Mean Velocity and Decay Characteristics of the Near-and Far-Wake of a Compressor Rotor Blade of Moderate Loading

A. RAVINDRANATH
Graduate Assistant

B. LAKSHMINARAYANA
Professor
Aerospace Engineering
Dept. of Aerospace Engineering,
The Pennsylvania State University,
University Park, Pa.

This paper reports the experimental study of the three-dimensional characteristics of the mean velocity in the wake of a moderately loaded compressor rotor blade. The measurements were taken with a three-sensor hot-wire probe rotating with the rotor. The wake was surveyed at several radial and axial stations. The loading was found to have substantial effect and this was reflected not only in the axial and tangential components, but also in the radial component. The radial velocities were found to be high very near the trailing-edge and this exhibits the characteristics prevalent in a trailing vortex system. The static pressures across the wake were measured using a direction insensitive spherical head static-stagnation pressure probe. The static pressure was found to be higher inside the wake. These and other measurements are reported and correlated in this paper.


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ABSTRACT

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NOMENCLATURE

- \( n \): number of coefficients in the Fourier series
- \( P \): stagnation pressure
- \( p \): static pressure
- \( R \): radius ratio \( = \frac{r}{r_t} \)
- \( r \): local radius
- \( U_t \): blade tip speed
- \( u_d \): normalized velocity defect \( = \frac{u_d}{u_{max} - u_{min}} \)
- \( u_{max} \): maximum velocity in the wake
- \( u_{min} \): minimum velocity in the wake
- \( w_t, W_0, W_z \): relative velocity in the radial, tangential and axial directions
- \( W \): relative total velocity
- \( \psi \): outlet air angle
- \( \Theta \): rotor efficiency (Euler) \( = \frac{\psi}{\Theta_{Euler}} \)
- \( \tilde{\Theta} \): speed of rotation of rotor blade
- \( \tilde{\Theta} \): distance between wake center line and the location where \( u_d = \frac{u_d}{2} \)
- \( \tilde{\tilde{\Theta}} \): outlet air angle

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INTRODUCTION

The flow through a compressor rotor blade row is very complex and deviates considerably from the two-dimensional flow field because of a number of phenomena including vortex shedding, variation in spanwise circulation, radial flow in the boundary-layers, hub- and annulus-wall boundary-layer growth, etc. The boundary-layer on the blades is shed out from the pressure and suction surfaces, and they combine very near the trailing-edge to form the viscous wake referred to as the rotor-wake. Study of the rotor-wake should yield a comprehensive understanding of the flow and acoustic field induced by this phenomena, which is essential to progress in reducing the noise, losses, and in improving the mechanical and aerodynamic performance of rotors.

The rotor-wakes, which represent the flow field downstream of the rotor, are not only influenced by upstream flowfield conditions but are also controlled by such parameters as blade spacing, solidity, hub-tip ratio, blade loading, distance from the blade trailing edge, and Centrifugal and Coriolis forces induced by curvature and rotation. These effects make the flow highly three-dimensional, which introduces many complications in the experimental and theoretical investigations of the rotor wake.

This paper reports some results of a comprehensive investigation of the wake of such a moderately loaded compressor rotor conducted at the Turbomachinery Laboratory of the Pennsylvania State University. The immediate objectives of the program include a thorough experimental investigation of the flow ahead of, within, and downstream of the rotor. In the present paper two aspects of the study will be discussed; a detailed study in the trailing-edge region outside the hub- and annulus-wall boundary layers, and a description of the flow field in the near-wake region.

The general structure of the rotor-wakes was studied by Raj and Lakshminarayana (1) and Reynolds et al. (2). These data were taken from a lightly loaded axial flow fan at Penn State. In the above studies it was realized that loading has a considerable effect on the nature of the mean velocities and turbulence quantities inside the wake. It was also noted that the annulus- and hub-wall boundary-layers affect the development and decay of rotor wakes near the wall regions and that the wake behavior in the trailing edge region is quite different from that in the near- or far-wake regions. This investigation was undertaken to study these effects, and thereby provide the acousticians and the aerodynamicists with the nature of the flow very near the blade trailing-edge, near-wake and far-wake region of a moderately loaded compressor blade. This information is essential for the prediction of the noise levels, optimum rotor-stator spacing, and losses in a compressor. In this paper only the wake data outside the annulus- and hub-wall boundary-layers will be presented.

EXPERIMENTAL EQUIPMENT AND PROCEDURE

The experiments described in this paper were conducted using the axial-flow compressor facility at the Department of Aerospace Engineering, the Pennsylvania State University. The measurement procedure used was similar to that described in reference (2). A tri-axial hot-wire probe rotating with the rotor was used to measure the characteristics of the rotor-wake. A modified version of the traverse gear used in reference (3) was employed for traversing the probe across the wake, while the rotor was in motion.

Description of the Facility

The axial-flow compressor facility used in this investigation is shown in Fig. 1. The general description of the stage is given in reference (4) and the operating characteristics in reference (5). At a fixed rotor speed the operating conditions can be varied by a throttle located downstream of the auxiliary fan. To remove dust and other particles from the flow which enters the test section a large dust screen was placed around the bellmouth inlet.

