A SIMPLIFIED SCHEME FOR SCHEDULING MULTIVARIABLE CONTROLLERS AND ITS APPLICATION TO A TURBOFAN ENGINE

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ABSTRACT
A simplified scheme for scheduling multivariable controllers for robust performance over a wide range of plant operating points is presented. The approach consists of scheduling only the output matrix of a dynamic controller, thus significantly reducing the number of parameters to be scheduled. Given a robust controller at a nominal design point, designed such that it gives a stable closed-loop system at various off-design operating points, the parameters of the controller output matrix are optimized such that the closed-loop response at the off-design points closely matches the design point closed-loop response. The optimization problem formulation for the synthesis of controller scheduling gains is discussed. Results are presented for controller scheduling for a turbofan engine for a conceptual Short Take-Off and Vertical Landing aircraft. The simplified controller scheduling is shown to provide satisfactory response for engine models corresponding to significant gross thrust variations from the nominal design point.

INTRODUCTION
The traditional approach to designing feedback control laws for complex nonlinear systems, such as high performance aircraft and aircraft engines, is to design linear control laws at various operating points of the system and then "gain schedule" the control laws through some kind of curve fitting of the various controller gains with the critical operating point variables as the independent scheduling parameters. This engineering approach has been successfully applied to currently operating aircraft and engines with control laws based on classical single-input / single-output (SISO) techniques. Many studies have been done applying similar gain scheduling approaches with multi-input / multi-output control design techniques, see for example Kapasouris et al. (1984) and Polley et al. (1988) on full-envelope engine control designs. However, none of these or similar aircraft control design studies have led to implementation of such multivariable control laws on an operational vehicle. One of the difficulties with implementing the gain scheduled multivariable control laws is the complexity of such control laws. For a controller of order $n$ with $r$ inputs and $m$ outputs, there will be $n(1+m+r)+mr$ controller parameters to schedule even with the controller represented in a minimal parameter state-space form as in Ly et al. (1985). More recently, multivariable control design techniques have been developed that either attempt to design a robust global controller which can operate over a wide range of plant operation without gain scheduling, see for instance Perez and Nwokah (1991), or directly synthesize a gain-scheduled controller using multiple models of the plant in the control design (Ostroff, 1992 and Reichert, 1992). The difficulty with both these approaches is that it is not apparent how to change the controller gains or the scheduling gains for any given operating point if the performance of the global or directly gain-scheduled controller is found to be unsatisfactory during implementation on the real plant.

The fundamental objective in using modern robust multivariable control design techniques is to reduce the controller complexity while guaranteeing the desired performance and stability robustness characteristics. A simplified controller scheduling scheme is presented in this paper that consists of scheduling only the output matrix of a nominal controller which is designed to give a robustly stable closed-loop system over a wide range of plant operating conditions. This scheduling scheme is of the form

$$K(s) = K_f K^s(s)$$

where $K(s)$ is the scheduled controller, $K_f$ is the controller output
scheduling matrix and $K'(s)$ is the nominal controller. The symbol $(s)$ represents the Laplace Operator. This idea of controller scheduling is shown in the block diagram of Fig. 1. Note that such a controller scheduling will reduce the number of design points for which the multivariable control synthesis has to be performed and will also significantly reduce the number of controller parameters that need to be scheduled. For a controller with $m$ outputs, the number of parameters to be scheduled will be at most $m^2$ considering $K_s$ to be a full matrix. The basic approach to be presented in this paper consists of applying parameter optimization techniques to synthesize the scheduling gains $K_s$ for the off-design control loop such that the transfer function matrix with the loop broken at the controlled outputs $(ii)$ in Fig. 1(b)) "closely" matches the output loop transfer matrix of the nominal control loop. The control selector ideas developed in Shaw et al. (1988) are somewhat similar in concept to the controller scheduling scheme of Eq. (1). However, the control selector idea accounts for changes only in the plant control effectiveness matrix, and the parameters of the multivariable controller also have to be scheduled to account for changes in the plant dynamics.

Note that although the proposed scheduling scheme is much simpler than scheduling all the parameters of the controller, this simplicity comes at the expense of reducing the scheduling degrees-of-freedom. The controller eigenvalues are not affected by the proposed scheduling scheme and it is possible that this simplified scheduling scheme might not provide adequate performance for significant variations in plant poles and zeros. The control designer has to trade-off the scheduling simplicity versus the performance requirements, and the suggested approach is to do a multiple set of "nominal" control designs for significantly varying operating conditions and apply the simplified scheduling scheme in a "large neighborhood" of these "nominal" designs.

In the following, the parameter optimization formulation for the synthesis of the controller scheduling gains is first discussed, followed by the application of the controller scheduling scheme to linear models of a multi-nozzle turbofan engine designed for operation in a conceptual Short Take-Off and Vertical Landing (STOVL) aircraft.

