Improving Dynamic Response of a Single-Spool Gas Turbine Engine Using a Nonlinear Controller

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Abstract: This paper describes the design of a closed-loop nonlinear controller to improve the dynamic response of a single-spool gas turbine engine. The nonlinear controller is obtained by scheduling the gains of multivariable compensators as a function of engine non-dimensional shaft speed. The compensators, whose outputs are fuel flow and nozzle area, are designed using optimal control theory based on a set of linear models generated from a nonlinear engine simulation. Investigations are also made into developing simple algorithms to obtain an analytical expression for the compressor given its characteristic. The detailed process of developing a nonlinear simulation model for the engine is also described. The open-loop fuel controller is studied using the digital simulation.

1 INTRODUCTION

The control designers of modern aircraft gas turbine engines use a variety of sensors and actuators in transient and steady-state operations. Classical designs have relied on hydro-mechanical fuel controls, i.e., a speed governor for gross transient response with more sophisticated control action used in steady-state regulation and trim. The use of a multivariable control system in gas turbine engines allows an integrated control action to meet steady-state and transient performance requirements. It is thus possible to apply optimal control theory to the design of a closed-loop nonlinear control system intended for operation over the entire power range. The advantages of this control system are DeHoff and Hall (1976):

- Enhanced performance from cross-coupled controls
- Maximum use of engine variable geometry
- A systematic design procedure that can be applied efficiently.

2 MODELING APPROACH

The method of modeling that allows greater insight into the dynamic behavior of a gas turbine engine is based upon the characteristics for each of the engine components. A gas turbine engine comprises a number of components such as compressors and turbines, and the behavior of each of these is well understood. The interactions between the components are fixed by the physical layout of the engine. Thus, for a given engine, if all component characteristics and the engine layout are known, then the gas turbine engine is precisely defined and its dynamic behavior can be expressed mathematically (Fawke and Saravanamuttoo, 1971).

2.1 MODELING THE COMPRESSOR

The performance of the compressor in a gas turbine engine is very important, especially during transients. In the past, the steady-state characteristics have been used to model this component and computer simulation therefore involves calculating the mass flow or pressure ratio by various interpolation techniques. It is desirable to develop algorithms to enable an analytical expression to be obtained given the compressor characteristics. Unfortunately, relative little work has been done in this regard. In this section, two simple algorithms are proposed. The first is specific to the present application, and the other is useful for theoretical modeling.

Presented at the International Gas Turbine and Aeroengine Congress and Exposition Cologne, Germany June 1-4, 1992

NOMENCLATURE

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<td>cₚ</td>
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SUBSCRIPT

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<tr>
<td>ss</td>
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2.1.1 USING THE POLYNOMIAL CURVE FITTING

The direct approach is to fit a polynomial to the given characteristic. Examining the shape of a typical compressor characteristic, it is clear that a second order polynomial will be adequate to describe this set of curves. Suppose the pressure ratio can be related to the non-dimensional mass flow as:

\[ z = a_1 x^2 + a_2 x + a_3 \]  

(1)

and the coefficients \( a_i \), \( i = 1, 2, 3 \) are further assumed to be a second order polynomial of non-dimensional speed:

\[ a_i = b_{i1} y^2 + b_{i2} y + b_{i3} \]  

(2)

Substituting eqn. 2 into eqn. 1, a general nonlinear polynomial, relating pressure ratio to non-dimensional mass flow and shaft speed, can be expressed as:

\[ z = c_1 x^2 y^2 + c_2 x^2 y + c_3 x^2 + c_4 x y + c_5 x + c_6 y + c_7 \]  

(3)

where \( x \) stands for non-dimensional mass flow, \( y \) for non-dimensional shaft speed, and \( z \) for pressure ratio. The coefficients \( c_i \) in the equation 3 are obtained from \( a_i \) and \( b_i \). By using the nonlinear least-squares fitting method, the coefficients \( c_i \) can be determined. The fitting result is shown in Fig.1.

2.1.2 USING AN ODD GENERIC POLYNOMIAL

An odd-polynomial compressor characteristic is defined by the function Harris and Spang (1991);

\[ f(x) = \frac{2n^3 + 1}{2n^3} x - \frac{1}{2n^2} x^{2n+1} \]  

(4)

where \( x \) and \( n \) are the scaled non-dimensional mass flow and speed respectively. All such functions have the same value at \( x = 1 \), as shown in Fig.2.