The rotor with 21 cambered and twisted blades is housed between an inlet guide vane (IGV) assembly and a stator assembly. The IGV has a swirl distribution varying linearly from zero at the hub to 0.3 at the tip and the IGV trailing edge is located about 1.8 rotor chords upstream of the rotor leading edge at mid-radius. The rotor tip diameter is 0.9322 m (36.700 in.) with a hub to tip ratio of 0.5. The rotor tip clearance is 2.26 x 10^-3 m (0.089 in.). The rotor is driven by a variable speed 37.3 kw (50 hp) motor. The rotor was operated at 1066 rpm and the speed is measured by a photocell attached to the rotor shaft. The inlet axial velocity before the inlet guide vanes is approximately 96 ft/sec at the operating flow coefficient of 0.56 (based on blade tip speed).

All measurements reported in this paper were carried out behind the rotor at various axial and radial distances and are tabulated in Table 1. The blade profiles are of modified NACA 65-010 basic profile with a circular arc camber. The blade has a varying stagger angle and blade chord along the radius and the details are given in reference (4). The rotor was operated at a flow coefficient of 0.56 and the performance curve for the compressor with its operating point is shown in Fig. 2. This operating point was chosen to provide a reasonable stall margin as well as good efficiency.

Rotating Probe Traverse Mechanism

The rotating probe traverse mechanism used in reference (3) was modified to suit the requirements for the rotor-wake measurements. This new traverse gear achieves the objectives of the gear used in reference (2) in a much simpler fashion.

Fig. 3 shows the main features of the new gear. It has a SLO-SYN motor of 225 in-oz torque driving a 3.81 x 10^-2 m (1.5 in.) diameter shaft through a palnetary reduction gear train. The gear train steps down (1.8 deg/step) circumferential stepping to (0.09 deg/step). The shaft carries a rider which can be traversed axially. The rider in turn carries a pair of mounting blocks which can be used to hold the probe. These mounting blocks give the probe two degrees of freedom; (a) rotation about the probe axis; this motion is used to align the hot-wire sensors in any desired direction. (b) Movement in the radial direction; the probe can be traversed from hub-to-tip of the machine.
Table 1 Wake Measuring Stations

<table>
<thead>
<tr>
<th>R</th>
<th>Chord m(inch)</th>
<th>Axial Location From Trailing Edge Z</th>
<th>Inlet Turbulence</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9595</td>
<td>0.150(5.90)</td>
<td>0.0104 0.0417 0.124 0.240 0.458</td>
<td>0.5% (Axial)</td>
</tr>
<tr>
<td>0.9324</td>
<td>0.147(5.80)</td>
<td>0.0104 0.0412 0.125 0.250 0.458</td>
<td>Intensity /</td>
</tr>
<tr>
<td>0.8615</td>
<td>0.141(5.56)</td>
<td>0.0104 0.0417 0.125 0.208 0.417</td>
<td>Axial Vel.</td>
</tr>
<tr>
<td>0.7973</td>
<td>0.137(5.40)</td>
<td>0.0104 0.03125 0.146 0.219 0.375</td>
<td></td>
</tr>
<tr>
<td>0.7297</td>
<td>0.134(5.26)</td>
<td>0.0104 0.0208 0.1354 0.240 0.458</td>
<td></td>
</tr>
<tr>
<td>0.6581</td>
<td>0.130(5.12)</td>
<td>0.0104 0.1166 0.125 0.308 0.458</td>
<td></td>
</tr>
<tr>
<td>0.5676</td>
<td>0.128(5.02)</td>
<td>0.0104 0.1166 0.125 0.308 0.458</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 1 PSU axial flow compressor facility

Fig. 2 PSU axial-flow compressor rotor performance curve
The probe holder, the stepping motor, and the probe are mounted inside the rotating hub of the machine. The probe which is located behind the blade row traverses in a slot in the hub. The motor is powered and controlled by a stationary traverse indexing device through an eight-channel brush slip-ring unit. The traverse unit and the probe are locked, except when indexed. Though the circumferential traverse can be accomplished when the rotor is rotating, the radial and axial motions have to be accomplished manually when the rotor is stationary.

Probes, Instrumentation and Data Transmission System

A tri-axial hot-wire probe, similar to that used in references (1 and 2) was employed for all velocity measurements reported in this paper. The probe has been found to measure the three components of velocity and turbulence intensities accurately. The probe sensors were built out of tungsten wire and had a length to diameter ratio of 270. The sensor resistances were around 5 ohms. The probe calibrations are corrected for the variations of temperature and aging of the wire. The effect of rotation on heat transfer properties of the wire was found to be negligible (6). The signals from the rotating hot-wire, rotating with the rotor, was taken through a ten channel slip-ring unit to a stationary data-processing system which was similar to the one employed in references (1 and 2).

To evaluate the accuracy of the hot-wire measurements and to find the static pressure gradients across the wake a spherical head static-stagnation pressure probe was rotated with the rotor. The probe was found to be fairly insensitive (within ±20 deg) to the large pitch or yaw angles that were encountered in the three-dimensional flow in the wake. The probe consists of 0.3175 cm (1/8 in.) sphere with trips mounted on the sphere to stabilize the wake region. The wake pressure and the total pressure were measured to evaluate the static pressure upstream of the sphere. A complete description of the probe is given in reference (7). The probe is calibrated in a known uniform flow and, therefore, the calibration would include the aerodynamic interference effects. But the errors due to the probe immersed in shear gradients are not included.

Fig. 4 illustrates the instrumentation used with the tri-axial hot-wire probe. The choice of the instrumentation was made such that the raw data from the experiment could be used directly in the data processing program developed by Gorton and Lakshminarayana (8). Consistent with the assumptions made in reference (8), a streamline coordinate system was chosen and the hot-wire axis was aligned along the blade stagger. The data was then transformed to the r-θ-z coordinate system where the r-direction is in the direction of the machine radius, z in the axial direction, and θ in the tangential direction.

