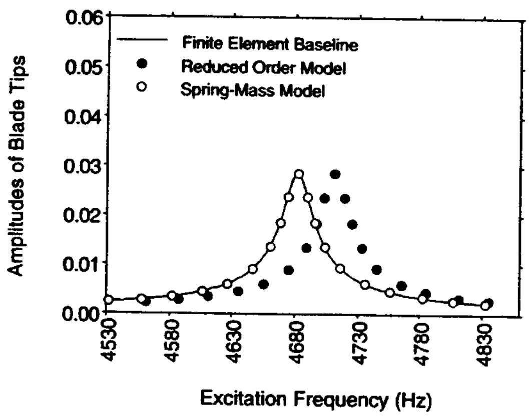


Figure 6: Tuned response curves

In comparing the frequencies that were calculated, it is clear that, overall, the two methods agree reasonably well. However, the reduced order approach tends to predict slightly higher frequencies probably because of the assumption of rigid blade base motion which tends to overconstrain the system. Note that the reduced order model represents the structure's response over a wide range of frequencies and nodal diameters. Results from the spring-mass model are not shown since the natural frequencies of the tuned system are part of the input parameters to the method and not calculated results.

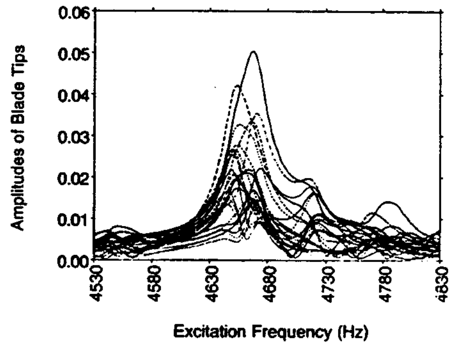
The nominal properties of the spring-mass model were calculated using the approach given by Griffin (1988). Initially, the spring-mass model predicted a spurious second mode which had to be eliminated by varying some of the parameters. Figure 6 shows representative response curves of the tuned system as predicted by the three different methods. For this particular case, the bladed disk is subjected to a fourth engine order excitation and the blades vibrate predominantly in their first bending mode. It appears that the three methods agree well with each other though the response predicted by the reduced order model is at a slightly higher frequency.

4.2 Mistuned Response

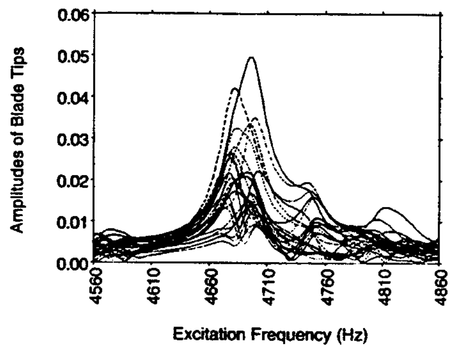
The finite element model is mistuned by varying the elastic modulus of the material in each blade. This same effect is simulated in LMCC by changing the natural frequencies of each blade when calculating its receptance. In the case of BLDVIB the stiffness, k_{1i}, of each blade was altered to produce the same frequency distribution. Figure 7 shows representative response curves of the blades as predicted by the three methods. In this case the bladed disk is subjected to a fourth engine order excitation and the blades predominantly vibrate in their first bending mode. Clearly, the agreement of all three methods is quite good. In general, it was observed that the reduced order model and the spring-mass model give comparable results when the blades respond predominantly in their first bending modes.

Figure 8 depicts representative response curves when the blades are excited in their first torsional mode. It is clear from the plots that the reduced order model works much better than the spring-mass model in representing the response of the system. This case differs from the bending mode case in that the disk is effectively much stiffer. This stiffness effect is discernible from the fact that the percentage change in the frequencies of the four and five diameter modes of the tuned system (Figure 5) is an order of magnitude smaller for the torsional modes, 0.075%, than for the first bending modes, 1.5%. For a stiff disk, the spring-mass model predicts that the blades essentially respond as isolated mistuned blades on a rigid foundation. However, both the finite element and LMCC results indicate that there is still significant interaction between the blades that results in complex system dynamics that is not captured by the simple spring-mass models.[4]

