A METHOD FOR THE CALCULATION OF THE TIP CLEARANCE FLOW EFFECTS IN AXIAL FLOW COMPRESSORS.
Part I: Description of Basic Models.

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ABSTRACT
Theoretical models for the investigation of the phenomena connected with the tip clearance are presented. The mass flow rate through the gap, the formation and evolution of the leakage vortex and the losses occurring inside and downstream of the gap are considered.

Firstly, a model was developed for the description of the flow through the gap, which uses different simple velocity profiles at the gap exit. The model recognizes the basic flow characteristics inside the gap.

A new method is proposed for the calculation of the shed vorticity and the formation of the leakage vortex. The moment of momentum equation is used along with the conservation of mass, in order to provide the circulation of the leakage vortex.

A diffusion model for the vorticity distribution is used for the calculation of the pressure deficit field, so that the total pressure losses due to the presence of the leakage vortex, are derived. Theoretical results are compared to experimental ones for compressor and turbine cascades as well as for single rotors. The agreement between theory and experiment is good.

NOMENCLATURE

- $C_D$: discharge coefficient (equation (9))
- $C_p$: static pressure coefficient (equation (8))
- $c$: chord
- $e$: tip clearance
- $h_1$: characteristic length of vortex core control volume (fig. (2))
- $i,j,k$: orthogonal Cartesian coordinates (fig. (2))
- $K$: $K = 1 - \frac{\Gamma}{\Gamma_B}$
- $l$: characteristic length of vortex core control volume (fig. (2))
- $p$: static pressure
- $P_t$: total pressure
- $r$: radial position
- $R$: vortex core radius
- $t$: time
- $u,v,w'$: vortex radial, peripheral and axial velocity components
- $\bar{V}$: flow velocity at the control volume of the vortex core
- $\bar{W}$: flow velocity inside the tip gap
- $W$: free stream axial velocity
- $w$: $w = W - w'$
- $z$: distance along vortex axis

Greek symbols

- $\Gamma$: leakage vortex circulation
- $\Gamma_B$: blade bound circulation
- $\Delta E$: rate of gap losses (equation (10))
- $\Delta s$: finite length along the blade camber line
- $\Lambda$: Owen's constant for the calculation of the eddy viscosity
- $\nu$: kinematic viscosity
- $\nu_e$: eddy viscosity
- $\rho$: density
- $\sigma$: contraction ratio (equation (1))
- $\phi, \theta$: angles of vector $n_3$ (fig. (2))
- $\omega$: vorticity

Subscripts

- $i,j,k$: orthogonal Cartesian coordinates (fig. (2))
- $\text{max}$: maximum value of velocity on the corresponding profile (fig. (1))
- $n$: normal to the surface direction
- $N$: normal to the blade camber line direction

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S direction along the blade camber line
1,2,3,4 positions along the blade passage (fig. (1)), or surfaces of the control volume of fig. (2)
∞ free stream conditions

INTRODUCTION

The axial compressor is one of the major components of an aero engine, and it is well known that tip leakage has a significant effect on its performance. Mechanical constraints determine the magnitude of tip clearance, which is associated with a reduction in efficiency as well as in the range of stable operation of the machine. The prediction of the losses associated with the tip clearance, as well as the estimation of the induced kinematic field, is essential in the matching of the successive stages of the compressor, in order to optimize its performance.

Extensive experimental and theoretical investigations have been carried out by various researchers, in order to provide a better understanding of the complicated phenomena, taking place inside and downstream of the tip clearance. Different approaches were adopted for the modelling of the tip clearance effects. Rains (1954) first realized that the pressure difference across the blade tip was dominant compared to the one along the blade on both surfaces. Consequently he proposed an order of magnitude theory where the clearance gap flow is modeled as being two-dimensional in planes normal to the mean camber line with the longitudinal momentum component being conserved through the gap. An expression for the losses due to the tip clearance presence was obtained assuming that all the secondary kinetic energy of the leakage flow would be dissipated during the mixing processes.

Lakshminarayana and Horlock (1965) used the lifting line theory to predict tip clearance effects. Their observation that the strength of the trailing vortex is less than the blade's bound vorticity, resulted in the adoption of an empirical relation between the retained lift at the blade tip and the tip clearance height. An approximate expression for the induced drag on the blade was proposed, based on lifting line theory. A further development of the model, with the adoption of a Rankine vortex core formation for the leakage vortex, provided good predictions of the flow angle. Vavra (1960) extended Rains’ method to a more general form. Booth et al. (1982) used Rains basic assumptions to propose a tip gap model employing an iterative procedure for the calculation of the tip clearance jet magnitude and angle.