### CONTROLLER SCHEDULING DESIGN METHODOLOGY

For a control loop of the form shown in Fig. 1, the results of Doyle and Stein (1981) have shown that the loop transfer function matrix provides a good measure of the closed-loop system performance and robustness characteristics. Based on these results, most of the multivariable control design techniques emphasize shaping of the loop transfer function matrix with the loop broken at the plant output or the plant input, $G^*(s)K'(s)$ or $K'(s)G^*(s)$ respectively from Fig. 1(a). For the purposes of this paper, the controller scheduling will be discussed with respect to loop shaping at the plant output. The approach to be presented can be applied to the loop at the plant input also.

Many papers have been published on testing for robustness to plant variations for control loops of the type in Fig. 1(a). Ref. [9] lists the robustness tests for different structures for the plant modelling error. Although these robustness tests are developed assuming that the controller does not change, these tests are based on variations in the loop transfer functions and can hence be applied to the case of control loop of Fig. 1(b) where both the controller and the plant are different from those for a nominal control loop. Applying Theorem 3 of Lehtomaki et al. (1981), the closed-loop stability of the scheduled off-design control loop of Fig. 1(b) is guaranteed if the characteristic polynomial for the off-design loop transfer function matrix $G(s)K'(s)$ has the same number of right half plane zeros as the characteristic polynomial for the nominal loop transfer function matrix $G^*(s)K'(s)$, and the off-design loop transfer function matrix satisfies the following condition

$$
\sigma(g(i\omega)K_sK'(i\omega) - G^*(i\omega)K^*(i\omega)) < \sigma[I + G^*(i\omega)K^*(i\omega)] \forall \omega
$$

In condition (2), $\sigma[\cdot]$ denotes the maximum singular value of $[\cdot]$, $\sigma[\cdot]$ denotes the minimum singular value of $[\cdot]$, and $I$ is an appropriately dimensioned identity matrix. Defining the loop transfer function difference as $E_{\omega}(j\omega) = G(j\omega)K_sK'(j\omega) - G^*(j\omega)K^*(j\omega)$, the robust stability condition (2) can be rewritten as

$$
\left| \frac{\sigma(E_{\omega}(j\omega))}{\sigma[I + G^*(j\omega)K^*(j\omega)]} \right| < 1
$$

where $M_{\omega}$ denotes the maximum value of $H(j\omega)$ over all $\omega$. A heuristic extension of the robust stability condition is that if the normalized loop transfer difference maximum singular value, as on the left side of condition (3), is << 1, then the off-design control loop performance and robustness characteristics will be similar to those for the nominal control loop. This heuristic extension suggests that an approach to synthesizing the controller scheduling gain matrix is to select $K_s$ to minimize the normalized loop transfer difference maximum singular value, i.e.

$$
\min_{K_s} \frac{\sigma(E_{\omega}(j\omega))}{\sigma[I + G^*(j\omega)K^*(j\omega)]}
$$

where

$$
E_{\omega}(j\omega) = \frac{\sigma(E_{\omega}(j\omega))}{\sigma[I + G^*(j\omega)K^*(j\omega)]}
$$

The normalized loop transfer difference maximum singular value, $E_{\omega}(j\omega)$, is a complex nonlinear function of the scheduling gains.
CONTROLLER SCHEDULING DESIGN EXAMPLE

The turbofan engine considered in this paper is a modified version of an existing engine for powering a conceptual Short Take-Off and Vertical Landing (STOVL) aircraft. The two-spool turbofan engine provides thrust through three nozzles - ejectors on the aircraft wing to provide propulsion lift at low speeds and hover; a 2D-CD (two-dimensional convergent-divergent) aft nozzle; and a ventral nozzle for aircraft pitch control and lift augmentation at low speeds. The flow to the ejectors is controlled via a butterfly valve and mixed flow is used for all three thrust ports. Details of the propulsion system are available in Adibhatla et al. (1994). Linear small perturbation models of the engine were generated from a real-time Component Level Model simulation. The linear models have the form:

\[ \dot{x} = Ax + Bu \quad ; \quad z = Cx + Du \]  

where the state vector is

\[ x = [N2, N25, Tmhpc, Tmpc, Tmhpt, Tmlpt]^T \]  

with

- \( N2 \): Engine fan speed, rpm
- \( N25 \): High pressure compressor speed, rpm
- \( Tmhpc \): High pressure compressor metal temperature, °R
- \( Tmpc \): Burner metal temperature, °R
- \( Tmhpt \): High pressure turbine metal temperature, °R
- \( Tmlpt \): Low pressure turbine metal temperature, °R

The control inputs are

\[ u = [WF, A8, ETA, A79]^T \]  

with

- \( WF \): Fuel flow rate, lbm/h
- \( A8 \): Aft nozzle area, in\(^2\)
- \( ETA \): Ejector butterfly angle, deg
- \( A78 \): Ventral nozzle area, in\(^2\)

The controlled outputs are

\[ z = [FG9, FGE, FGV, N2]^T \]  

with

- \( FG9 \): Aft nozzle gross thrust, lbf
- \( FGE \): Ejector gross thrust, lbf
- \( FGV \): Ventral nozzle gross thrust, lbf

and \( N2 \) as defined earlier.