EXPERIMENTAL DATA AND INTERPRETATION

Rotating hot-wire measurements were made to study the rotor-wake characteristics in the trailing-edge, near- and far-wake regions. Trailling-edge region is defined as the region very close to the blade trailing-edge where the radial velocities show trends similar to that of a trailing vortex system; a near-wake is defined as the region where the defects in velocity are of the same order of magnitude as the free-stream velocity. Far wake is defined as the region where velocity defects are very small. With these measurements, the mean velocity and turbulence characteristics of the
rotor-wake were found. Similarity rules and decay laws were established for the mean velocities. Static-pressure variations across the wake were also measured. In this paper only the mean velocity characteristics of the rotor-wake outside the annulus- and hub-wall boundary-layers are presented. The turbulence data and data inside the annulus- and hub-wall boundary layers will be presented and interpreted in a subsequent paper.

Mean Velocity Profiles

All velocities discussed in this section are relative velocities and are normalized by the corresponding free-stream axial velocity, unless otherwise specified. The data at three radial locations, \( R = 0.7973, 0.7297 \) and \( 0.6581 \), are presented in this paper.

Axial velocity. Variations of axial velocity across the wake and at various downstream locations for the radii \( R = 0.7973 \) and \( R = 0.6581 \) are shown as a three-dimensional plot in Figs. 5 and 6. The tangential distance is normalized by the blade spacing.

Fig. 5 Three-dimensional plot of axial velocity profiles, \( R = 0.7973, \phi = 0.56 \)

Fig. 6 Three-dimensional plot of axial velocity profiles, \( R = 0.6581, \phi = 0.56 \)

Fig. 7 shows the variations of the axial velocity profiles for the radius \( R = 0.7297 \). The wake profiles are asymmetrical about the wake center indicating differential growth of the boundary layers on the two surfaces of the blade. This is true at all the radii (Figs. 5, 6, and 7) and near the trailing edge region. Because of wake spreading and mixing with free-stream as well as interchange of momentum and energy on either side of the wake, the wake will become nearly symmetrical at far downstream locations.

In the trailing-edge region, for the radius \( R = 0.7297 \), the wake defect is 0.88 which reduces to 0.2 at \( Z = 0.5313 \). For the wake at \( R = 0.7973 \), a defect of 0.71 at \( Z = 0.03125 \) reduces to 0.24 at \( Z = 0.5625 \). As the hub wall is approached the decay rate decreases as shown at the radius \( R = 0.6581 \) (Fig. 6) where a defect of 0.66 at \( Z = 0.1146 \) reduces to 0.33 at a \( Z \) location of 0.6875. The same trend is observed even when the annulus-wall boundary-layer is approached. So the wakes decay differentially in the radial direction.

Not only is the decay different at each radius so are the velocity profiles. Because of radial transport of mass, momentum, and energy, wakes at the hub are expected to be thinner gradually increasing towards the tip. But in the present case the wakes at lower radius are as thick as those at higher radii. The probable reason for this is that the drag coefficient at the root is higher than that at tip due to larger turning, and this may offset the effect discussed earlier and yield results contrary to what is expected. Also,
Axial velocity profiles show a lower value of the free stream on the pressure side and higher value on the suction side. This is a consequence of the existence of pressure gradients (inviscid effects) across the passage immediately downstream of the trailing-edge. This effect is less pronounced beyond Z = 0.045.

The variation of the tangential gradients of axial component of velocity indicate the changes in profile shape downstream of the rotor. For the wake in the trailing-edge region the gradients are very large and can be clearly seen in Fig. 7. This characteristic for the wake results from the development of the flow as it moves over the rotor blade and transforms into the wake at the trailing-edge from a boundary-layer. In the far-wake region the gradient becomes considerably smaller because of wake spreading and mixing with the free-stream. The steep gradient in the trailing-edge region represents highly unstable and developing flow conditions. There is another interesting feature that can be observed in the trailing-edge region. Here the pressure-surface boundary-layer is slightly thicker than the suction-surface boundary layer (Figs. 5, 6, and 7). One probable cause of this is that radial inward velocities on the pressure surface (reported later) will cause a larger accumulation of boundary-layer on the pressure-surface compared to that on the suction-surface. Exact cause of this phenomena is not known. Measurements presently carried out to determine the boundary-layer characteristics near the trailing-edge (z<0) should provide an explanation of this phenomena. As the wake travels downstream the radially inward velocities decrease, and beyond 0.15 chord downstream, we have only radially outward flow (as discussed later). The wake shows a tendency to become symmetrical further downstream under these conditions.

The effect of blade loading can be observed in the near-wake plots of Fig. 7. The wakes become highly unsymmetrical because of loading and is noted by comparing Fig. 7 with Figs. 4 and 5 of reference (2). From reference (2) we note that the wake becomes approximately symmetrical at 0.271 chord downstream where as in Fig. 7 for the present case the wake still tends to be asymmetrical at half-a-chord downstream. Thus the effect of blade loading is to sustain the asymmetry to a much larger extent downstream of the blade trailing-edge.

Blade loading not only makes the profiles asymmetrical, it also increases the velocity defect and slows down the decay rate. Thus at mid-radius, comparing Fig. 7 with Figs. 4 and 5 of reference (2) we note that at about the same Z location (say Z = 0.0208) a heavily loaded blade gives much deeper wake compared to that of a lightly loaded blade used in reference (2). The decay of the heavily loaded rotor-wake is much slower with a defect of 0.16 at half-a-chord downstream compared to that of a lightly loaded rotor-wake where the defect is 0.2 at Z = 0.271.