A combined two-dimensional potential flow and mixing model was proposed by Tilton (1986) for the calculation of the pressure distribution through the gap and the evaluation of the discharge coefficient.

Yaras et al. (1989) used a variation of Rains’ model to determine the magnitude and the direction of the leakage jet. Midspan pressure differences, shifted rearward, along with a linear rise in pressure difference at the leading edge, are used as an input to the calculation. A loss model (Yaras et al. 1990a) was developed as an extension to Vavra’s (1960) method.

Chen et al. (1990) adopted a slender body approximation for the decomposition of the tip clearance velocity field into independent through-flow and cross-flow, based in the dominance of normal pressure gradients as Rains proposed. In order to reconstruct the leakage vortex evolution the 2-D unsteady clearance flow was considered instead of a 3D-steady one, using a Galilean transformation.

The present work presents theoretical models for the investigation of the phenomena related to the tip clearance presence. In order to facilitate the development of such models, the work of previous researchers has been reviewed and used as a starting point. However, a different approach was adopted and additional work has been done, leading to relatively simple models which can provide a good description of the complicated tip leakage flow.

GAP FLOW MODEL

Rains (1954) introduced the assumption that the flow through the tip gap of a compressor blade may be simulated on the basis of two-dimensional considerations. He acknowledged the predominant role of the pressure difference across the gap against the pressure gradient along the blade chord on both sides of the blade and he assumed that the gap exit flow results from this pressure difference in the direction normal to the tip blade camber line. According to Rains' two-dimensional model, the flow across the blade is characterised by the formation of a vena contracta at the gap entrance, followed by a mixing region, with the longitudinal momentum component being conserved through the gap.

Wadia and Booth (1982), Booth et al. (1982) as well as Moore...
and Tilton (1988) validated Rains' basic model and improved it, providing good results for turbine cases. Very good agreement between the Rains' model and experiment was obtained also by Storer (1991) for a compressor linear cascade.

The simple model adopted here and schematically presented in fig. (1), is very similar to the ones mentioned above and uses Rains' basic assumptions in order to calculate both the mass flow rate through the gap and the corresponding total pressure losses. Following the formation of the vena contracta, a loss producing region appears, characterised by one of the simple profiles present in fig. (1). The profile shape at different gap positions is assumed to depend on the value of the dimensionless distance (normalized with the gap height) from the entrance of the gap.

This simple model, predicts quite accurately the mean exit jet velocity (and the corresponding mass flow rate of the tip clearance jet). It is emphasized that this is critical for the prediction of the features of the tip clearance vortex. The model also provides an acceptable estimate of the gap losses.

According to this model, the gap exit jet flow depends on both the static pressure at the gap exit and the total pressure at the gap entrance. Alternatively, if conservation of the longitudinal momentum is considered, it could be stated that the exit jet flow is driven by the static pressure difference existing between the pressure and the suction sides. A potential flow field is assumed to prevail near the entrance of the gap, leading to the formation of the "vena contracta". This no loss region is followed by a mixing region, which is responsible for the non uniform jet velocity profiles at the gap exit. The presence of a vena contracta has been identified among others by the experimental investigations of Sjolander and Amrud (1986) and Yaras et al. (1989) for turbine blades. Storer's (1991) experiments on a compressor cascade suggest that the length scale of reattachment inside the clearance gap depends heavily on height of the tip gap. Similar results were obtained by Moore et al. (1989) by performing laminar and turbulent flow calculations for an idealized two-dimensional tip gap geometry. These results, compared with Graham's (1985) measurements for different gap heights, suggest that the ratio of the tip gap height to blade thickness could be a useful criterion for the reconstruction of the jet profile at exit of the tip clearance. The adoption of different simple profiles, based on this ratio, is used in the present model to determine the mass flow rate through the gap. As far as the losses are concerned, mixed out losses are only considered. These can be computed by applying momentum considerations to each of the profile shapes adopted. It is, thus, assumed that all the losses are taking place inside the mixing region after the vena contracta.

The potential solution of a flow at rest entering a slot, provides the contraction ratio \( \sigma \) (fig. (1)), as

\[
\sigma = \frac{\pi}{\pi + 2} = 0.611
\]  

Eq. (1) has been derived using the analysis of Milne-Thomson (1968) for symmetric slots, where the centerline of the slot represents the endwall (see for a comprehensive derivation Moore et al. (1989)). The experimental results of Moore and Tilton (1988) and their analysis provide support for the existence of a vena contracta with a value of the contraction ratio close to the one given in eq.(1).

