MULTIVARIABLE ADAPTIVE CONTROL USING ONLY INPUT AND OUTPUT MEASUREMENTS FOR TURBOJET ENGINES

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ABSTRACT

Current and future aircraft engines are increasingly relying upon the use of multivariable control approach for meeting advanced performance requirements. A multivariable adaptive control (MRAC) scheme is proposed in this paper. The adaptation law is derived using only input and output (I/O) measurements. Simulation studies are performed for a two-spool turbojet engine. The satisfactory transient responses are obtained at different operating points from idle to maximum dry power within the flight envelope. These show insensitivity of the design to engine power level and flight condition. Simulation results also show high effectiveness of reducing interaction in multivariable systems with significant coupling. Using the multivariable MRAC controller, the engine acceleration time is reduced by about 19 percent in comparison with the conventional engine controller.

NOMENCLATURE

- $A_e$: Exhaust nozzle area, cm$^2$
- $e$: Error vector
- $e_1$: Output error
- $H$: Altitude, Km
- $I/O$: Input and output
- $M$: Mach number
- MRAC: Model reference adaptive control
- $n^*$: Relative degree
- $N_L$: Low rotor speed, rpm
- $PCN_L$: Percent of low rotor speed
- PI: Proportional-integral
- $r$: Demand for closed-loop control
- $s$: Laplace transform operator
- $SM_h$: High pressure compressor stall margin
- SPR: Strictly positive real
- $t$: Time, second
- $u$: Control variable of closed-loop control
- $v$: Output of filter
- $V$: Lyapunov function
- $W$: Transfer function
- $W_f$: Main fuel flow, Kg/Sec
- $x$: State space vector
- $y$: Output variable
- $\gamma$, $\delta$: Adaptive gain
- $\theta$: Adjustable control gain (vector)

INTRODUCTION

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Integrated flight and propulsion control systems for the next generation of fighter aircraft will require the propulsion control system to vary the engine inlet and exhaust nozzle variables simultaneously. The benefits to the engine will include improved acceleration performance, thrust response, and engine efficiency while maintaining compressor stall protection (Burcham et al., 1984, Yonke et al., 1987, Shaw et al., 1985).

The integrated flight and propulsion systems are dictating advanced requirements for propulsion systems over wider operational envelopes. To satisfy these performance requirements, variable geometry components have become an integral part of advanced aircraft engines. Future variable—cycle engines may incorporate variable fan, compressor, turbine, and exhaust nozzle geometries to improve overall performance. As a result, modern turbojet engines become multi—input multi—output, nonlinear, time—varying systems with significant coupling. As a result, it is difficult to design controllers for these advanced engines.

Classical control synthesis techniques, which involve the analysis and design of single—input, single—output control loops, have worked quite well for the older, simpler engines. Unfortunately, such techniques yield poor transient and static performance when they are applied to the more complex, multivariable engines.

In recent years, several multivariable control design techniques have been used to design control systems for aircraft engines (Sain, 1977, Athans et al., 1986, Peil et al., 1986). The MRAC has made promising advances (Lindorff et al., 1966, and Monopoli, 1981). The design of MRAC schemes using the Lyapunov direct method has been more appealing, because of its inherent assurance of stability. However, there have been a limited number of multivariable control designs for full flight envelope operation (Polley et al., 1988).

Huang and Sun (1993a) have proposed an MRAC scheme using the full state vector for a multivariable two — spool turbojet engine control system within the full flight envelope. However, one difficulty in applying the MRAC approach is the need for the knowledge of the full state vector. The use of a state observer is, of course, a natural step towards the relaxation of this condition. However, no substantial results on this subject have yet been obtained.

In this paper, using an accelerated gradient method, a PI adaptation law of MRAC using only input and output measurements of the plant is proposed. The proposed adaptive scheme is applied to the design of the multivariable control system of turbojet engines. The proposed scheme ensures good transient responses at different flight conditions and satisfactory acceleration responses through large ranges in the flight envelope. It also shows high effectiveness of reducing interaction in multivariable systems with significant coupling. Simulations of the multi—variable engine control system show significant reductions of engine acceleration time and increases in engine thrust.