Relative tangential velocity. Figs. 8 and 9 show the three-dimensional plot of relative tangential velocity for the radii R = 0.7973 and R = 0.6581 while Fig. 10 shows the tangential velocity variation with downstream distance for the radius R = 0.7297.

In general, the tangential velocity defects were found to be greater than axial velocity defects. At R = 0.7297 and Z = 0.0104 the defect is almost 0.9 $\%$ dropping off to about 0.2 $\%$ in half-a-chord downstream. The same type of trend is observed for the other radii R = 0.7973 and 0.6581. The tangential velocity profiles are also asymmetrical because of differential growth of boundary-layers on the two surfaces.

The variation of the decay of tangential velocity defect in the radial direction show an interesting trend. At R = 0.7973 and at a downstream location of 0.5630 chords the defect in tangential velocity is 0.13 and this defect continuously increases towards the hub. Probable cause for this is the larger turning near the hub and hence increased drag coefficient. Furthermore, the presence of hub wall boundary layer and secondary flow may produce this effect. As the defect is large near the lower radius
Fig. 10  Tangential velocity profiles, R = 0.7297, \( \phi = 0.56 \)

as compared to higher radius, the decay rate is correspondingly slower at the lower radius. But compared to axial velocity, the tangential velocity defect decays much more rapidly. Loading not only increases the defect and asymmetry, but it also slows down the decay. Comparison of Fig. 10 with that of Figs. 6 and 7 of reference (2) will confirm this argument. Tangential velocities also show considerable asymmetry in the trailing-edge and near-wake regions. Blade loading does make a significant contribution to the asymmetry and its effect persists up to half-a-chord downstream.

Tangential velocities are greatly influenced by the radial component of velocities. In the trailing-edge and near-wake regions for R = 0.7297, it is observed that there are very large radial velocities and a small component of negative relative tangential velocities. This implies that the absolute and relative velocities are in the same direction and that there is a very large change in angle in these regions.

Radial velocities. Radial velocities are caused by an imbalance in the radial pressure gradients and the centrifugal forces. Physically at either surface of the trailing-edge of a compressor rotor blade, the radial velocities must be zero while the maximum radial outward velocities occur slightly away from the blade surface. This was confirmed by measurements on an inducer blade by Lakshminarayana, et al. (9). Raj and Lakshminarayana (1) and Reynolds, et al. (2) measured the radial velocity profiles in the wakes from a lightly loaded rotor. The behavior of the radial velocities were very much different in these cases compared to that of reference (9). In the present case radial velocity profiles for the radii R = 0.7973 and 0.6581 are shown as three-dimensional plots in Figs. 11 and 12, respectively, while Fig. 13 shows the radial velocity variation with downstream distance for R = 0.7297.

Fig. 11  Three-dimensional plot of radial-velocity profiles, R = 0.7973, \( \phi = 0.56 \)

As mentioned earlier the radial velocities are caused by an imbalance of centrifugal forces and the radial pressure gradients inside the wake. Depending on which of the above forces dominates, there will be a radially inward or outward flow. Very near the trailing-edge there might be one other reason why
a radially inward or outward flow exists. A trailing vortex system associated with the circulation might give a radially inward flow on one side and radially outward flow on the other side of the blade. Referring to Fig. 13 we note that at the wake center as well as in the free stream we have nearly zero radial velocity and radially inward flow near the pressure-surface and radially outward flow near the suction side. This situation is possible if the trailing vortex dominates the flow in the trailing-edge region while the boundary-layer flow dominates the flow in the near- and far-wake regions. This complex flow phenomenon is not fully understood and a knowledge of the flow on the blades (blade boundary-layer) could assist in explaining the nature of the flow. Further experiments in this region along with flow visualization studies have been planned with a view towards better understanding and interpretation of this data.

In the near- and far-wake region we have a radially outward flow with the maximum velocity occurring slightly away from the wake center and towards the suction-surface. This flow phenomena can be explained on the basis of the earlier concept of imbalance between the centrifugal forces and radial pressure gradients.

The nature of radial velocities differs considerably in the radial direction. Wall constraints imposed on the flow near the tip would force the flow to have an inward direction (9). In addition, secondary flow and tip leakage flow would greatly influence the nature of flow in this region. Consequently, we will have mostly radial inward flow at these locations. So the radial velocities would be maximum near the mid-radius decreasing in magnitude towards the tip of the machine (Figs. 11, 12 and 13). Far downstream, the radial velocity is inward at \( R = 0.6581 \) and outward at \( R = 0.7973 \) and 0.7297.

Very near the trailing edge, for \( R = 0.7297 \), the maximum radial velocity \( \left( \frac{w_R}{w_{\infty}} \right) \) is 0.7 dropping to about 0.2 in half-a-chord downstream. The decay of the radial velocity seems to follow the same trend as that of the axial and the tangential velocities. The magnitude of radial velocity for the other two radii \( R = 0.7973 \) and 0.6581 is considerably less, being consistent with the reasoning mentioned earlier. The large radial component of velocities indicates the highly three-dimensional nature of the rotor-wake. The radially inward or outward flow in the wake will result in an increased dissipation of energy. The spread of the radial velocity profile downstream of the blade is not as marked as in the case of axial or the tangential component of velocity.