Applying the Bernoulli's theorem from position 1 to position 3 (fig. (1)) and considering that the momentum of the flow through the gap is conserved in the direction of the mean camberline one gets

\[
P_i = P_3 + \frac{1}{2} \rho W_{N3}^2
\]  

where subscript \( N \) denotes the normal to the camberline direction.

Applying the continuity equation from station 3 to station 4 one gets

\[
W_{N3} \sigma = \int_{S_3}^{S_4} W_{N4} \, dy
\]  

For the various gap exit profile shapes considered different expressions result from the above equation.

\[
\frac{W_{N4}}{W_{N3 \text{max}}} = 1 + \frac{2}{3} \left( \frac{1}{\sigma} - 1 \right)
\]

Applying the momentum equation in the normal to the mean
cambere line direction from position 3 to position 4 (fig. (1)) one gets

\[ (P_3 - P_4) e_{\zeta} = - \rho \int_0^{\infty} W_{\infty}^2 dy \]  

\[ \text{(5)} \]

For the "parabolic" profile considered, this expression yields

\[ P_3 - P_4 = - \rho \sigma \int_0^{\infty} W_{\infty}^2 dy \left[ 1 \pm \frac{1}{2} \left( \frac{1}{\sigma} - 1 \right) \right] \]  

\[ \text{(6)} \]

Combining eqs. (2),(3),(5) and considering the conservation of the longitudinal component of momentum, a relation between the jet velocity at the gap exit and the pressure difference between suction and pressure sides at the tip region is derived. For the case of the "parabolic" profile this relation reads

\[ \frac{W_{\infty}}{W_0} = 0.92 \sqrt{\text{C}_{\text{P}_{24}}} \]  

\[ \text{(7)} \]

where

\[ \text{C}_{\text{P}_{24}} = \frac{P_2 - P_1}{\frac{1}{2} \rho W_1^2} \]  

\[ \text{(8)} \]

For the mean value of the normal component of jet velocity at the gap exit, a similar relation may be derived, which reads

\[ \frac{W_{\text{Neff}}}{W_0} = C_D \sqrt{\text{C}_{\text{P}_{24}}} \]  

\[ \text{(9)} \]

where \( C_D \) is the discharge coefficient. Its value depends on the shape of the adopted gap jet profile. For the profile shapes considered in this work the \( C_D \) values are:

<table>
<thead>
<tr>
<th>Profile Shape</th>
<th>( C_D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>uniform</td>
<td>0.84</td>
</tr>
<tr>
<td>parabolic</td>
<td>0.8</td>
</tr>
<tr>
<td>triangular</td>
<td>0.76</td>
</tr>
<tr>
<td>inverse parabolic</td>
<td>0.72</td>
</tr>
</tbody>
</table>

The pressure difference used for the calculation of the mass flow rate through the gap is the one modified by the presence of the leakage vortex. It is further modified near the leading edge region of the blade, as proposed by Yaras et al. (1989), using a linear variation of the pressure difference for the first thirty percent of the blade chord.

In order to calculate the losses inside the gap it is assumed that the loss production is occurring at the mixing region downstream the vena contracta. Then, one gets for the losses

\[ \Delta \tilde{E} = \left[ \frac{1}{2} \Delta \sigma \int_0^{\infty} W_{\infty}^2 dy \right] \left[ \frac{1}{2} \int_0^{\infty} \frac{P_2 - P_1}{\rho} \Delta \sigma e W_{\infty} \right] \]  

\[ \text{(10)} \]

where \( \Delta s \) is a finite distance along the mean camber line of the blade. The work due to the stresses in the shear layers near the wall has been ignored in the above equation. For the "parabolic" profile, equation (10) reads

\[ \Delta \tilde{E} = \left[ \frac{1}{2} \Delta \sigma \int_0^{\infty} W_{\infty}^2 dy \right] \left[ \frac{1}{2} \int_0^{\infty} \frac{P_2 - P_1}{\rho} \Delta \sigma e W_{\infty} \right] \]  

\[ \text{(11)} \]

Taking into account the moment of momentum equation and the conservation of momentum in the direction of the blade camber line, equation (11) finally yields

\[ \Delta \tilde{E} = \frac{1}{2} \Delta \sigma e W_{\text{Neff}}^2 B \left( - \text{C}_{\text{P}_{24}} \right)^{3/2} \]  

\[ \text{(12)} \]

where \( B \) is a coefficient depending on the profile shape. Values of \( B \) for the various profile shapes are shown below

- uniform: -0.24
- parabolic: -0.19
- triangular: -0.15
- inverse parabolic: -0.11

The mixed out losses inside the tip clearance can be computed using equation (12) with constant \( B \) taking the above values, according to the shape of the profile at the gap exit.

