NUMERICAL SIMULATION FOR AEREOELASTICITY IN TURBOMACHINES WITH VORTEX METHOD

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ABSTRACT

A new method for quantitatively predicting the interaction between the unsteady aerodynamics and structural dynamics was developed. The unsteady flow with large scale separation is simulated numerically with the developed Discrete Vortex Method. In this calculation process, the instantaneous unsteady force and moment of force acting on the blades can be obtained at each time step. On other hand, the cascade is considered as an elasticity system with damping including the effects of the interblade phase angle. The blades are excited to vibrate by the unsteady force and moment. To deal with the above two respects, the resulting code is established in an ASME Journal. Papers are available from ASME for 15 months after the meeting. Copyright © 1994 by ASME

1 INTRODUCTION

Stall flutter is considered as a challenging problem in the design of advanced turbomachinery. Many numerical studies, as well as theoretical and experimental investigations, have been attempted to illuminate the mechanism of the stall flutter and the quantitative relationship with the parameters. In fact, the problem still remains not sufficiently understood. It requires a great deal of further research effort in the unsteady aerodynamics, flow induced vibration of the blades and, especially, the strong interaction between these two phenomena, i.e. the aerelasticity problem. Since the complicacy of the phenomena, some hypotheses and/or approximations were introduced in some calculations, for example, simplified harmonic vibration of the flow field, stable vibration of flow field along with the solid etc. Further investigations have demonstrated that these hypotheses are in disagreement with the real situation. The reason is that the unsteady separated flow is not a simply ordered motion but a typical nonlinear process. Some simplified hypotheses limit or lose the colorful nature characters of the problem.

Recently, the Discrete Vortex Method has been developed for the numerical simulation of two-dimensional, incompressible and separated flows at high Reynolds Number. When propagating stall occurs, the blades of axial-flow turbomachinery may be excited to vibrate in each model of bending and torsional vibration. Using this method, a series of numerical tests have been carried out and some important results have been obtained. All these studies have provided sufficient information about the non-linear behavior of the stall flutter. The applicability of the method is demonstrated numerically.
2 MODEL OF ELASTIC SYSTEM

For the sake of concerning the basic mechanism of the complicated stall flutter and saving the needed computational time, the linear cascade of oscillating blades is considered as a two—dimensional, linear elastic system with spring and damper as shown in Fig. 1. The model of vibration consists of bending and torsion excited by the unsteady lift and moment respectively. Because of the difference of vibration phase between the neighboring blades in both bending and torsional vibration, the effects of elastic force, moment and damping produced by the difference of phase angle between the neighboring blades, must be considered. It embodies the functions of construction of the cascade, such as the part—span damper of blade etc.

As an example, for bending vibration, the dynamic equations of the linear elastic system can be written in dimensionless form as follows:

\[
\begin{align*}
D_i' &= \frac{D_i + 2D_L}{m} \\
C_i' &= \frac{C_i + 2C_L}{m} \\
C_i'' &= \frac{C_i + D_i}{m} \\
L_i &= \frac{L}{m}
\end{align*}
\]

\[
\begin{bmatrix}
\dot{\xi}_i \\
\dot{\eta}_i \\
\dot{\zeta}_i \\
\dot{\nu}_i
\end{bmatrix} =
\begin{bmatrix}
D_i' & -D_i'' & -D_i'' & -D_i'' \\
-C_i' & -C_i' & -C_i' & -C_i' \\
-C_i' & -C_i' & -C_i' & -C_i' \\
-C_i' & -C_i' & -C_i' & -C_i'
\end{bmatrix}
\begin{bmatrix}
\dot{\xi}_i \\
\dot{\eta}_i \\
\dot{\zeta}_i \\
\dot{\nu}_i
\end{bmatrix}
\]

In the Eq. (1), the \( p \) is the number of blades in the spatial period of propagating stall, which is related to the number of stall balls. For the torsional vibration, the form of the dynamic equation is similar to that for bending vibration, but the mass \( m \), force \( L \), displacement \( \xi \) will be replaced by the torsional moment of inertia \( J \), moment of force \( M \) and twist angle \( \theta \) respectively.

Here it should be noted that in the Eq. (1), the mass \( m \), elastic and damping coefficients \( C \) and \( D \) are written as the same value for every blade, it is just for simplicity in the presentation. If need be, the coefficients for each blade in the Eq. (1) also can be taken as the different and it does not affect the solution procedure.