The effect of blade loading on the radial velocities can be discerned by comparing the results of this paper with those of reference (2). The radial velocities are dependent on the pressure gradient, centrifugal forces and Coriolis forces arising due to rotation and the strength of the trailing vortex. All these effects increase with increased blade loading. Referring to Fig. 13, it can be noted that for this blading at \( R = 0.7297 \) and at \( Z = 0.5313 \) the magnitude of the maximum radial velocity is nearly 0.2, while for a lightly loaded rotor used in references (1 and 2) the radial velocities have dropped to a value of around 0.1 even at 0.2 chords downstream. So the effect of loading is to induce higher radial velocities, and these decay at a slow rate.

Total mean relative velocity. The total mean relative velocity which is the resultant of the axial, tangential and radial velocities is shown plotted in Fig. 14a for \( R = 0.7297 \). A comparison of the velocities derived from the hot-wire and static-stagnation pressure probe is also shown. The velocities are normalized by the corresponding free-stream velocity and the abscissa represents the distance along the span as a fraction of blade spacing with \( Y \) equal to zero at the wake centerline. A maximum wake defect of 0.8 at \( Z = 0.0104 \) reduces to about 0.2 at \( Z = 0.5313 \). The data seems to confirm that the velocities are very low at the blade trailing edge and the defect decays very rapidly in this region (0 < \( Z < 0.13 \)). In the case of the rotor-wakes, this is probably the closest station to the trailing-edge (\( Z = 6.25 \times 10^{-2} \) in. or 1.588 \( \times 10^{-2} \) cm) where measurements are available. Furthermore, the defects measured are larger than any reported so far. The asymmetry in the velocity profiles, which arises due to differential growth of boundary-layers and the effect of loading in sustaining the asymmetry to beyond half-a-chord downstream can also be observed in Fig. 14a.

Velocity derived from the hot-wire and the pressure measurements are shown plotted in Fig. 14a. The agreement between the two measurements beyond \( Z = 0.1354 \) is very good indicating pressure measurements are as accurate as hot-wire measurements. In the trailing-edge and near-wake measurements there is slight discrepancy between the two measurements. The pressure measurements do show the inviscid effects which are not reflected in the hot-wire measurements. A point of concern is that the velocity measured by hot-wire is greater than that measured by the pressure probe and the difference between the two is as much as 10 percent at the wake center. Although the spatial resolution of the hot-wire is better than that of the
Fig. 14a Total relative velocity profiles, R = 0.7297, \( \phi = 0.56 \)

pressure probe it is not possible to assume that the hot-wire has better accuracy than the pressure probe. Two most probable errors at the trailing edge location are (a) wall vicinity effects in the case of pressure probe and (b) distortion of cooling characteristics of the wire because of vicinity of the wall. No attempt is made in this paper to evaluate the relative merits of the two systems in the near-wake and trailing-edge region as the source of error is not properly understood.

The variation of the free stream total velocity \((W_0)\) with downstream distance is shown in Fig. 14b. The free stream velocity increases by 5 percent in the trailing edge region \(0 < Z < 0.2\). Beyond this region, the variation is negligibly small at all radii.

Wake-Width Variation

The variation of the semi-wake width normalized by blade spacing is shown in Fig. 15a for the radii \(R = 0.7973, 0.7297\) and \(0.6581\). The wake-width increases rapidly in the trailing-edge region and then the growth becomes very gradual in the near- and far-wake region. As the interchange of energy, mass and momentum is continuous on the two sides of the wake as it travels downstream, the growth should also be continuous, and this behavior is observed in Fig. 15a, which is consistent with those observed in the isolated airfoil and cascade wake studies. The wake-width varies considerably in the radial direction. It increases towards the hub and annulus-wall which is due to the complex interaction of the wake, hub- or annulus-wall boundary-layer, secondary flow (tip vortex in the annulus-wall case).

Wake-width, besides other factors, depends on the aerodynamic properties of the blade, and this should vary with radius in conformity with the variation of the drag coefficient. Normalizing the wake-width, as a function of \(C_D\), should collapse all the points onto a single curve. Fig. 15b shows this correlation, where \(C_D^{0.5}\) has been used as the normalizing parameter. A functional dependency for the growth of wake-width \((L/C_D)\) was tried as a function of the downstream distance. An equation of the type, 

\[ L/C_D = k \left( \frac{Z}{\cos \beta_0} - \frac{Z_0}{\cos \beta_0} \right)^n \]

can be used to represent the present data quite accurately. The constant \(k\) and the exponent \(n\) take on different values in the
Curvature can also be thought of as the variation in flow angle at the wake center with downstream distance. The flow angle can be calculated from the velocity triangles at each of the downstream locations. This plot is shown in Fig. 16b. In the trailing-edge region the flow is still developing where the trailing vortex system may dominate the wake. As the axial component of velocity is very small and the radial velocities are large, the flow deflection is quite considerable resulting in small negative tangential relative velocities. This implies that the relative and the absolute velocities will be in the same direction. As the wake travels downstream the tangential relative velocity reverses its sign and becomes positive. The reversal of tangential velocity in its direction introduces a large angle variation which is reflected in the wake curvature in the trailing-edge region. Beyond $Z > 0.1354$ the flow angle variation is very small and consequently the curvature of the wake is also small.

The effect of loading is to increase the curvature of the wake and is noted by comparing Fig. 16 with Fig. 10 of reference (2).

**Similarity of Velocity Profiles**

Experimental evidence has shown that similarity exists for the mean velocity profiles. Raj and Lakshminarayana (1) have shown that these velocity components follow the Gaussian profile given by $e^{-0.693n^2}$. Raj and Lakshminarayana (1) and Reynolds et al. (2) examined this for a lightly loaded fan blade. In this section the same is tried for a heavily loaded compressor blade.