With the controlled outputs \( z \) defined as in (10), the control objective is to provide decoupled command tracking of the gross thrust from the three exhaust ports and the engine operating point defined by the fan speed. The engine control design to be considered for this study was part of an integrated flight propulsion control design for operation of the STOVL aircraft in
transition flight phase. The transition flight regime is the low speed region where the control of the aircraft is transitioning from aerodynamically generated forces to propulsive forces. Three linear models of the engine were generated corresponding to aircraft trim speeds of 60, 80 and 100 Knots and PLA settings of 79, 75 and 66 respectively. Note that apart from PLA variations, the three models correspond to different trim settings for the three thrust ports with propulsive lift supporting 30% of the aircraft weight at the 100 Knot trim condition and 80% of the aircraft weight at the 60 Knot trim condition. For the rest of the discussion in this paper, the aircraft trim speeds are used as the reference for the three engine models.

One of the measures of model variations that is often used in literature on robust control is the multiplicative error, \(L(s)\), defined by

\[
G(s) = [I + L(s)]G^0(s) \tag{11}
\]

With \(G^0(s)\) and \(G(s)\) known, (11) can be rewritten as

\[
L(s) = [G(s) - G^0(s)][G^0(s)]^{-1} \tag{12}
\]

With the 80 Knot model as the nominal model \(G^0(s)\), the maximum singular value of the multiplicative error, \(\sigma[L(j\omega)]\), for the 60 and 100 Knot models is shown in Fig. 2. The fact that \(\sigma[L(j\omega)] > 1\) for both the 60 and 100 Knot models across the frequency range of interest indicates that there are significant variations between these and the nominal design model. Shown in Fig. 3 is the response of the 60, 80 and 100 Knot models to a step fuel flow \((WF)\) input of 1000 lbm/h. We note significant variations in the engine response for the three operating conditions. The variation in the basic engine rotor dynamics is apparent from the different rise time for the engine fan speed \((N2)\) and the speed of response for the three thrusts. The variations in control effectiveness due to different trim conditions is also apparent from the thrust responses. For the low speed 60 Knot model, the aft nozzle is mostly blocked, i.e., the trim aft nozzle area \(A8\) is close to zero, so fuel flow has little effect on aft thrust \(FG9\). For the higher speed 100 Knot model, the ventral nozzle is mostly blocked, i.e., the trim ventral nozzle area \(A78\) is close to zero, so fuel flow has much less effect on ventral thrust \(FGV\) for the 100 Knot model as compared to the two other models.

A robust controller was designed for the 80 Knot design model such that it provided the desired command tracking performance with the 80 Knot model and provided stable closed-loop system with the other two models. The nominal controller was obtained by partitioning a centralized integrated flight propulsion control design as discussed in Garg (1993). The nominal controller did not provide adequate performance for the 60 Knot and 100 Knot engine models, so the controller scheduling optimization was applied for these models. For both the off-design engine models, the frequency range was selected to be \(0.1\) to \(100\) rad/s with 20 points per decade resulting in \(N = 81\), and the desired minimum value for the normalized loop transfer difference singular value was chosen to be \(\sigma_0 = 0.1\). The controller scheduling matrix \(K_s\) was chosen to be the full 4x4 matrix resulting in 16 parameters to be optimized. Using the numerical optimization algorithm of Anon. (1989), convergence to an optimal value for the scheduling gains was obtained within 4 major iterations for both the off-design models. In both cases, the optimal cost was non-zero indicating that the normalized loop difference maximum singular value, \(\tilde{E}_{\text{ap}}(s)\), will exceed the desired minimum value of 0.1 over portions of the frequency region of interest. Figure 4 shows a comparison of \(\tilde{E}_{\text{ap}}(s)\) for the 60 Knot model with the nominal controller and with the optimized gain scheduling matrix \(K_s\). Fig. 5 shows the same comparison for the 100 Knot model. In both cases the simplified controller scheduling significantly decreased the difference between the off-design and the nominal loop transfer function. Although from Figs. 4 and 5, the stability condition (3) is not satisfied for the 60 and 100 Knot models with the nominal controller, it is important to note that condition (3) is only a sufficient condition. With the optimized scheduling gains, \(\tilde{E}_{\text{ap}}(s) < 0.4\) for the 60 Knot model, and \(\tilde{E}_{\text{ap}}(s) < 0.8\) for the 100 Knot model, thus ensuring stability of the closed-loop system with the optimized scheduling gains and also improved matching with the nominal loop transfer function.