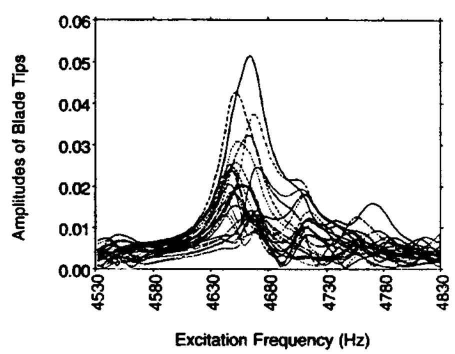
[4] Simulations were also made of the response of the mistuned system in first bending with a stiffer disk. In this case all three methods predicted that the blades would respond as isolated blades. Again, the spring-mass model worked quite well for first bending.



a. Finite element baseline

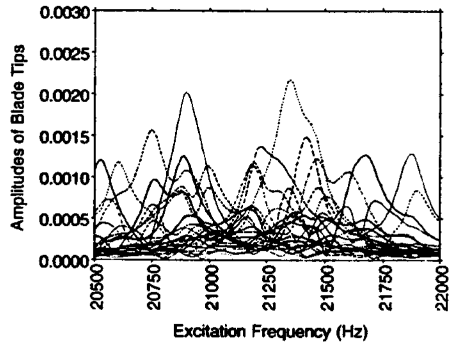


b. Reduced order model

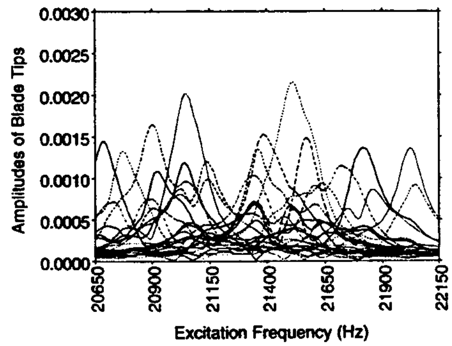


c. Spring-mass model

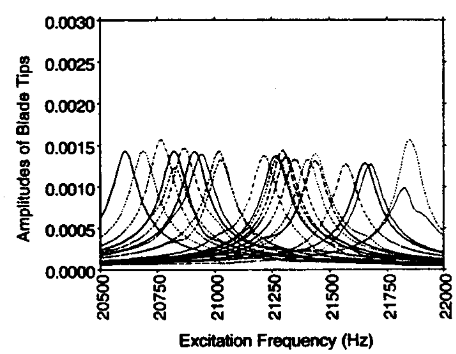
Figure 7: Mistuned response curves for blades in first bending



a. Finite element baseline



b. Reduced order model



c. Spring-mass model

Figure 8: Mistuned response curves for blades in first torsion

4.3 Limitations of the Reduced Order Model

Additional simulations were made with both thicker and thinner disks. In general, finite element and LMCC results agreed quite well except for one case when the thickness of the disk is halved. In the case in question, the tuned system exhibits frequency veering for the third and fourth modes when j is in the range of 2 to 6, Figure 9. As a result, the bladed disk has two families of modes that are close together and that respond to the excitation. Because of the closeness of the frequencies, the resulting vibration contains significant amounts of both blade torsion and disk modes. Figure 10 shows the response curves predicted by the three different models. It appears that neither the reduced order model nor the spring-mass model successfully predict the correct results. One problem with the LMCC model is that it does not predict the separation in the frequencies of the two families of modes very accurately when $\mathbf{K}_{oo}$ is infinitely stiff, Figure 9. This problem was addressed by introducing a finite value of the blade attachment stiffness matrix so that the frequency difference for the tuned system's response more closely approximates that of the finite element model, Figure 9. Figure 10(d) shows the response curves predicted by the reduced order model when the attachment stiffness has been adjusted. The results from the reduced order model agree somewhat better with the finite element results, but not as well as in the other cases examined.