3 UNSTEADY FLOW FIELD

As in the Refs.3—4, the unsteady flow past the cascade of oscillating blades with large separation is solved numerically by the Discrete Vortex Method.

The vortex dynamical equation is:

\[
\vec{D}_i \cdot \omega = \nabla \vec{\omega}
\]

The Discrete Vortex Method represents the vorticity field as the sum of large number of vortex blobs

\[
\omega = \sum \omega_k
\]

The nonpenetration condition on the moving bodies is:

\[
\vec{U} \cdot \vec{n} = \vec{\nu} \cdot \vec{n}
\]

The solution procedure was described in the References\(^3—4\) in detail. In the References\(^3—4\), a series of numerical test was carried out to simulate the propagating stall occurred in the cascade of oscillating blades. Even if the blades vibrate with the simple harmonic mode, the force acting on the blades still exhibits the non—linear characteristics due to the occurrence of the propagating stall. Therefore the results predict the complexity of the aeroelasticity problem.

In the References\(^3—4\), the mode, frequency, phase and amplitude of blade vibration must be specified at the outset of the computation. While, in this paper, they are not given and determined automatically by the coupling function between the excited aeroforce and the structural dynamics. At each computational step, not only the instantaneous flow field and the vorticity pattern, but also the excited unsteady force and moment of force acting on the blades can be obtained. On other hand, in the computational process, the blades are permitted to be in any motion which is not limited to the simple harmonic motion. The boundary conditions at the moving blades can be satisfied at each computational step, the method is very effective for the numerical simulation of the unsteady flow with the high Reynolds Number and large scale separation.

In the Reference\(^3—4\), the long time sequence of force acting on the blades was determined for the various parameters and the relevant spectral analysis was then obtained. From the spectral analysis, it is known that even if the blades vibrate in the simple harmonic motion, the force acting on the blades is not a harmonic oscillation. It illustrates the nonlinear phenomena, and reveals the complexity of the coupling function of the aeroelasticity.

4 AEROELASTICITY COUPLING

The model of the aeroelasticity coupling is given in the Fig. 2. It includes two parts: (1) the elastic system is excited to vibrate by the unsteady force and moment of force. The mode of vibration consists of the bending and torsion. (2) the numerical computation for the unsteady flow field with arbitrary motion of the blades. For the propagating stall occurrence in the engine, there are several stall balls. Therefore it is a reasonable hypothesis that a spatial period of stall consists of \( p \) blades. So, in the computational process, there are only \( p \) blades in the computational domain, and thus a vast amount of the needed computer time can be saved. We point out that this hypothesis is not the limitation of the present method, if necessary, the computational domain can consist of arbitrary number of blades, even full cascade.

As described above, the effect of the difference of the vibration phase between the neighboring blades has been considered in the present work. The different phase in the forces and moments of force acting on the neighboring blades certainly causes the different phases in the excited vibration. It leads to the relevant elastic and damping effects.

The two basic subprograms for the flow field and structure dynamics respectively are linked with a main program to construct a coupled computational procedure. At every time step, from the calculation of the unsteady flow field, we can obtain the unsteady force and moment which will excite the blades to vibrate. On other hand, in the computation of the structure dynamics, once the unsteady force and moment is determined, the displacement, velocity and acceleration of the blades can then be obtained. These data will be used to specify the boundary conditions for the calculation of the unsteady aerodynamics at next time step. This data exchange procedure is carried out successively step by step. It makes the extensive time—marching possible, and the non—linear problem is then solved in a discrete continuation fashion. Since this coupled procedure is implemented through the link of the two basic subprograms, thus a lot of needed computer time can be saved.
The ordinary differential equations (1) is usually solved by the Runge–Kutta Algorithm. But numerical tests indicate that GIL Algorithm shows better accuracy, and is then chosen to be employed in the present work. The main GIL Algorithm are given below:

\[ Y_{r, \alpha} = Y_r + \frac{1}{6} \left[ K^1 + (2 - \sqrt{2}) K^2 + (2 + \sqrt{2}) K^3 + K^4 \right] \]

(5)