**VOXER MODEL**

The calculation of the induced kinematic field as well as the estimation of the losses downstream of the gap exit is based mainly on the correct computation of the strength and position of the leakage vortex. The formation and the evolution of this vortex is quite similar to that of the finite wing case and previous researchers insisted on using prediction methods tailored to this last case.

Lifting line theory, originally developed to predict lift forces on finite wings, was introduced by Lakshminarayana (1964) and others in the turbomachinery field, in order to predict the effects of the leakage vortex in the kinematic field and the amount of loss occurring inside the blade passage. The fact that the tip clearance vortex strength is not equal to the blade bound vorticity, led Lakshminarayana and Horlock (1965) to introduce a factor \( K \) which expresses the percentage of the bound vorticity retained inside the tip region. The rest of the
bound vorticity is shed into the tip vortex. An experimental relation was proposed by Lakshminarayana and Horlock (1965), relating $K$ to the clearance-to-chord ratio. A similar approach was adopted by Lewis and Yeung (1977), who proposed a relation for calculating the retained lift (and thus the vortex strength), which gives different results from the ones of Lakshminarayana’s and Horlock (1965). Additional experiments exist which confirm that the circulation of the leakage vortex is a fraction of the blade’s bound circulation but do not indicate directly the validity of the "retained lift" theory, which stipulates that the shed out vorticity is produced by part of the vortex lines of the blade, the rest going towards the endwall and producing the existing pressure difference along the gap.

On the other hand various researchers, such as Yamamoto (1989), Inoue and Kurumaru (1989), Sjolander and Amrud (1986) and others demonstrated that the strength of the leakage vortex seems to increase with increasing tip clearance height. This increase in the amount of vorticity of the leakage vortex is followed by an increase of the vortex diameter. In addition, the data of Inoue and Kurumaru (1989) demonstrate that the shed circulation depends also on the relative wall speed. In the case of a compressor this relative wall motion increases the shed out vorticity, while in the case of a turbine it decreases the amount of vorticity shed into the blade passage. This fact was confirmed by the experimental work of Yaras et al (1991) for a turbine cascade, where the relative wall motion was simulated by a moving belt. The above experimental evidence seems to suggest that the key factor for the determination of the amount of shed out vorticity is the mass flow rate through the tip clearance. In the present work a model was developed which takes into account the above experimental evidence for the behaviour of the shed out vorticity.

The flow field near the endwall is characterised by the interaction of two opposing flows. The secondary flow, having a direction from the pressure to the suction side of the passage, and the leakage jet flow, coming out of the gap at an angle to the main flow. As a result of this interaction very close to the endwall, the leakage flow rolls down away from the endwall, along a separation line, and forms the leakage vortex. In the outer flow, this vortex interacts with the tip-side passage vortex, rotating contrary to the leakage vortex. At the same time, as we march downstream, the additional mass from the tip clearance jet, entering the vortex, increases both its radius and its strength. The complexity of this flow situation suggests that some drastic assumptions should be adopted in order to derive a simple model.

In order to formulate this model, experimental evidence was used. In fact, flow visualisation suggests that the mass leaving the gap exit enters in its quasi-totality inside the leakage vortex. The performed measurements (Lakshminarayana (1970), Inoue et al (1989), Yaras et al (1990), etc) indicate that the vortex nearly has a solid body rotation structure. These features are retained in the adopted model. In order to follow in a schematic way the flow situation depicted by flow visualization and available experimental evidence, it was assumed that the mass flow coming out of the gap is wrapped around the existing solid body rotation vortex, increasing its radius and moment of
moment of momentum theorem to the control volume $R$ of fig. (2), for steady state conditions and ignoring gravity effects. Considering the component of the moment of momentum equation in the direction of the axis of the solid body rotation vortex and neglecting the contribution of the shear stresses we have

$$\omega_1\int_0^{2\pi R} \rho x^2 V_{z1} \, dr \, d\phi - \omega_2\int_0^{2\pi R} \rho x^2 V_{z2} \, dr \, d\phi = \Delta p_{i2} \cdot \phi $$

$$+ \int_0^{R} \rho V_{z1} \, V_{z0} \, ds =$$

$$= \int_0^{R} \left( \frac{1}{2} \cos \phi + h \sin \phi \cos \theta \right) P \, dh \, ds$$

with $V_{3n}$ being the velocity component in the direction of $\vec{\pi}_3$.