**Axial and tangential velocity profiles.** Maximum velocity defect was used as the non-dimensionalizing velocity scale for the local velocity defect. The wake width at half the wake depth was taken as the characteristic length scale. Corresponding values $L_o$ and $L_p$ were taken for the pressure- and suction-surfaces of the wake. Using this technique, similarity was examined for the axial and tangential velocity profiles and are shown in Figs. 17 and 18. It is seen that the similarity does exist even in the case of a heavily loaded blade following the Gaussian distribution given by $e^{-0.693n^2}$. As seen from Figs. 17 and 18, the discrepancy from the Gaussian curve for the wake in the trailing-edge region can be attributed to the fact that we are not only dealing with the wake but are also dealing with the trailing vortex system out of the trailing-edge of the blade. There is a substantial scatter from the Gaussian curve, in the outer regions of the flow. The discrepancy is marked particularly for a heavily loaded blade (compare Figs. 17 and 18 with that of 11 and 12 of reference 2). Townsend's (11) approach of inclusion of an intermittency factor into the Gaussian law in the form

$$\frac{U_d}{U_{dc}} = \exp\left\{-C_1 n^2 \left[1 + \frac{1}{3} n \frac{U_d}{U_{dc}}\right]^4\right\}$$

might yield better results. Here $U_d$ is the velocity defect, $C_1$ and $n$ are constants.

**Radial velocity profiles.** Radial velocities show two different trends, one trend in the trailing-edge region and the other in the near- and far-wake regions. For the far-wake region the similarity is quite good and is shown plotted in Fig. 19a. The maximum radial outward velocity is taken as the normalizing velocity. The similarity for the wakes for $R = 0.6581$, 0.7297, and 0.7973 is quite good except for the scatter in the outer regions of flow. This indicates that even the radial velocities follow the trend of the Gaussian function.
The radial velocities in the trailing-edge region exhibit two different types of velocity distribution with radially inward flow on the pressure side and radially outward flow on the suction side of the wake. Here again maximum radial velocity defect was used as the normalizing velocity (on the suction surface the absolute magnitude of the velocities were considered). The plots are shown in Fig. 19b. The agreement with the Gaussian distribution is fairly good. It has to be realized that at these Z-locations we are not only dealing with the boundary-layer flow but also trailing vortex system. Similarity for such complex flow phenomena has not been proven before and this is a first step in that direction. It is quite probable that the Townsend's approach might yield a better correlation with the experimental data.
Decay of Velocity Defects

Decay of axial and tangential velocities are shown plotted in Fig. 20a. Since the free stream velocities on the pressure and suction side are different it is argued that the two velocity defects decay differentially. This is clearly seen at the three radii $R = 0.7973$, $0.7297$, and $0.6581$. The variations between the two surfaces are small.

The general trend observed for the decay of the defect in axial and tangential velocity is that they decay very rapidly in the trailing-edge region and the decay tends to be less rapid in the near- and far-wake regions. The very rapid decay in the trailing-edge region is attributed to very large turbulence intensities, the pressure gradients, and three dimensional effects. The tangential velocity defect decays much more rapidly than the axial velocity defect. The contributing factor to the very rapid decay of the tangential velocities is the radial component of velocity as well as the radial distribution of free stream tangential and axial velocities (10).

The decay of the maximum radial velocity at the three radii are shown in Fig. 20b. It is clear that the decay is extremely rapid in the trailing-edge region and slows down considerably in the near- and far-wake regions. Even at 0.6 chords downstream it is seen that there is a significant radial velocity component.

1 In all the plots showing the decay characteristics, the decay quantity is plotted against $Z/\cos\beta_0$, since this quantity represents the streamwise distance from the trailing edge.
This not only indicates that the decay of radial velocities is less rapid compared to the defect in axial and tangential velocities but also reflects the radial migration of flow in the rotor-wakes. The very rapid decay in the trailing-edge region can be attributed to not only the high turbulence intensities but also the decay of the trailing-vortex system which dominate the flow in this region. Also plotted in Fig. 20b is the data on the maximum radial velocity from other sources (1, 2, 12, and 13). Comparison of the data from various sources indicates the effect of blade loading on the radial velocity. Higher blade loading induces larger radial velocities. The data in references (1, 2, and 13) correspond to low blade loading, the authors at moderate loading and those of reference (12) are at higher Mach numbers.

The decay of the total relative velocity, which is the resultant of axial, tangential and the radial velocity components is shown plotted in Fig. 20c.

Some very interesting features are noted in the total velocity decay which was not reflected in the axial and tangential velocity decay. For the radius \( R = 0.7297 \) and at \( Z = 0.0104 \) the defect in total velocity is 0.69 which increases to 0.735 at \( Z = 0.0208 \) and from on it monotonically decays to a value of 0.13 at half-a-chord downstream. The data of reference (14) for a transonic rotor also follows the trend of the authors data (Fig. 20c). The decay of the total velocity conforms more closely to the decay of the axial velocity. It is seen that at half-a-chord downstream, the defect in total velocity \( (w/W_0) \) is of the same order as that of axial velocity defect \( (w_{xc}/W_{0x}) \) while the tangential velocity defect \( (w_{xc}/W_{0x}) \) is much less than either the axial or the total velocity.

The initial wake characteristics differ considerably with radius and hence the decay rates are also different. The main controlling factor in the development of a wake is the section drag coefficient of the blade element. So normalizing the decay by the drag coefficient should collapse all the data points onto a single decay curve. This plot is shown in Fig. 21 where \( \sqrt{C_D} \) has been used as the normalizing parameter. It is evident that the decay data correlates well with \( \sqrt{C_D} \).