Where

\[ K^1 = \Delta t \cdot f(t_r, y_r) \]
\[ K^2 = \Delta t \cdot f(t_r \frac{\Delta t}{2}, y_r + \frac{K^1}{2}) \]
\[ K^3 = \Delta t \cdot f(t_r + \Delta t, y_r - (\sqrt{2} - 1) \frac{K^0}{2} + (2 + \sqrt{2}) \frac{K^3}{2} ) \]
\[ K^4 = \Delta t \cdot f(t_r \frac{\Delta t}{2}, y_r + (\sqrt{2} - 1) \frac{K^0}{2} + (2 + \sqrt{2}) \frac{K^3}{2} + (1 + \frac{\sqrt{2}}{2} K^4) \]

Then equations (1) can be written as follows

\[ \dot{x} = -p(x) - q(x) \dot{x} + (L) \]

(6)

Let

\[ \dot{q} = x \]

(7)

\[ \{x \} = -p(x) - q(x) \dot{x} + (L) \]

(8)

The detailed solution procedure can be found in the relevant references.

6 NUMERICAL TESTING

The testing cascade consists of the double–circular–arc airfoils with 10° camber and 10% thickness, the stagger angle of the airfoil is 0°. As the incidence study presented in the Ref. [2], the evolution of unsteady separation in a cascade of oscillating blades is charted as incidence increases toward stall. For example, the propagating stall occurs when the incidence is larger than about 40°, and further more the incidence is increased to about 55°, the fully developed propagating stall is taken place obviously. In order to emphasize the occurrence of the propagating stall, the selected incidence is larger than 55° in the present calculations. The selected dimensions parameters are: mass of the blade \( m = 350 \), elasticity coefficient \( C = 420 \), and the damping coefficient \( D = 100 \). The dimensionless time interval of 0.025 between the two computational steps and the spatial interval of 0.04 between the two neighboring points of vortex creation are compatible to satisfy the CFL condition for the stability of numerical calculation.

The calculation is started from the initial state when the blades are stationary. For each case, a large number of computational steps, such as 5000 steps or 10000 steps, is carried out to understand the long term behavior of non–linear phenomena of stall flutter, and is also required for Fourier Analysis in the post–processing of the obtained data. The displacements of the blades and the lift and/or moment acting on the blades are recorded at each computational step. In the post–processing, the spectrum analysis is in progress with the Fast Fourier Transform (FFT). If need be, the patterns of streamline and distribution of the vortices also can be plotted at any time steps.

The numerical tests include: comparison between the cases with and without damping, incidence study, propagating stall and vibration responses etc. They are described below respectively:

A. Without Damping

At first, we studied the simplest case where the damper is absent for the purpose of understanding the sufficient development of vibration of the blades excited by the propagating stall. The displacements of the blades as function of the time are plotted in the Figs. 3a, 3b. The computational step size is selected as 0.025, thus 9600 steps have been taken to complete a simulation of total time 240. During the first two third of the time, as shown in Fig. 3a, the displacement oscillated and the amplitude did not increase even without damping. After that, the amplitude increases rapidly. This phenomena characterizes the non–linear behavior of the aerelasticity problem. The sequence of displacement of the neighboring blades is shown in Fig. 3b. We can find that there are obvious difference between these two neighboring blades in both the amplitude and phase of vibration.

For different time intervals and different blades, the spectra for the displacements of the blades are shown in the Figs. 4 respectively:

(a) The spectrum for the first half of the total time of the simulation.

(b) The spectrum for the second half time period.

(c) The spectrum for full time length.

The value of frequency for the vibration response is same as the value shown in Fig. 4b, but the peak value is less than that shown in Fig. 4b.

(d) The spectrum for displacement of the neighboring blade.

This plot is similar to the Fig. 4c, though the phases of displacement are different for the neighboring blades.

B. With Damping

(1) Displacement

Now we consider the damping effect in the calculation. The displacements of neighboring three blades during the long time period are shown in Figs. 5a, 5b, 5c respectively. Obviously, it is not the simple extension along with the increase in time. They express again the colorful nonlinear behaviors and can not replaced by the hypotheses of simply periodic oscillation. The amplitudes have been limited less than 1% of the chord of the airfoil due to the effect of the selected value of damping coefficient. In order to compare the phases of vibration, the displacements of these neighboring blades during the beginning period are enlarged in Figs. 6a, 6b and 6c respectively. The motions of all blades are started from the stationary state, but the phase differences between these blades occur gradually and automatically. The spectrum of displacement of the blade is shown in Fig. 7.