The mass balance for the control volume $R$ reads

$$\int \rho dz_1 \, V_{z1} + | \int \rho dz_2 \, V_{z2} | = \int \rho dz_2 \, V_{z1}$$

Assuming that the density is constant and uniform for the velocity magnitude on the surfaces $(S_1)$ and $(S_2)$, the following equations are derived from equations (13) and (14) respectively

$$\rho \left[ \omega_1 V_{z1} \frac{R_1^4}{2} - \omega_2 V_{z2} \frac{R_2^4}{2} - V_{z2} \Delta x \frac{(h_1 + h_2)}{2} \right]$$

$$= P_3 \int \left[ (\cos \phi + h \sin \phi \cos \theta) \right] dh \, ds$$

$$\pi R_2^2 V_{z2} = \pi R_1^2 V_{z1} + V_{z2} \Delta x$$

The circulation at a radius $R$, associated with a solid body rotation of speed $\omega$ is

$$\Gamma = \oint_c d\vec{r} \cdot 2 \pi \omega R^2$$

Combining eqs (15), (16), (17) yields the expression of the vortex strength at station 2 $\Gamma_2$, expressed in terms of the conditions at station 1

$$\Gamma_2 = -\frac{1}{V_{z2} R_2^2} \left[ \Gamma_1 V_{z1} R_1^2 + 2 V_{z2} \Delta x (h_1 + h_2) \right] -$$

$$- \frac{4}{V_{z2} R_2^2} \int \left( \frac{1}{2} \cos \phi + h \sin \phi \cos \theta \right) dh \, ds$$

While equation (18), may be used for the calculation of the strength of the vortex, the mass balance equation (16) provides
FIG.(5): CALCULATION OF THE MIXED OUT LOSSES INSIDE THE TIP CLEARANCE.

a basis for the calculation of the vortex core radius.

THE DIFFUSION OF THE LEAKAGE VORTEX

The rolling up of the tip clearance jet and the formation of the leakage vortex is followed by a diffusion of vorticity, leading to an increase of the vortex radius and a reduction of the maximum value of vorticity downstream the trailing edge. Calculations performed using the simple model described above, demonstrate reasonable agreement with experiment. However, discrepancies were present, especially in the region downstream of the blading. For this reason an effort was made to incorporate the diffusion process mentioned above in the model, so to make more realistic estimates for the total pressure losses occurring inside the blade passage due to the presence of the leakage vortex. The corresponding theoretical reasoning is described in this section.

During the evolution of the leakage vortex, a reduction of static pressure is produced inside it. This modification of the pressure field can be easily distinguished in the measured static pressure difference distributions at the tip region of the blade suction side. The diffusion of the leakage vortex produces a rise in the static pressure along the centerline of the vortex and the pressure deficit is reduced downstream of the trailing edge (Yaras et al. (1990), Inoue et al. (1989)).

Yaras et al. (1990), (1991) used Lamb's (1932) formulation for the diffusion of a line vortex, for the calculation of the induced kinematic field of the leakage vortex, with very promising results. Storer (1991) used Newman's (1959) formulation for the same problem, to calculate the blockage caused by the presence of the leakage vortex. His measurements of the axial velocity and total pressure loss distributions, downstream the trailing edge, suggested that the adoption of the simple model of a line vortex can describe the phenomenon adequately.

The models developed by Lamb (1932) and Newman (1959) for the diffusion of a line vortex are used in the present work to simulate the diffusion of the leakage vortex and predict the pressure disturbance as well as the total pressure loss distribution due to the tip clearance presence. The corresponding formulation, described in the Appendix is involved also in the computation of the shed out vorticity, as the vortex radius is increasing due to the diffusion process and,
RESULTS AND DISCUSSION

In order to validate the proposed flow model, the experimental data of Storer (1991), Yaras et al. (1989) and Inoue et al. (1985) were used.