The functional dependency of the defects in axial and tangential velocities, and the radial velocity with streamwise distance \( (Z/cos\beta_0) \) is tried as shown in Fig. 21. The functional relationship for the decay of the axial and tangential velocity defect as well as radial velocity can be represented in the trailing-edge and near-wake region in the form

\[
\frac{w_{xc}}{W_{0x}} = K_1 \left( \frac{Z}{cos\beta_0} \right)^n
\]

where the values of \( K_1, Z_0 \) and \( n \) take different values for the three component of velocities. These relationships for the data presented in Fig. 21 are given by (in the near-wake region)

\[
\begin{align*}
\frac{w_{xc}}{W_{0x}} &= 0.592 \left( \frac{Z}{cos\beta_0} + 0.35 \right)^{-2.39} \\
0.03 &< (Z/cos\beta_0) < 0.2 \\
\frac{w_{th}}{W_{0x}} &= 0.778 \left( \frac{Z}{cos\beta_0} + 0.35 \right)^{-2.39} \\
0.03 &< (Z/cos\beta_0) < 0.2
\end{align*}
\]
velocities were found to vary linearly with $Z$ and can be represented by a general equation of the type

$$\frac{w_c}{w_0} = K_2 \left(\frac{Z}{\cos \theta_0}\right)^{-1} + K_3$$

(8)

where again the constants $K_2$ and $K_3$ take different values for the three different velocity components and are as follows:

$$\frac{w_c}{w_0} = 0.39 \left(\frac{Z}{\cos \theta_0}\right)^{-1} + 0.984$$

(9)

$$\frac{w_{\theta}}{w_0} = 0.221 \left(\frac{Z}{\cos \theta_0}\right)^{-1} + 1.492$$

(10)

$$\frac{w_z}{w_0} = 0.221 \left(\frac{Z}{\cos \theta_0}\right)^{-1} + 1.20$$

(11)

The decay rates as represented by the above equations are shown plotted in Fig. 21. It is seen that the data correlates quite well with these expressions.

Static Pressure Variations Across the Wake

In order to get a complete understanding of the flow in the wake, time averaged static-stagnation surveys were done in a relative frame of reference. A spherical head static-stagnation pressure probe built at Penn State and described earlier was employed for all the pressure surveys reported in this section. Variation of static pressure rise coefficient $(C_p)$ across the wake of the rotor is shown plotted in Fig. 22. Some of the traditional models of an inviscid core flow plus a viscous wake appear in these plots. Static pressure not only varies across the wake, but also in the wake edge near trailing edge due to the inviscid effects, which is not reflected in the stagnation pressure plots. In the trailing-edge region the static pressure variation across the wake is as large as 40 percent which drops off to about 5 percent in half-a-chord downstream. The measurement seems to confirm the trend observed by Thompkins and Kerrebrock (12) who have reported a 25 percent variation in static pressure across the wake at 10 percent of the axial chord downstream of a transonic rotor. In the present case the first measurement station is very close to the trailing-edge and very large static pressure gradients were measured. However, the data must be reviewed with some caution. As reported earlier, the pressure probes are subjected to wall vicinity effects and also the large angle variations reported earlier would adversely effect the values in the trailing-edge region.

As seen from Fig. 22, the static pressure increases towards the wake center and that the peak pressure lie outside the center of the wake towards the pressure surface. It is also observed that the minimum stagnation pressure and the maximum static pressure are off-set by a small tangential distance. At this point it is difficult to comment whether the error arises from the spatial error of the static-stagnation ports or whether it is because of the nature of the flow itself. Further analysis in understanding this flow situation is being pursued.

It is logical to assume that similarity exists in pressure profiles as with velocity profiles. The plot of such a similarity is shown in Fig. 23. The data seems to follow the Gaussian distribution fairly closely and only in the outer regions of the flow do we see substantial discrepancy from the curve $e^{-0.693n}$. This might be due to interference effects or the effect of the inviscid region located just outside the 'wake' region. The data due to Thompkins and Kerrebrock (12) also seem to show the existence of similarity (Fig. 23) in static pressure distribution.

Experimental evidence indicates that the static-pressure increases at the wake center and as the wake travels downstream this difference decreases. For the data presented for $R=0.7297$, the ratio of the static pressure at the wake center to the corresponding static pressure in the free-stream decays rather rapidly in the trailing-edge region and the decay is markedly slow in the near- and far-wake regions. This very rapid decay at the trailing-edge regions is attributed to the very rapid decay in velocity defect and turbulence intensity.

The existence of pressure gradients across the wake can be understood by examining the equation of motion in a rotating coordinate system in a direction normal ($n$) to the streamlines ($s$). The equation can be approximately written as

$$-\frac{1}{\rho} \frac{\partial P}{\partial n} = 2\Omega \frac{\partial v_n}{\partial r} \cos \theta + \frac{w_s^2}{R_c} - \frac{\partial}{\partial n} \left(\frac{w_n^2}{n^2}\right)$$

(12)

where $n$ is the direction normal to streamline, $w_n$ is the streamwise velocity, $R_c$ is the radius of curvature of the streamline and $w_n^2$ is the turbulent fluctuation in n direction. It is evident from the above equation that, in addition to Coriolis and centrifugal forces, the gradient of turbulent intensity in the n direction has an appreciable influence on the pressure gradient $\partial P/\partial n$. In the wake center, near the trailing-edge region, the turbulence intensity terms dominate and hence the static pressure gradients can exist even in the absence of curvature $(R_c)$. Qualitative analysis of this effect is given by Reynolds (15).