This unique feature of the equation 4 aids in obtaining the compressor characteristic by dynamic scaling. At each non-dimensional speed, the values of pressure ratio and non-dimensional mass flow are scaled from the reference surge values. The reference scaling value (the surge point) changes as the compressor speed is varied.

The resultant map is shown in Fig.3. There is no significant physical meaning for the region, where at a speed, the non-dimensional mass flow is less than the surge line value.

2.2 THERMODYNAMIC MODEL OF THE ENGINE

Using the above approach to modeling the compressor (Fig.1), a nonlinear simulation model could be developed for the engine (Fig.4).

The modeling procedure using the intercomponent volume method Ewker and Nett (1991) is described in this section.

Component A is the inlet. In this component, it is assumed that flow is adiabatic. Using an isentropic efficiency to account for the pressure loss, the conditions at the inlet to the compressor can be calculated using the following equations:

\[ T_{01} = T_{am}(1 + \frac{7 + 1}{2} Ma^2) \]  

(5)

\[ T_{101} = T_{am} + \eta_{is}(T_{01} - T_{am}) \]  

(6)

\[ P_{01} = P_{am} \frac{T_{01}^{1.4}}{T_{am}^{1.4}} \]  

(7)

Component B is the compressor. It is modeled empirically using the fitted polynomial, which gives the non-dimensional mass flow across the compressor:

\[ \frac{n_t \sqrt{T_{01}}}{P_{01}} = F_n(N, P_{02}, P_{01}, T_{01}) \]  

(8)

The total temperature rise is found by using the isentropic efficiency factor \( \eta_t \) in the following manner:

\[ T_{02} = T_{01}(1 + \frac{1}{\eta_t}((P_{02}^{1.4}/P_{01}^{1.4}) - 1)) \]  

(9)

Figure 1: CHARACTERISTIC FITTING USING THE POLYNOMINAL

Figure 2: AN ODD POLYNOMIAL CURVE

Figure 3: THE CHARACTERISTIC GENERATED USING ODD-POLYNOMIAL

Figure 4: SINGLE-SPOOL ENGINE
Component C is a volume representing the volume of the compressor plus combustor. Here it is assumed that the flow is adiabatic and the flow properties were spatially uniform. The discretized continuity equation combined with the isentropic relation results in:

\[
\frac{d}{dt} P_03 = \frac{\gamma R T_02}{V_3} (m_u - m_v - m_n + m_f) \tag{10}
\]

Component D is the compressor bleed valve. In this engine, a bleed valve at compressor exit has been introduced. It is modeled with an empirical orifice relation which gives:

\[
m_b = A_1 \sqrt{\frac{P_{out}}{R T_{out}} (P_03 - P_{in})} \tag{11}
\]

Component E is the combustor. In the combustor, the total pressure loss is assumed to be a fixed percentage of its inlet pressure \( P_{in} \), which gives

\[
P_03 = k_{it} P_02 \tag{12}
\]

The total temperature rise is given by the steady-state energy equation in the combustor:

\[
T_03 = T_02 + \frac{\eta_f m_f Q_f}{m_u (m_u + m_f - m_v c_p)} \tag{13}
\]

Component F is the turbine. It is modeled empirically with the steady-state turbine performance map, which gives the nondimensional mass flow across the turbine, thus:

\[
m_t = \frac{P_{out}}{\sqrt{T_{out}}} \frac{P_03}{P_04} \tag{14}
\]

The total temperature drop is found by using the isentropic efficiency factor \( \eta_T \) in the following manner:

\[
T_04 = T_03 (1 - \eta_T (\frac{P_{out}}{P_03})^{1/\gamma - 1} - 1) \tag{15}
\]

Component G is the volume of the turbine plus nozzle. As in the case of the volume C, the exit total pressure \( P_04 \) is modeled using the dynamic equation as:

\[
\frac{d}{dt} P_04 = \frac{2 R T_04}{V_4} (m_t - m_u) \tag{16}
\]

Component H is the propulsion nozzle. Here it is assumed that the flow is adiabatic. By using an isentropic efficiency factor to account for flow loss, the algebraic continuity equation can be put into the form:

\[
\frac{m_u \sqrt{T_04}}{P_4} = \frac{u_s}{\sqrt{T_04}} \frac{A_p P_{crit} T_{crit}}{R P_04} \frac{T_{crit}}{T_04} \tag{17}
\]

where

\[
\frac{u_s}{\sqrt{T_04}} = \sqrt{2 c_p m_0 (1 - \frac{P}{P_04})^{\gamma - 1/\gamma}} \tag{18}
\]

and

\[
T_{crit} = 1 - \eta_T (1 - (\frac{P}{P_04})^{1/\gamma - 1}) \tag{19}
\]