The first case (Storer (1991)) concerns a linear compressor cascade with a circular arc camber line and 5% maximum thickness-to-chord ratio. The inlet velocity of 24m/s corresponds to an inlet Mach number of 0.03. Five different tip gap heights (besides zero clearance) were measured, with the smallest being 0.5% of the chord and the largest 4% of the chord. The mean jet velocity distributions normal to the blade's camber, are available only for three clearances (1% e/c, 2% e/c and 4% e/c) and these are used for the validation of the model. The comparison presented in figures (3a) to (3c), between theory and experiment for the mean jet velocity distribution normal to the blade's camber line is good. The adoption of linear variation of pressure for the first thirty percent of the chord had a positive influence especially in the case of 2% e/c (fig. (3b)).

FIG.(7): PREDICTION OF THE PRESSURE MODIFICATION AT THE TIP CLEARANCE LEVEL, BASED ON THE EXPERIMENTAL VALUES OF PRESSURE DIFFERENCE AT THE TIP LEVEL FOR THE CASE OF ZERO CLEARANCE.
Better results were obtained for the second test case, which concerns to the turbine linear cascade, tested by Yaras et al. (1989). The blades have a maximum thickness-to-chord ratio of 0.0984 and the chord based Reynolds number is 4.3x10^5, with the corresponding inlet velocity being 30 m/s. Figures (4a) and (4b) present comparisons of the calculated and measured gap exit mean jet velocity for equal to 0.02 e/c and 0.032 a/c. Especially for the smaller tip clearance, the present flow model is predicting the details of the jet velocity distribution quite well, which is essential for the correct prediction of the shed pressure difference, including the vortex influence, is known, the presented calculations were performed using as an input the gap exit mean jet velocity for equal to 0.02 e/c and 0.032 a/c. The corresponding inlet velocity being 30 m/s. Figures (4a) and (4b) present comparisons of the calculated and measured total pressure loss through the gap, defined as

\[ \int W_{n}(s) (P_{r}(s) - P_{w}(s)) ds \]  \hspace{1cm} (19)

Comparisons between theory and experiment are presented in figures (5a) and (5b). The existing differences can be partially explained by the fact that the present model does not include the losses due to the endwall boundary layer inside the gap. However, the trends are well predicted and the fact that this part of the losses is small compared to the total losses attributed to tip clearance effects, renders this level of comparison adequately accurate for the complete calculation. It should be noted that the coefficient \( C_{b} \), appearing in equation (9) has been derived according to (9a), while the predictions of other workers seem to have been computed on the basis of the value of \( C_{b} \) which produced the best fit with experiment. The significance of mixing losses upon the value of \( C_{b} \) has been already pointed out by Moore and Tilton, (1988).

Inoue's (1985) data were used in order to validate the ability of the vortex model to predict the circulation of the leakage vortex correctly. The corresponding rotor comprises of 12 blades of NACA 65 series profile, with 6% of the chord maximum thickness at the tip and 10% at the hub. Five different clearances were measured (0.5, 1.0, 2.0, 3.0 and 5.0 mm) keeping constant the blade geometry at tip section and changing the inner diameter of the casing.

The computation of values of the shed vorticity into the leakage vortex may be performed only when the tip clearance model described above has been implemented in a secondary flow calculation. The details of such a complete calculation procedure are described in Part II of this paper. In order to give indications concerning the validity of the model proposed, comparisons between the complete computational procedure (including the diffusion model) and experimental results of Inoue et al (1985) are presented in fig. (6) of this part of the paper, for various tip gap heights. In the same figure, the fraction of the bound vorticity which is shed into the leakage vortex calculated with the models of Lakshminarayana (1970) and that of Lewis and Yeung (1977) are also included. The agreement between our model and the experiment is quite good.

It is emphasized that the present model is capable of reproducing a considerable range of values for the ratio \( \Gamma/\Gamma_{b} \), starting from very small ones corresponding to small gap values up to the value of unity (shed vorticity equal to the bound vorticity). More calculation results, which prove the validity of the present model are presented in Part II of the paper.

The ability of the diffusion model to describe the corresponding process and improve the predictive capabilities of the simple model presented initially, is examined in two different ways. The prediction of the modified static pressure distribution at the tip clearance region, due to the leakage vortex presence is checked firstly, followed by the estimation of the total pressure loss distribution downstream of the trailing edge. The diffusion analysis, on which both the above mentioned predictions are based, is outlined briefly in the Appendix. The expression for the static pressure distribution, which is used in the present work reads

\[ \frac{\partial P}{\partial r} = \frac{\Delta P}{4 \pi r^3} \left[ 1 - 2 \exp \left( \frac{-r^2}{R^2} \right) + 2 \exp \left( \frac{-2r^2}{R^2} \right) \right] \]  \hspace{1cm} (20)

while the total pressure loss is computed as

\[ \Delta P_{t} = \Delta P + \frac{1}{2} \rho \left[ 2W(w - v^2 - w^2 - u^2) \right] \]  \hspace{1cm} (21)

In the above equations \( u,v,w \) are the corresponding velocity components, whose expressions are given in the Appendix.