Closed-loop tracking performance with the gain scheduled controllers was evaluated for both the off-design engine models. For all four commanded variables, the tracking performance was much improved with the controller scheduling. Fig. 6 shows a comparison of the closed-loop responses for a step aft gross thrust command, \(FG9_c\), for the nominal 80 Knot model, 60 Knot model with nominal controller and 60 Knot model with the optimized scheduling gains. The variables shown correspond to perturbations from the trim conditions. As discussed earlier, for the 60 Knot model the trim requires very low thrust from the aft nozzle, so the trim aft nozzle area, \(A8\), is significantly lower than that for the nominal 80 Knot model. Thus the response of the 60 Knot model with the nominal controller for \(FG9\) is very sluggish as seen from Fig. 6. The controller gain scheduling restores the \(FG9\) command tracking response to closely match the nominal system response while also maintaining the desired decoupling with the other responses.

Fig. 7 shows a comparison of the closed-loop responses for a step ventral gross thrust command, \(FGV_c\), for the nominal 80 Knot model, 100 Knot model with nominal controller and the 100 Knot model with the optimized scheduling gains. Again as discussed earlier, for the 100 Knot model the trim requires very low thrust from the ventral nozzle, so the trim ventral nozzle area, \(A78\), is significantly lower than that for the nominal 80 Knot model. Thus the response of the 100 Knot model with the nominal controller for \(FGV\) is very sluggish as seen from Fig. 7. The controller gain scheduling restores the \(FGV\) command
tracking response to closely match the nominal system response while also maintaining the desired decoupling with the other responses.

The example results shown in Fig. 6 and 7 demonstrate the feasibility of using the simplified scheduling scheme of Fig. 1 with robust nominal control designs. The numerical data for these examples is available by contacting the author. The scheduling designs discussed above were implemented in a nonlinear integrated flight propulsion control (IFPC) design for the STOVL aircraft as discussed in Garg and Mattern (1994). In the IFPC design, the flight controller was also scheduled using the simplified controller scheduling scheme over the transition flight envelope from 120 Knots to 60 Knots. The IFPC design with the simplified controller scheduling was successfully "flying" by pilots in a fixed-base piloted simulation as reported in Bright et al. (1994). This simplified scheduling scheme has also been successfully applied recently to an advanced concept General Electric turbofan engine as part of a technology familiarization study (Frederick and Adibhatla, 1996). The results presented in this study demonstrate that satisfactory performance can be achieved over a wide range of engine power codes by using the simplified scheduling scheme.

CONCLUSION
A simplified controller scheduling scheme that exploits the robustness of a nominal multivariable control design and requires scheduling of only \( m^2 \) parameters, where \( m \) is the number of controller outputs, was presented. The scheduling scheme is based on optimizing the \( m^2 \) parameters to match the off-design control loop transfer matrix with the nominal loop. The cost function to be minimized is based on heuristic extensions of well known stability robustness conditions. The feasibility of the scheduling scheme was demonstrated for a turbofan engine control design example wherein a nominal controller with the simplified scheduling scheme provided satisfactory closed-loop performance over a wide range of operating points. The simplified scheduling scheme has been applied to an integrated flight propulsion control design for a Short Take off and Vertical Landing aircraft and successfully demonstrated in fixed-base piloted simulations.

REFERENCES
Figure 1 Block Diagram Showing the Simplified Scheduling

(a) Nominal Control Loop

(b) Off-Design Control Loop

Figure 2 Maximum Singular Value of Multiplicative Error, $\hat{\sigma}([L(\omega)])$, for 60 Knot and 100 Knot Engine Models

Figure 3 Response of Open-Loop Engine Models to Step Fuel Flow Input, $WF = 1000$ lbm/h
Figure 4 Normalized Loop Difference Maximum Singular Value, $\tilde{E}_d(\omega)$, with Nominal and Gain Scheduled Controllers, $K_0(s)$ and $K_gK_0(s)$, for 60 Knot Model.

Figure 5 Normalized Loop Difference Maximum Singular Value, $\tilde{E}_d(\omega)$, with Nominal and Gain Scheduled Controllers, $K_0(s)$ and $K_gK_0(s)$, for 100 Knot Model.

Figure 6 Closed-Loop Response to Step $FG_9 = 100$ lbf - Nominal 80 Knot Model, 60 Knot Model with Nominal Controller, and 60 Knot Model with Gain Scheduling.
Figure 7  Closed-Loop Response to Step $FGV_c = 100$ lbf - Nominal 80 Knot Model, 100 Knot Model with Nominal Controller, and 100 Knot Model with Gain Scheduling