The preceding equations hold only if the nozzle is not choked. If the nozzle is choked, i.e. \( P_04 < P_03 \), then:

\[
\frac{m_u \sqrt{T_{crit}}}{P_{crit}} = \frac{u_s}{\sqrt{T_{crit}}} \frac{A_p P_{crit} T_{crit}}{R P_{crit} T_{crit}} \tag{20}
\]

where

\[
\frac{P_{crit}}{P_04} = (1 - \frac{1}{\eta_T} \frac{\gamma - 1}{\gamma + 1})^{1/\gamma - 1} \tag{21}
\]

and

\[
\frac{T_{crit}}{T_04} = \frac{2}{\gamma + 1} \tag{22}
\]

and

\[
\frac{u_s}{\sqrt{T_04}} = \sqrt{2 c_p} \tag{23}
\]

Using the momentum equation applied to the entire engine and ignoring the unsteady terms which are negligible compared to the momentum flux term, an estimate of thrust of the engine is calculated:

\[
F = n_t u_s - n_u u_0 + A_p (P_3 - P_{in}) \tag{24}
\]

Component I is the compressor/turbine spool. This dynamic equation comes from a power balance on the spool which gives:

\[
J N \frac{d}{dt} N = W_t - W_e \tag{25}
\]

where \( W_t = m_t c_p (T_03 - T_01) \) and \( W_e = m_u c_p (T_{04} - T_01) \). The governing equation is therefore

\[
J N \frac{d}{dt} N = m_t c_p (T_03 - T_04) - m_u c_p (T_02 - T_01) \tag{26}
\]

An information flow diagram for the model described above is given in Fig.5, which shows where the parameters and inputs enter into the model structure.

Figure 5: INFORMATION FLOW DIAGRAM FOR SINGLE-SPool ENGINE

3 OPEN-LOOP FUEL CONTROLLER

When using the open-loop fuel controller, it has been a common practice to meter fuel flow to the engine as a function of compressor pressure ratio. In this method, as the fuel at a fixed compressor pressure ratio will change with the engine inlet condition. As a result, acceleration rate will vary as a function of the inlet condition.

This could be avoided by scheduling fuel non-dimensionally during transients. Therefore, the open-loop fuel control group usually incorporates the inlet pressure and temperature, the nondimensional fuel flow group becomes \( m_f / (P_1 \sqrt{T_1}) \).

Using such a group will involve the measurement of inlet temperature which is generally slow in response compared with the measurements of speed and pressure. A alternative non-dimensional parameter could be obtained by dividing the group above by non-dimensional speed \( N/\sqrt{T_1} \), which gives

\[
\frac{m_f}{P_1 \sqrt{T_1}} \frac{N}{\sqrt{T_1}} = \frac{m_f}{P_1 N} \tag{27}
\]

In the common open-loop controller, using the fuel group of this form, only shaft speed and inlet pressure need to be measured, and is made a function of the compressor pressure ratio.

4 GENERATING LINEAR MODELS

Before linearizing the nonlinear model, it is necessary to normalize the nonlinear engine model. The nonlinear model includes the physical units of the performance variables and scaling of their physical units is necessary to make their interactions easier to compare and understand. For this single-spool engine, the physical maximum limits of the variables, shown in table 1, were used as scaling values.
state condition, i.e., constant input \( u'' \) producing constant state \( x'' \) and constant output \( y'' \), then the combination \( (u'', x'', y'') \) satisfies:

\[
0 = F(x'', u'') \\
\dot{x} = F(x'', u'') \\
y = G(x'', u'')
\]

(29)

The point \( (u'', x'', y'') \) is termed an equilibrium operating point of the gas turbine engine. Perturbing the control input with \( \delta u \) results in state and output perturbation \( \delta x \) and \( \delta y \), respectively and control input, state and output become \( u = u'' + \delta u, x = x'' + \delta x \) and \( y = y'' + \delta y \), and the equation 29 follows:

\[
\dot{x} + \delta \dot{x} = F(x'' + \delta x, u'' + \delta u) \\
\delta y = G(x'' + \delta x, u'' + \delta u)
\]

(30)