**MRAC USING ONLY I/O MEASUREMENTS**

Consider an unknown linear single input/single output plant described by the differential equations

\[ \dot{x} = A x + b u (t), \quad y = h x, \]

where \( u \) is the input, \( y \) is the output, \( x \) is the \( n \)th state vector, \( A \) is an \( n \times n \) matrix, and \( h \) and \( b \) are \( n \)—vectors. The transfer function of the plant \( W_p(s) \) may be represented as

\[ W_p(s) = h s^n / (s - 1)^m K_p R_p(s) \]

where \( R_p(s) \) and \( Z_p(s) \) are monic polynomials of order \( n \) and \( m(\leq n - 1) \) respectively and \( K_p \) is a constant gain parameter.

A model represents the behaviour desired from the plant when it is augmented with a suitable controller. The model has a reference input \( r(t) \) and an output \( y_m(t) \). The transfer function of the model, denoted by \( W_m(s) \), may be represented as

\[ W_m(s) = K_m Z_s(s) / R_m(s) \]

where \( R_m(s) \) and \( Z_m(s) \) are monic polynomials of order \( n \) and \( m \) respectively and \( K_m \) is a constant. The purpose is to find \( u \) such that the output error

\[ e = y - y_m \]

tends to zero asymptotically for arbitrary initial conditions and arbitrary piece—wise continuous and uniformly bounded reference signals \( r(t) \).

The following usual assumptions are made: a1) the plant is strictly proper with relative degree \( n^+ (= n - m) = 1 \) and \( W_m(s) \) has the same relative degree; a2) only the plant input and output are used to gen-
erate \( u \); 3) the sign of \( K_p \), the 'high frequency gain', is known, and we assume it positive without loss of generality; 4) \( W_m(s) \) is strictly positive real (SPR); 5) the plant is supposed to be completely observable and controllable; 6) \( W_m(s) \) is minimum phase.

As given by Narendra et al. (1978), the following input and output filters \( F_1 \) and \( F_2 \) respectively are used

\[
\begin{align*}
l_1 &= A_1 v_1 + g_1 y_1, \quad w_1 = c^T v_1 \quad (F1) \\
l_2 &= A_2 v_2 + g_2 y_2, \quad w_2 = d_2 y_2 + d^T v_2 \quad (F2) \quad (5)
\end{align*}
\]

with \( c_1[1(t), \ldots, c_{n-1}(t)], \quad d_1^T = [d_1(t), \ldots, d_{n-1}(t)] \), \( g_1 = [0, \ldots, 0, 1] \), \( v_1, v_2 \in \mathbb{R}^{n-1} \) and \( \Delta \) such that \( Z_m(s) = \det(sI - \Delta) \).

We define the 'regressor' vector as \( \theta = [c_1 T, d_1 T, \ldots, c_{n-1} T] \) and the \( 2n \) adjustable parameters denoted by a vector \( \theta = [\theta_1, \theta_2, \ldots, \theta_{2n}]^T \). In the usual adaptive control scheme, the control \( u \) is structured as

\[
u = \theta^T(t) \omega(t) \quad (6)
\]

It is well known that under the above assumptions there exists a unique constant vector \( \theta^* = [c^* T, d^* T, K_p]^T = [\theta_1^*, \theta_2^*, \ldots, \theta_{2n}^*]^T \) such that the plant matches \( W_m(s) \) exactly, i.e., the transfer function of the closed-loop plant, from \( r \) to \( y \), is \( W_m(s) \). Of course, \( \theta^* \) can only be known if \( W_m(s) \) is known. When this is not the case, \( \theta \) is adapted so that \( e(t) \to 0 \) as \( t \to \infty \) and eventually (under some signal richness condition) \( \theta \to \theta^* \).