The decay of the maximum static pressure difference across the wake $(p_{max} - p_0)$, shown in Fig. 24, is rapid in the trailing edge region and reaches nearly the free stream values within half-a-chord downstream.

Fourier Decomposition of Rotor-Wake

In the aerodynamic and aeroacoustic analysis of rotors it is convenient to represent the wake velocities in terms of the Fourier series. This representation is possible because the velocity distributions are periodic and continuous. The velocity defect at a position $(r,\theta,z)$ can be represented as

$$u_{dN}(r,\theta,z) = A_0 + \frac{m}{n=1} \left[ A_n(r,\theta) \cos \frac{n\theta}{L} + B_n(r,\theta) \sin \frac{n\theta}{L} \right]$$

(13)

This type of representation was tried for the data of Raj and Lakshminarayana (1) and Reynolds et al. (2) and for the present data. The method is based on the recursive technique of Ralston (16). In the analysis, the velocity defect is normalized by the wake centerline defect and all the angles by the angle at half the wake-width.
Fig. 22 Static and stagnation pressure variation across the wake

Fig. 23 Static-pressure profile similarity for the wake

From the Fourier series we know that,

\[ A_n = \epsilon_n \frac{cn}{2\pi} \int_{0}^{2\pi} \frac{U_d}{U_dN} \cos \frac{n\theta}{\theta_L} d(\theta/\theta_L) \]  

(14)

where: \( \epsilon_n \) = Neumann's factor (=1 for \( n = 0 \); = 2 for \( n > 0 \))

2\( \pi \) = interval of integration (in the analysis only the wake portion is considered) for \( \theta/\theta_L \). Value of \( \theta/\theta_L \) at the wake center is \( \pi \) and at edges 0 and 2\( \pi \).

\( A_0 \) represents the average value across the wake and since the defect is normalized by the corresponding wake centerline defect, \( A_0 \) should approximately remain constant with downstream distance. This is shown plotted in Fig. 25 for the data from reference (1) and for the present case for the radii \( R = 0.7973, 0.7297 \) and 0.6581. Data of reference (1) is at zero loading. The effect of loading is clearly seen in these plots. Higher the loading, higher is the defect and correspondingly larger values of \( A_0 \).

The harmonic content in the wake is represented by the Fourier cosine coefficients and the asymmetry of the wake by Fourier sine coefficients. The first coefficient (\( A_1 \)) is shown plotted in Fig. 25. As normalized velocity defects were used in the analysis the values of \( B_n \) were found to be very small indicating the axisymmetric nature of the normalized wake. The average values of the harmonics are shown in Fig. 26 for the authors' data. Fig. 26 shows the plot of the scatter in the first four coefficients for the data under consideration. The Fourier curve correlated quite well with the data when the first four coefficients were considered in the analysis, with the correlation coefficient for the entire wake being about 0.998.

CONCLUSIONS

The measurements reported in this paper represent the first set of data in the trailing-edge region of a moderately-loaded compressor rotor blade. The results reveal the highly three-dimensional nature of the rotor-wake, with radial velocities dominating the flow in the trailing-edge region. It is also seen that the effect of loading has a substantial influence on the wake width and the decay of the velocity defect. Some of the major conclusions of the present investigation are:

1. In the trailing-edge region the radial velocities are found to be very large and are influenced by the trailing-vortex system. Large velocity defects are measured in the trailing edge region.
Fig. 24 Decay of the difference in static pressure with downstream distance

Fig. 25 Variation of Fourier coefficients $A_0$ and $A_1$ with downstream distance

Fig. 26 Average Fourier coefficients and scatter of harmonic content in rotor-wake
Higher blade loading results in not only higher velocity defect but also slows down the decay rate. Tangential velocities are found to decay much more rapidly in the trailing-edge region than the other velocity components. In the far-wake region, the decay of all the three components of velocities are of the same order. Decay laws are presented for all the velocity components.

3 The wake curvature is considerable in the trailing-edge region and beyond 0.2 chords downstream of the wake become very small.

4 The variation of the wake width with downstream distance is found to be very rapid in the trailing-edge region. In the near- and far-wake the growth is slow.

5 Mean velocity profiles in the trailing-edge and near-wake regions are unsymmetrical and tend to become symmetrical in the far-wake region. The effect of loading has been found to sustain the asymmetry to a much longer downstream distance as compared to a lightly loaded case investigated earlier. All the components of velocity become symmetrical and similar when the defect is normalized by its value at the wake center and the lengths by corresponding length scale. All the similarity profiles follow the Gaussian function given by \( e^{-0.693n} \). The wake profiles are also represented by Fourier series. The magnitude of the coefficients decrease rapidly, with the first three coefficients being the dominant ones.

6 Static-pressure in the wake is found to increase towards the wake center. The decay of static-pressure difference across the wake is found to be rapid in the trailing-edge region and it slows down in the far-wake region. The static-pressure profiles also exhibit the similarity, except in the trailing-edge region, and the similarity follows the Gaussian distribution, \( e^{-0.693n^2} \). The discrepancy in the trailing-edge region from the Gaussian might be due to the complex interactions of the boundary-layer flow and the trailing-vortex systems.

7 The wake decay and wake-width differ in the radial direction. The main controlling factor seems to be the section-drag coefficient of the blades. The width as well as the decay correlate well with \( \sqrt{C_D} \).

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