Because of the continuity requirements imposed on the functions of \( F \) and \( G \), equation 31 can be expanded in a Taylor series about the point \( (u'' = u'', x'' = x'') \), ignoring the higher order terms and noting \( i'' = 0 \), \( y'' = 0 \), we obtain:

\[
\dot{x} = Ax'' + Bu'' \\
\delta y = Cx'' + Du''
\]

(31)

The constant matrices \( A, B, C, D \) have the dimensions of \( n \times n, n \times m, r \times n, r \times m \) respectively and are given by:

\[
A = \frac{\partial F_i}{\partial x_j} \quad B = \frac{\partial F_i}{\partial u_j} \quad C = \frac{\partial G_i}{\partial x_j} \quad D = \frac{\partial G_i}{\partial u_j} \quad i = 1, 2, ..., n \quad j = 1, 2, ..., r
\]

(32)

This equation approximates the dynamic behavior of the nonlinear gas turbine engine in a small region about the operating point \( (u = u'', x = x'') \).

**Remark 1:** Linear models of a gas turbine engine can be generated in many ways, e.g., numerically from the nonlinear simulation, or directly from engine test data. A common method is to utilize a nonlinear hybrid or digital engine simulation. This procedure is computationally efficient; but often, linearized behavior is dependent on the perturbation size.

**Remark 2:** Linear models generated numerically do not contain the most convenient parameterization of dynamics and sometimes contain far too complex a description to be practically utilized for control design. It is likely that a design model including engine, actuator and sensor dynamics could be a high order system. This case, model reduction will be the first important step to establish an effectively simple model which includes only elements important to the desired control function.

### 4.2 COMPARING LINEAR, NONLINEAR MODEL

After nine linear models have been obtained, it is necessary to compare their performance with those of the nonlinear model. Time-domain simulation has been conducted in which linear and nonlinear model are subject to a series of the equal step change in fuel flow. The results for operating point 1 are shown in Fig.18. It is shown that linear models have approximately same dynamics as the nonlinear model at the most of the operating point.

### 5 NONLINEAR CONTROLLER DESIGN

The nonlinear control synthesis procedure derived here is based on linearizing the engine nonlinear model at a set of closely-spaced steady-state operating points and applying the linear optimization method to the linear models. The nonlinear control problem is thereby reduced to a series of linear control problems. This permits the use of established analytical and numerical methods associated with linear optimal control theory. At each operating point, an optimal linear feedback controller is generated by minimizing a quadratic performance criterion. Weighting factors within each performance criterion enable the control designer to satisfy performance specifications by trading off system response against control actuation rate. Nonlinear feedback control is then constructed by combining the series of linear controllers into a single nonlinear controller whose feedback gains vary with states.

### 5.1 CHOICE OF CONTROLS AND OUTPUTS

The nonlinear model for this engine have four states. In order to improve the dynamic response, the final nozzle area is used as control input as well as fuel flow. In the present design, fuel flow is used to control the engine shaft speed and nozzle area control jet pipe pressure, which will indirectly control the engine thrust.

### 5.2 LINEAR QUADRATIC REGULATOR(LQG)

Given a linearised model corresponding to a engine operating point, the standard linear optimal design technique can be used to determine full state feedback gains by minimizing a quadratic performance index Bryson and Ho (1969):

\[
J = \frac{1}{2} \int_0^\infty (\delta x^T Q \delta x + \delta u^T R \delta u) dt
\]

(33)

and solving the corresponding Riccati equation;

\[
A^T P + PA + PBR^{-1}B^T P - Q = 0
\]

(34)

The state feedback gains are;

\[
\delta u = -R^{-1}B^T P \delta x
\]

(35)

To satisfy the steady state performance, the control actuation rates are introduced by augmenting the linearised model with \( m \) integrators. Fig.6 depicts the resultant structure. It should be noted that \( \delta \) has been omitted in the Figures.

![Linearised Model Augmented by M Integrators](image1)

**Figure 6: Linearised Model Augmented by M Integrators**

The augmented system equation becomes;

\[
\dot{\delta x} = Ax'' + Bu'' \\
\delta u = \delta w \\
\delta y = Cx'' + Du''
\]

(36)

Note that the order of the overall system i.e. the system whose input is \( \delta w \) and output is \( \delta y \), is \( n+m \). The original control variable \( \delta w \), which is now the output of integrators, becomes a state vector. The vector \( \delta w \) is the new \( m \times 1 \) control vector, and \( \delta \dot{x} = [\delta \dot{x}]^T \) is the new \( (n+m) \times 1 \) state variables. The overall perturbational system is then given by;

\[
\dot{\delta x} = Ax'' + Bu'' \\
\delta u = \delta w \\
\delta y = C\delta x + Du''
\]