The plant (1) plus the filters (5) can be rewritten as

\[
\dot{x} = \bar{A} x + \bar{b} (u - \theta^* \omega) + b r, \quad y = h^T x \quad (7)
\]

where

\[
\bar{A} = \begin{bmatrix} A & d_1 & b & h \\ d_2^T & \Delta + gc^T & gd^T & \end{bmatrix}, \quad \bar{b} = \begin{bmatrix} b^* \theta_1^* T, & b^* T, & 0 \end{bmatrix}, \quad \bar{b} = \begin{bmatrix} A & d_1 & b & h \\ d_2^T & \Delta + gc^T & gd^T & \end{bmatrix}
\]

\[
(A, b, h) \text{ is a non-minimal realization of } W_m(s) \quad (Narendra et al., 1978). \]

The representation of the reference model can be written in the form

\[
\dot{x}_m = A x_m + b r, \quad y_m = h^T x_m \quad (9)
\]

Defining the state error vector as \( e = x - x_m \), denoting \( \phi = \theta(t) - \theta^* \), and subtracting (9) from (7), the error equations between the model and the plant can be expressed as

\[
\dot{e} = \bar{A} e + \bar{b} \phi \omega, \quad e_1 = h^T e \quad (10)
\]

A typical adaptation law is the gradient law in the form of integral as

\[
\theta = -\Gamma \int e \omega \, dt, \quad \Gamma = \Gamma^T > 0 \quad (11)
\]

which can be shown to yield a globally stable adaptive system. Here, we introduce a 'proportional + integral' adaptation law (Huang et al., 1993a) as

\[
\theta = -\Delta e \omega - \Gamma \int e \omega \, dt, \quad \Delta = \Delta^T > 0, \quad \Gamma = \Gamma^T > 0 \quad (12)
\]

Since \( W_m(s) \) is SPR, there exists \( P = P^T > 0 \) and \( Q = Q^T > 0 \) such that

\[
A^T P + PA = -2Q, \quad Pb = h \quad (13)
\]

The global asymptotical stability of the adaptive system using \( u \) given by (6) and (12) can be proved with the Lyapunov function

\[
V(e, \phi) = \frac{1}{2} \left[ e^T P e + \frac{1}{\theta^T_{2n}} (\phi + \Delta e \omega)^T \phi \right] \quad (14)
\]

Calculating the time derivative of \( V \) with respect to (10) and (11), one has (recall \( \phi = \theta^* \))

\[
\dot{V} = -e^T Q e + \omega^T P e + \frac{1}{\theta^T_{2n}} \frac{d}{dt} (\phi + \Delta e \omega)^T (\phi + \Delta e \omega) \quad (15)
\]

Noting that \( \bar{b} = b / \theta^T_{2n}, Pb = h, \he = e_1 \), we obtain

\[
\dot{V} = -e^T Q e - \frac{1}{\theta^T_{2n}} e_1^T e_1 \omega^T \Delta \omega \quad (16)
\]

The \( V \) in (14) is positive definite and \( \dot{V} \) in (16) is negative semi-definite in the \( (e, \phi) \) space. Global stability of the origin is implied and moreover \( e(t) \to 0 \) as \( t \to \infty \).

### MultiVariable Control System for Turbojet Engines

In a conventional engine control system, a certain engine stall margin is required based on the consideration of stability. However, the engine stall margin varies with different operating conditions. For example, the stall margin is small during certain flight maneuvers, engine acceleration or deceleration, and firing. In the current engines, the engine stall margin is determined to avoid stall based on the worst case. Operation with this large stall margin requires the engine thrust to be reduced over what would be obtained if a smaller stall margin could be used. Thus, the performance of the propulsion system is limited.

Additional thrust is obtained by uptrimming the...
engine pressure ratio in some operating conditions and in parts of the flight envelope where excess stall margin and trubine temperature margin exist (huang et al., 1993a).

In this paper, the objective of a nonlinear multivariable controller for a two—spool turbojet engine is to control low rotor speed ($N_L$) and high pressure compressor stall margin ($SM_h$) during accelerations and decelerations while ensuring that predetermined limits on stall margins, speeds and temperatures are not exceeded. The multivariable controller design developed uses low rotor speed and high pressure compressor stall margin as the sensed variables, and fuel flow ($W_i$) and exhaust nozzle area ($A_s$) as the control variables.