(2) Lift

Fig. 8 presents the spectral analysis of the unsteady force acting on the blades. Besides some other components there is an obviously highest peak of frequency 0.19. These excited forces are due to the occurrence of the propagating stall. The value of frequency 0.19 is agreed with the result presented in Ref. [3]. To compare Fig. 8 with Fig. 7, the frequency responses of displacement are consistent with the frequency responses of lift. On the spectrum of displacement, there is a peak at the nature frequency of the system, though this peak is not remarkable on the spectrum of lift.

(3) Incidence Study

The incidence study is carried out at a series of incidence: 55°, 60°,
60°, 63°, 65°, 67°, 68°, 70°, 71.5°. For each case, a large number of computational steps are performed, and the spectra of lift and displacement are obtained. For example, the spectra of displacement for incidence 60° and 65° are shown in Figs. 9 and 10 respectively. To synthesize these results, the response of frequencies and their peak values are plotted as the function of incidence in Fig. 11 and 12 respectively which indicate that the response frequency of highest peak is decreased as the incidence increases. In the Fig. 12, there is a peak at the incidence 60° that means the frequency of the propagating stall response is close to the nature frequency of the structure system.

7 CONCLUSION

Due to the complicacy of the stall flutter occurred in turbomachinery, the understanding of the non-linear characteristics of the aerelasticity problem has to be further improved, and thus is a demanding area which attracts many investigators in numerical, theoretical and experimental studies. The developed Discrete Vortex Method provides an useful tool to numerically simulate the two-dimensional, incompressible, unsteady flow with large scale separation at high Reynolds Number. The main purpose of the present study is to develop an numerical method to simulate the interaction between the unsteady flow field and structural dynamics. The resulting code consists of two basic subprograms: one describing the unsteady aerodynamics and the other one concerning the structural vibration. Both subprograms are utilized by a main program. As a progress of the present work, the mode, amplitude, frequency and interblade phase angle of vibration are not specified at the outset of the computation, but determined automatically in the computational procedure. A series of numerical tests confirmed our evidence that this method is suitable to the practical application, and the obtained results are reliable.

Numerical results reveal the strong non-linear behaviour of the stall flutter. As a matter of fact, the fully developed stall flutter is unstable, and the stable state can never be approached no matter whatever large number of computational steps have been performed. Recently the development of nonlinear science provides us new idea and point of view. On the other hand, the research on the aerelasticity problem is also a contribution to the development of the nonlinear science.

The parameters study helps us understanding of the effects of some parameters, for example, the incidence plays an important role in developing stall flutter. The value of incidence affects the occurrence of the stall flutter, and affects the frequency of the propagating stall response as well. Response frequency is decreased with the increase of the incidence.

Unfortunately, as we have seen, there are short of sufficient experimental data to check the obtained numerical results, since it is quite difficult to do the detailed measurement for the stall flutter.

The method still contains some deficiencies. The most egregious of these is the hypothesis of the spatial period of the propagating stall which must be specified at the outset of the computation. Otherwise, the computational domain will consist of the full cascade and it needs too much computational time.

The suggested method is also applicable to other aerelasticity problems, such as the wind vibration of high building and the flow induced vibration in the ocean engineering.

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REFERENCES

Where:

- \( n < m \) denotes structural coupling effect of blade No. \( n \) on blade No. \( m \), generally:
- \( n < m \) means \( m \) affects \( n \)
- \( AC \) denotes active controller.
- \( P_s(s,t) \) denotes pressure on \( n \)-th blade as a function of surface coordinate \( s \) and time \( t \).
- \( h \), \( \dot{h} \) denote bending and torsional displacements respectively.
- \( \dot{h} \), \( \ddot{h} \) denote bending and torsional velocities respectively.

Fig. 3a Displacement of the Blade (without damping)

Fig. 3b Displacement of the Neighboring Blade (Without damping)

Fig. 4a Spectrum of Displacement (655 - 4800 step)

Fig. 4b Spectrum of Displacement (4820 - 9600 step)
Fig. 4c Spectrum of Displacement (590–9600 step)

Fig. 4d Spectrum of Displacement for Neighboring Blade

Fig. 5a Displacement of the 1st Blade (with damping)

Fig. 5b Displacement of 2nd Blade (with damping)

Fig. 5c Displacement of 3rd Blade (with damping)

Fig. 5a Displacement of the 1st Blade (enlarged)

Fig. 6a Displacement of 1st Blade (enlarged)
Fig. 11: Variation of Frequency of Response With Incidence.

Fig. 12: Variation of Peak Value With Incidence.