(37)

The matrices \( A, B \) and \( C \) are \( (n+m)\times(n+m), (n+m)\times m, \) and \( \times(n+m) \) and can be partitioned into;

<table>
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<tr>
<th>Variable</th>
<th>Scaling Value</th>
<th>Physical Unit</th>
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<td>8193.80</td>
<td>rpm</td>
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<tr>
<td>( P_2 )</td>
<td>1005.70</td>
<td>KN/m²</td>
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<tr>
<td>( T_0 )</td>
<td>1250.80</td>
<td>K</td>
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<tr>
<td>( P_1 )</td>
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<td>KN/m²</td>
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<td>( P )</td>
<td>59243.0</td>
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</tr>
<tr>
<td>( m_l )</td>
<td>1.40000</td>
<td>kg/s</td>
</tr>
<tr>
<td>( A_p )</td>
<td>0.32000</td>
<td>m²</td>
</tr>
</tbody>
</table>

**Table 1: Scaling values for the single-spool turbojet engine**
Applying LQG to the augmented model and minimizing the quadratic performance index lead to the state feedback gains;

\[ \delta w = -K \delta z = [K_{11} \ K_{12}] [\delta u \ \delta z] \]

The resultant control structure is shown in Fig. 7. The gain matrix \( M \) has been added to the closed-loop system to permit the consideration of command inputs \( \delta r \) and an equation defining \( M \) will be derived later. This standard linear optimal control structure may therefore be termed the output \( \delta y \) tracking the commanded input \( \delta r \) despite the addition of \( m \) integrators to the linearised system equation. This is because the control commands are not generated by the integral of the error between the desired and the actual output (Athans, et al 1986).

\[ A = \begin{bmatrix} 0 & 0 \\ A & B \end{bmatrix}, \quad B = \begin{bmatrix} I \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} C & D \end{bmatrix} \]

\[ (A, B, C, D) \]

(38)

5.3 OPTIMAL INTEGRAL CONTROL

An integral control structure drives a dynamic system to its proper state using control commands defined by integrating the errors between the desired and actual output (Szuch et al 1977), Rosenblad (1990). Consequently, an optimal integral controller is an improvement over standard linear optimal control because it ensures zero steady-state errors even in the presence of plant parameter variations. Given an optimal feedback gain \( K \), it is possible to derive an equivalent optimal integral control structure.

\[ \delta u(t) = \int_0^t H(x)(r(t) - z(t)) dt \]

(45)

For any command \( \delta r \), it is desired that the control modes in Fig. 7 and 8 yield identical state and control variable time responses. This equivalence is accomplished by equating the \( \delta z \), \( \theta u \) and \( \delta r \) coefficients on the right-hand sides of the above two equations, leading to;

\[ K_{11} = (HE + LB), \quad K_{12} = (HEC + LA), \quad M = H \]

(46)

Given the system matrices \( A, B, C, \) and the optimal feedback gain \( K \), the above equation provides an explicit means for uniquely determining the integral control gain matrix \( H \). If the system matrices \( A, B, C, D \) are perfectly known and \( H, L, \) and \( M \) are determined from the above equations, then the responses of these systems in Fig. 7 and 8 to arbitrary command input will be identical. Unlike the standard control mode in Fig. 7, however, the integral controller assures zeros steady-state output errors even when unknown parameter variations occur in the system equations.

6. NONLINEAR CONTROL SYNTHESIS

The optimal linear control synthesis method described in the preceding section is carried out at all the steady-state operating points, and as the result, a series of optimal incremental feedback controls \( \delta u \) can be obtained.

\[ Q = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]

(47)

for all the linear optimal compensator designs. Selection of weighting matrices \( Q \) and \( R \) in this way, along with the continuity requirement imposed upon \( F \) and \( G \) assure that the optimal feedback gains \( H \) and \( L \) vary smoothly from one operating point to another.

Since a operating point is defined by the state \( x \), \( H \) and \( L \) become functions of \( x \), i.e., \( H = H(x) \) and \( L = L(x) \). These matrices may be converted to continuous functions of \( z \) using any of several interpolation schemes. With \( H \) and \( L \) continuous functions of state, it is possible to develop a relationship defining an nonlinear optimal control \( u(t) \).