Considering the values of the static pressure in the free stream, it is possible to integrate numerically equation (20), attributing to \( R \) the value resulting from the combination of equations (16) and (27). Some remarks are necessary, concerning the utilization of equation (21). Besides the static pressure, the computation of which was outlined above, it must be remarked that the available experimental evidence shows that the radial component of the velocity is rather small and could be neglected. As for the two other components, the peripheral one can be computed using the information, which was already given above, while the axial component requires a value for constant \( A \) for its computation. This value may be obtained from the already known values of the mass flow rate inside the leakage vortex.

Comparisons of computation results with Storer's (1991) experimental data are presented in figures (7a) through (7d). The experimental values of the pressure difference for the case of zero clearance were used as input. The agreement between theory and experiment is good and the corresponding model seems to be adequate for the prediction of the deficit pressure field of the leakage vortex. The reason for the discrepancy existing near the leading edge has already been discussed.
Figures (8a) and (8b) present comparisons between mass averaged experimental data and peripherally mean predictions for the Storer's (1991) experiment. The theoretical results have been obtained using the complete calculation procedure outlined in Part II of this paper. The vortex losses were obtained by subtracting the losses for the zero clearance case. Figures (8c) and (8d) present comparisons between peripherally mean predictions and the circumferentially averaged Inoue's (1985) experimental data. The presented values were obtained by subtracting the losses for the smallest clearance of 0.5 mm. The errors introduced by this assumption seem to be negligible, as the losses for the case of 0.5 mm gap are very small compared to those for the large clearances.

The comparisons for both cases indicate good agreement between theory and experiment. Looking at the variation of the total pressure loss it may be seen that the position and strength of the leakage vortex is sufficiently accurately predicted.

CONCLUSIONS
Basic theoretical models for the prediction of the tip clearance effects were examined and presented in this paper.

The first model concerned the flow inside the tip clearance gap. Some of Rains' basic assumptions, confirmed even quite recently by various researchers, were used in order to construct the model. Addition of a simple modelling of the loss production mechanism inside the gap enables the model not
only to predict the gap losses but also to calculate the gap exit jet velocity with reasonable accuracy. The satisfactory prediction of the mass flow rate through the gap is emphasized, as this quantity monitors the shed vorticity from the gap into the leakage vortex.

The second model concerned the formation and the development of the leakage vortex. An alternative and simple way of describing the process of the rolling up of the jet issued from the gap in order to form the leakage vortex is presented, which provides an additional indication that the retained lift theory is invalid. The proposed model was improved by introducing a mechanism of vorticity diffusion. This provides the model with the ability to predict correctly not only the shed out vorticity, but also the distribution of the total pressure losses due to the tip clearance effects.

From the theoretical developments presented in this paper it may be seen that the two models are able to describe the basic tip clearance mechanisms essentially without any empirical information, other than the theoretical assumptions underlying them.

Although the present work was performed for compressor case, the validity of both models was tested against all available experimental results, including those obtained for turbines. The basic reason for this is the scarcity of detailed experimental data required for checking up the discussed models. Comparisons with experiment reasonably confirmed the validity of the models for the compressor case. For the turbine case it is concluded that more experimental data are necessary for testing the capabilities of models proposed.

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APPENDIX

Line Vortex Diffusion Model

The basic model for which Lamb (1932) gave a mathematical solution may be discussed briefly as follows. We consider a uniform and parallel to the axis \( r=0 \) flow (of velocity equal to \( W \)). At time \( t=0 \), the vorticity is zero everywhere, except along the axis \( r=0 \) where there exists a line vortex of strength \( \Gamma \). As the time passes vorticity is diffused radially and, if the problem is considered in successive planes normal to the vortex axis, a core is formed. Those planes lie at distances \( z \) equal to \( z=Wt \), where \( t \) is the time needed to reach the corresponding plane if we are moving with speed equal to \( W \). Then the distribution of vorticity reads

\[
\omega(r,t) = \frac{\Gamma}{4\pi r} \exp\left(-\frac{r^2}{4vt}\right) \tag{22}
\]

where \( \nu \) is the kinematic viscosity, \( r \) is the radial distance from the vortex axis and \( t \) is the time measured from the beginning of the diffusion process.