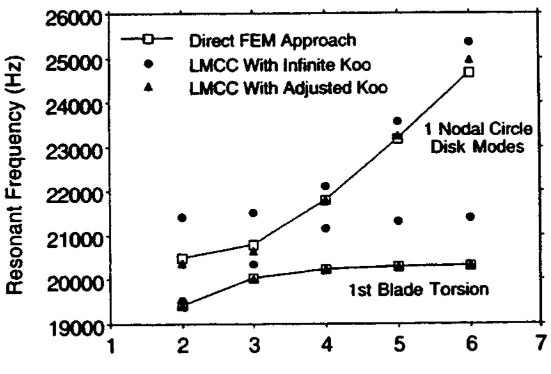


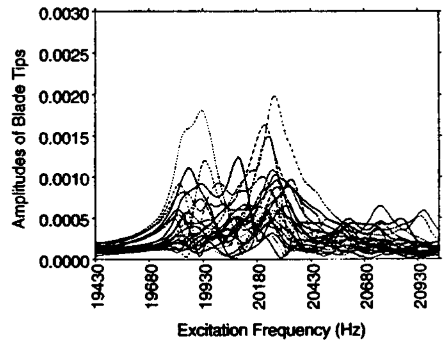
Figure 9: Tuned system frequencies when the disk thickness is reduced by two

4.4 Execution Time Comparison

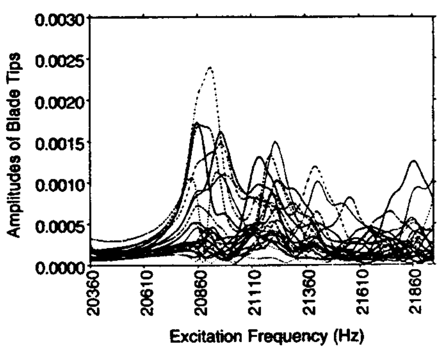
There are two principal advantages of the reduced order model approach which lead to increased computational efficiency. The first is that the number of degrees of freedom that must be determined (three or six per blade) in the forced response analysis is independent of the refinement of the finite element model of the blade and disk models that are used to calculate its input. Thus, LMCC is especially efficient when compared to a direct finite element solution of a bladed disk that has a large number of elements. Secondly, the most computationally expensive part of the calculation with LMCC is in calculating the modes of the clamped blade and tuned disk using finite elements. Since this needs to be done only once, LMCC becomes progressively more efficient when more mistuned bladed disks are simulated in a specific Monte Carlo analysis. An estimate of the relative run times of LMCC and a finite element analysis using modal superposition for a representative case indicates that LMCC would be two to three orders of magnitude faster for a one hundred bladed disk simulation.

CONCLUSIONS

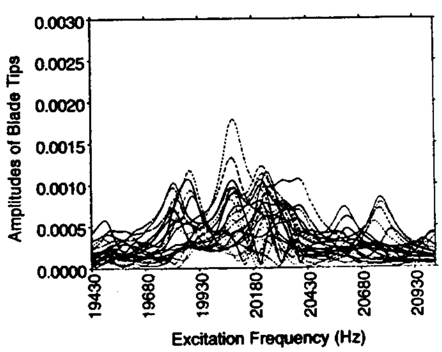
A new reduced order approach has been developed for analyzing the forced response of mistuned bladed disks. The reduced order model is based on the idea of decomposing the bladed disk into substructures and representing the response in



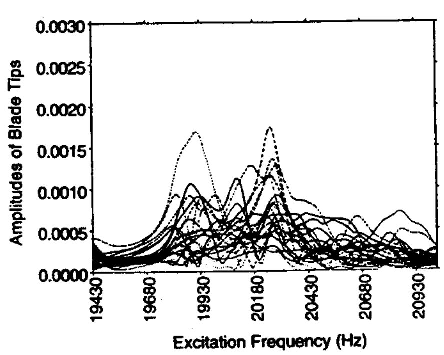
a. Finite element baseline



b. Reduced order moel



c. Spring-mass model



d. Reduced order model using adjusted K_{oo}

Figure 10: Mistuned response curves for coupled disk and torsion modes

terms of the degrees of freedom associated with their interfaces. It further assumes that the blade bases undergo rigid body-type motions. The blade vibration may then be expressed as a combination of blade base motion and cantilever blade modes. This approach results in only six equivalent degrees of freedom, three rotations and three translations, at each disk-blade interface. Because of the reduction in the number of unknowns, the reduced order model provides a formulation which can be solved efficiently by computers. An advantage of this formulation is that the reduced order model is directly calculated from finite element analyses of a single clamped blade and a tuned disk without any additional ad hoc assumptions.