The high pressure compressor stall margin is defined as

$$SM_h = \left(\frac{\pi}{\pi_c}\right)\left(\frac{G}{G_c}\right) - 1 \quad (17)$$

where $\pi$, and $G$, are critical pressure ratio and critical gas weight flow rate respectively and $\pi$ and $G$ are pressure ratio and gas weight flow rate, respectively. Since the high pressure compressor stall margin defined in (17) is not measurable directly, we removed the ratio $G/G_c$ from (17) to get the approximation of $SM_h$, i.e. $SM_h = (\pi/\pi_c) - 1$.

In order to derive the adaptive law, the engine model and reference model must be chosen. The multivariable adaptive control would be impractical to develop and implement with the nonlinear equations. Therefore, it has been customary to utilize the linearized model of the engine to simplify the mathematical expressions of the adaptive law. By using the decentralized control concept, the multivariable engine control system can be implemented as two subsystems — the rotor speed control subsystem and the stall margin control subsystem. Since the engine has second order dynamics, each of the subsystems can be served as a linear second order differential equation and the reference model of each subsystem is chosen as

$$W_m(s) = \frac{c_i(s + d_i)}{s^2 + a_i s + b_i}, \quad i = 1, 2 \quad (18)$$

The input and output filters of two subsystems from (5) are given by

$$\dot{v}_{11} = -d_1 v_{11} + W_f, \quad \dot{v}_{12} = -d_2 v_{12} + N_L$$

$$\dot{v}_{21} = -d_3 v_{21} + A_s, \quad \dot{v}_{22} = -d_4 v_{22} + SM_h \quad (19)$$

The adaptive control scheme is chosen as

$$W_f = \theta_1 v_{11} + \theta_2 N_L + \theta_3 v_{12} + \theta_4 N_L$$

$$A_s = \theta_5 v_{21} + \theta_6 SM_h + \theta_7 v_{22} + \theta_8 SM_h \quad (20)$$

where $\theta_i (i = 1, 2, j = 1, \ldots, 4)$ are eight adjustable control gains.

From (12), we can obtain the adaptive law in the form of a 'PI' law as follows

$$\delta_{11} = -\delta_{11} e_{11} - \gamma_{11} s e_{11} dt$$

$$\delta_{12} = -\delta_{12} e_{12} - \gamma_{12} s e_{12} dt$$

$$\delta_{13} = -\delta_{13} e_{13} - \gamma_{13} s e_{13} dt$$

$$\delta_{14} = -\delta_{14} e_{14} - \gamma_{14} s e_{14} dt$$

$$\delta_{21} = -\delta_{21} e_{21} - \gamma_{21} s e_{21} dt$$

$$\delta_{22} = -\delta_{22} e_{22} - \gamma_{22} s e_{22} dt$$

$$\delta_{23} = -\delta_{23} e_{23} - \gamma_{23} s e_{23} dt$$

$$\delta_{24} = -\delta_{24} e_{24} - \gamma_{24} s e_{24} dt$$

where $e_{11} = N_L - N_{Lm}, e_{21} = SM_h - SM_{hm}, \delta_{ij}, \gamma_{ij} (i = 1, 2, j = 1, \ldots, 4)$ are positive constant parameters.

Fig. 1 Multivariable adaptive control system for turbojet engines
The block diagram of multivariable adaptive system is shown in Fig. 1.

Compared to the integral adaptation law, the PI law has new properties of fast adaptation and perfect model-following. It is important to note that in the adaptation law (21) the integral term provides the memory of the adaptation, and the proportional term is introduced in order to accelerate the reduction of the output error at the beginning of the adaptation process. Therefore, the new adaptation law is also called "accelerated gradient law".

The ratio between the values of the proportional gain \( \delta_u \) and the values of the integral gain \( \gamma_i \) in the PI law (21) has an important influence on the speed of reduction of the output error. It has been shown that a high speed reduction of the error between model and plant is obtained for large value of \( \delta_u \) and \( \gamma_i \), but in counterpart, the control energy is higher. The simultaneous augmentation of proportional and integral gains improves the speed of adaptation, but the gains are limited by the saturations existing in the adaptation loop and by the imperfect characterization of the plant (Huang et al. 1993b).