The peripheral velocity distribution is then given by

\[
\nu(r,t) = \frac{\Gamma}{2\pi r} \left(1 - \exp\left(-\frac{r^2}{4vt}\right)\right) \tag{23}
\]

For \( r<\sqrt{4vt} \) the motion is a rigid body rotation with angular velocity \( \Gamma/8\pi vt \), while for \( r>\sqrt{4vt} \) the motion is irrotational, similar to the one of a line vortex. This leads to a vortex core radius of

\[
R = \sqrt{4vt} \tag{24}
\]

We consider the problem not as a three-dimensional steady one but as a two-dimensional unsteady one, assuming that the time steps of the diffusion process correspond to distances \( \Delta z \) between successive planes normal to the vortex axis, fulfilling the relation

\[
\Delta z = \Delta t W \tag{25}
\]

where \( W \) is the velocity of the mainstream along the vortex axis. From equation (24) we have for two successive time steps

\[
R^2 \Delta z = 4\nu \Delta t
\]

or from equation (25)

\[
\Delta R^2 = 4\nu \frac{\Delta z}{W} \tag{27}
\]

The turbulent nature of the tip vortex flow can be incorporated by replacing viscosity \( \nu \) with \( \nu + \nu_e \), where \( \nu_e \) is the appropriate eddy viscosity value. Yaras and Sjolander (1990) use Owen’s (1970) modelling for \( \nu_e \) which reads

\[
\nu_e = A \nu \frac{\Gamma}{\sqrt{\nu}} \tag{28}
\]

with \( A \) taking values between 0.7 and 1.2. The value of 0.95 has consistently been used throughout the present work. Equation
(27) provides a way of estimating the expansion of the leakage vortex core due to the diffusion of its vorticity.

A similar solution to the problem of a line vortex diffused radially was proposed by Newman (1959). Choosing cylindrical polar co-ordinates $r, \theta, z$, with Oz the axis of the line vortex, he used simplified forms of the three momentum equations accompanied by the continuity equation in order to provide the distributions of the three velocity components and pressure at successive planes normal to the vortex axis.

Denoting the radial velocity by $u$, the rotational velocity by $v$ and the longitudinal by $w'$ and putting $w' = W-w$ where $W$ is the axial velocity in the undisturbed fluid, the following relation is derived

$$v = \frac{\Gamma}{2\pi r} \left[ 1 - \exp \left( -\frac{w'^2}{4vz} \right) \right]$$

(29)

Considering equations (24) and (25) we have

$$v = \frac{\Gamma}{2\pi r} \left[ 1 - \exp \left( -\frac{r^2}{R^2} \right) \right]$$

(30)

which is identical to equation (23). Newman's analysis is interesting, because expressions for the other velocity components, as well as for the static pressure are developed. The two other velocity components may be finally expressed as

$$w = A \exp \left( -\frac{r^2}{R^2} \right)$$

(31)

$$u = -\frac{A}{2z^2} \exp \left( -\frac{r^2}{R^2} \right)$$

(32)

where $A$ is the constant of integration.

The expression for the static pressure reads

$$\frac{\partial P}{\partial r} = \frac{\rho \Gamma^2}{4\pi r^3} \left[ 1 - 2 \exp \left( -\frac{r^2}{R^2} \right) + \exp \left( -\frac{2r^2}{R^2} \right) \right]$$

(33)

The numerical integration of the above equation provides the distribution of pressure inside the vortex using as boundary condition the value of the undisturbed free stream static pressure.

If $P_{te}$ and $P_t$ are the total pressure in the free stream and inside the vortex respectively we have that

$$P_{te} = P_t + \frac{1}{2} \rho [W^2 - (w'^2 + v'^2 + u'^2)]$$

(34)

Substituting $w'$ in the above equation the total pressure loss is expressed as

$$\Delta P = \frac{\Delta P}{2} \rho [2Ww' - w'^2 - v'^2 - u'^2]$$

(35)

This provides the distribution of total pressure losses inside the vortex core which is diffused radially, using equations (30), (31), (32) and (33) in order to calculate the values of the various quantities appearing on the right hand side.

REFERENCES


Sjolander S.A., Amrud K.K., 1986, "Effects of Tip Clearance on Blade Loading in a Planar Cascade of Turbine Blades".
ASME paper 86-GT-245.