Comparisons of results from direct finite element simulations and the reduced order model indicate that, in general, the reduced order model agrees quite well with the finite element baseline. An exception to this agreement occurred when modes from two different families (one predominantly blade modes and one predominantly disk modes) were close in frequency and excited simultaneously. Comparisons were also made with results from spring-mass models. The spring-mass models predicted good results when the blades were responding predominantly in first bending, but did not predict the response when the disk was stiff and the blades were responding predominantly in first torsion.

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One of the authors, J. H. Griffin, wishes to acknowledge his discussions of reduced order models with Professor C. Pierre from the University of Michigan. Professor Pierre has on-going research in the area of reduced order modeling of mistuned systems. His approach is similar in some respects to that reported here. At the time of the discussions in January 1993 both approaches had been extensively and independently formalized. As a result of their discussion, they agreed that they would acknowledge each others work when publishing their results. The authors are not aware of any publication of Professor Pierre's work on reduced order modeling that has appeared in the open literature.

REFERENCES

Bishop, R. E. D. and Johnson, D. C., 1960, *Mechanics of Vibration*, Cambridge University Press.

Dye, R. C. F. and Henry, T. A., 1969, "Vibration Amplitudes of Compressor Blades Resulting From Scatter in Blade Natural Frequencies," *Transactions of the ASME*, pp. 182-188.

Ewins, D. J., 1988, "Structural Dynamic Characteristics of Bladed Assemblies," *ALGOR Manual on Aeroelasticity in Axial-Flow Turbomachines*, Vol. 2, Specialized Printing Service Limited, Loughton, pp. 15-1 to 15-37.

Fabunmi, J. A., 1980, "Forced Vibrations of a Single Stage Axial Compressor Rotor," *Journal of Engineering for Power, Transactions of the ASME*, Vol. 102, pp. 322-328.

Griffin, J. H. and Hoosac, T. M., 1984, "Model Development and Statistical Investigation of Turbine Blade Mistuning," *Journal of Vibration, Acoustics, Stresses, and Reliability in Design, Transactions of the ASME*, Vol. 106, pp. 204-210.

Griffin, J. H., 1988, "On Predicting the Resonant Response of Bladed Disk Assemblies," *ASME Journal of Engineering for Gas Turbines and Power*, Vol. 110, pp. 45-50.

Kaza, K. R. V. and Kielb, R. E., 1984, "Flutter of Turbofan Rotors with Mistuned Blades," *AIAA Journal*, Vol. 22, No. 11.

Menq, C. H., Griffin, J. H., and Bielak, J., 1986, "The Forced Response of Shrouded Fan Stages," *ASME Journal of Vibration, Acoustics, Stress, and Reliability in Design*, Vol. 108, No. 1, pp. 50–55.

Ottarsson, G. and Pierre, C., 1993, "A Transfer Matrix Approach to Vibration Localization in Mistuned Blade Assemblies," ASME Report No. 93-GT-115, presented at the International Gas Turbine and Aeroengine Congress and Exposition, Cincinnati, Ohio, May 24-27.

Rzadkowski, R., 1994, "The General Model of Free Vibrations of Mistuned Bladed Discs, Part I: Theory," *Journal of Sound and Vibration*, Vol. 173, No. 3, pp. 377-393.

Rzadkowski, R., 1994, "The General Model of Free Vibrations of Mistuned Bladed Discs, Part II: Numerical Results," *Journal of Sound and Vibration*, Vol. 173, No. 3, pp. 395-413.

Wagner, L. F., 1993, "Vibration Analysis of Grouped Turbine Blades," Ph.D. Thesis, Carnegie Mellon University, Pittsburgh, PA.

Yang, M.-T., 1994, "A Reduced Order Approach for the Forced Vibration Analysis of Mistuned Bladed Disk Assemblies," Ph.D. Thesis, Carnegie Mellon University, Pittsburgh, PA.