From the Fig.8, the optimal incremental control is defined by:

\[ dU(a) = L(x)dx(s) + H(z)(r(s) - z(s)) \]

(48)

The optimal control \( u(t) \) can now be obtained from the path integral:

\[ u(t) = u(0) + \int_{a(0)}^{a(t)} du \]

(49)

\[ u(t) = u(0) + \int_{a(0)}^{a(t)} L(x)dx + \int_0^t \int_{a(0)}^{a(t)} H(z)(r(r) - dx(r))dr \]

(50)

By definition, the gain matrix \( H(x) \) is a function of \( x \) alone, i.e., it is independent of \( r \) and \( z \), hence \( H(x) \) can be removed from the first integration, resulting in:

\[ u(t) = u(0) + \int_{a(0)}^{a(t)} L(x)dx + \int_0^t H(z)(r(r) - dx(r))dr \]

(51)

The above equations make use of the fact that \( r(t) = z(t) \) for a system controlled by the integral mode and initially in a steady state condition. In the computer implementation, each entry in the \( H \) and \( L \) is scheduled as a function of non-dimensional speed; for example, \( H(1, 1) \) is;
\[ H(1, 1) = f_1 \exp(-\lambda_1 \cdot N) + f_2 \exp(-\lambda_2 \cdot N) \] (52)

and the scheduling result is shown in Fig.9

![Graph showing non-dimensional speed vs. H(1,1)](image)

**Figure 9: VARIATION OF H(1,1) WITH NON-DIMENSIONAL SPEED**

### 7 SIMULATION RESULTS

In this section, we present the simulation results based on the open-loop fuel controller and the multivariable nonlinear controller.

Simulations shown in Fig.10 through 12 show that the pressures and shaft speed responses are much faster than that of the open-loop fuel controller. This is due to the fact that nozzle area has been opened at the start of the transient, while allowing more fuel flow to be injected to achieve fast response. The variations of fuel flow and final nozzle area are shown in Fig.13 and 14. It can be seen that for the open-loop fuel controller, the final nozzle area is fixed while in the nonlinear controller, this area is modulated with fuel flow. The gain of thrust response is shown in Fig.15 where 95% of rated thrust response is achieved in about 5 second while for the open-loop fuel controller, this time is about 7 second. In the Fig.16, the turbine inlet temperature response is shown to be improved significantly. There is no overshoot of the nominal design value and the initial jump in this temperature contributes greatly to the fast response in the shaft speed and engine thrust. Fig.17 indicates that the transient trajectory with nonlinear controller make more use of low speed surge margin than the open-loop fuel controller, increasing the overfueling capacity of the engine. In the high speed range, this trajectory lay well below that of the open-loop fuel controller, therefore avoiding the turbine temperature overshoot.

![Comparison of shaft speed response](image)

**Figure 10: COMPARISON OF SHAFT SPEED RESPONSE**

![Comparison of compressor exit pressure response](image)

**Figure 11: COMPARISON OF COMPRESSOR EXIT PRESSURE RESPONSE**

![Comparison of jet pipe pressure response](image)

**Figure 12: COMPARISON OF JET PIPE PRESSURE RESPONSE**

![Comparison of fuel flow of the controllers](image)

**Figure 13: COMPARISON OF FUEL FLOW OF THE CONTROLLERS**
Figure 14: COMPARISON OF NOZZLE AREA OF THE CONTROLLERS

Figure 15: COMPARISON OF ENGINE THRUST RESPONSE

Figure 16: COMPARISON OF TURBINE INLET TEMPERATURE RESPONSE

Figure 17: COMPARISON OF TRANSIENT TRAJECTORY ON COMPRESSOR CHARACTERISTIC

Figure 18: LINEAR AND NONLINEAR MODEL COMPARISON
8 CONCLUSIONS

In this paper, the work into modeling and control of a single-spool gas turbine engine has been described. The algorithms proposed for approximating a compressor enable an analytical expression to be obtained, therefore the simulation time step can be increased. Simulations have shown that a multivariable nonlinear controller could greatly improve the engine dynamic response. The results of this application demonstrate that the design tool is valid and the design method provides a systematic way for synthesizing a multivariable nonlinear controller. The selection of the weighting matrices is specific to the present study. Using non-dimensional speed as a scheduling variable is based on our understanding that this slow-changing variable will guarantee the stability of the nonlinear controller. These will need further study. Since the nonlinear controller is obtained based on the linear design method, it can not directly provide optimal performance for large perturbation transients of a nonlinear system.

9 ACKNOWLEDGMENT

The first author wishes to thank British Council and Chinese Government for the financial support during this research.

10 REFERENCES


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