SIMULATION RESULTS

A nonlinear time-varying and coupling model of a two-spool turbojet engine for full flight envelope operation is used in the simulation. The engine consists of low pressure compressor, high pressure compressor, combustor, high pressure turbine, low pressure turbine, augmentor, and exhaust nozzle. All these are represented by equations or table lookup. A Newton-Raphson iterative solution is used to ensure flow continuity between components and energy balance so that calculated pressure and temperature changes are consistent with enthalpy and entropy changes of real gases throughout the engine. The nonlinear model is a simplified model in which the volume dynamics in the engine are not included in this study. The dynamics of the engine are significantly different at different operating conditions. For example, response time ranged from 1 to 6 seconds at sea level from idle to maximum dry power.

The reference models of the two subsystems are chosen with \( a_i = 10, b_i = 25, c_i = 2, 5, d_i = 10, i = 1, 2 \). The choice of positive constant parameters \( \delta_u \) and \( \gamma_i \) plays a crucial role in the MRAC developed. The parameters are selected to obtain the best tracking property and are given as \( \delta_{t1} = 10^{-7}, \delta_{t2} = 10^{-2}, \delta_{t3} = 10^{-4}, \delta_{t4} = 10^{-7}, \gamma_{t1} = 5 \times 10^1, \gamma_{t2} = 5 \times 10^8, \gamma_{t4} = 5 \times 10^4, \delta_{t5} = \delta_{t6} = \delta_{t7} = 10^{-7}, \gamma_{t1} = 10^{-7}, \gamma_{t2} = 10^{-2}, \gamma_{t3} = 10^{-4}, \gamma_{t4} = 10^{-9} \).

Satisfactory response properties were obtained at various operating points within the full flight envelope. Fig. 2 shows step responses of \( N_L \) with MRAC.
at different operating points within the full flight envelope, i.e., at different altitudes and different Mach numbers. Fig. 3 shows the time responses of SMₜ using the MRAC scheme in the full flight envelope. The demand of SMₜ is a slope signal from the initial SMₜ to the desired value of 0.09 within 0.5 second. Fig. 3(b) shows that the exhaust nozzle area arrives at its minimum value of the geometry limit. The simulation results show that the control scheme can compensate very well for unknown and nonlinear time-varying engine dynamics.

Acceleration simulations with conventional control and with the multivariable MRAC proposed are performed (Fig. 4) to compare their acceleration performances. Both accelerations are run at sea level from PCNL₁ = 60% to PCNL₂ = 100.5%. The desired value of high pressure compressor stall margin SMₜ is 0.09. The adaptive controller controls SMₜ as close as possible to the desired value by manipulating exhaust nozzle Aₑ. Meanwhile, it keeps the turbine temperature within its limit. Using the multivariable MRAC controller, the engine acceleration time is reduced by about 19 percent (Fig. 4(a)), the engine thrust is significantly increased during acceleration, and is increased by about 8 percent at maximum dry power (Fig. 4(b)) as compared to the conventional engine controller. The improved thrust is achieved by decreasing exhaust nozzle area to uptrim engine pressure ratio. The stall margin decreases with uptrim. However, the minimum stall margin is not reduced during acceleration when acceleration performance is achieved using the adaptive controller in comparison with a conventional controller (Fig. 4(c)). Fig. 4 also shows satisfactory effectiveness of reducing the interactions in the multivariable control system with significant coupling and the insensitivity of the design to engine power level and flight condition.

CONCLUSIONS

This paper has summarized an approach for designing a multivariable nonlinear controller of MRAC for turbojet engines. The MRAC scheme is designed using only input and output measurements. The multivariable control system ensures good transient responses at different flight conditions and satisfactory acceleration and deceleration responses through
large ranges in the flight envelope. Using the multivariable adaptive controller, the engine acceleration time is reduced by about 19 percent in comparison with the conventional engine controller.

Compared the MRAC law proposed by Huang et al. (1993a), the multivariable adaptive scheme developed in this paper offers advantages as follows:

1) only the input and output measurements are used to generate the control signals;
2) satisfactory performance and the insensitivity of the design to engine power level and flight condition;
3) eliminating or substantially reducing interaction in multivariable system with significant coupling;
4) using the proposed MRAC controller, the engine acceleration time is reduced and the thrust is increased significantly in comparison with the conventional engine controller.

REFERENCES


