A METHOD FOR THE CALCULATION OF THE TIP CLEARANCE
FLOW EFFECTS IN AXIAL FLOW COMPRESSORS.
Part II: Calculation Procedure.

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ABSTRACT
An algorithm was set up for the implementation of the tip clearance models, described in Part I, in a secondary flow calculation method. A complete theoretical procedure was, thus, developed, which calculates the circumferentially averaged flow quantities and their radial variation due to the tip clearance effects.

The calculation takes place in successive planes, where a Poisson equation is solved in order to provide the kinematic field. The self induced velocity is used for the positioning of the leakage vortex and a diffusion model is adopted for the vorticity distribution.

The calculated pressure deficit due to the vortex presence is used, through an iterative procedure, in order to modify the pressure difference in the tip region. The method of implementation and the corresponding algorithm are described in this part of the paper and calculation results are compared to experimental ones for cascades and single rotors. The agreement between theory and experiment is good.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>c</td>
<td>chord</td>
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<tr>
<td>e</td>
<td>tip clearance</td>
</tr>
<tr>
<td>F</td>
<td>blade surface force per unit surface</td>
</tr>
<tr>
<td>m,n</td>
<td>Meridional and normal directions of the orthogonal curvilinear coordinate system</td>
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<tr>
<td>N</td>
<td>number of blades</td>
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<tr>
<td>P</td>
<td>static pressure</td>
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<tr>
<td>P_t</td>
<td>total pressure</td>
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<tr>
<td>r</td>
<td>radial position</td>
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<tr>
<td>R</td>
<td>vortex core radius and radius from the axis of the machine in equations 4,5,6</td>
</tr>
<tr>
<td>u_t</td>
<td>friction velocity</td>
</tr>
<tr>
<td>u,v,w'</td>
<td>vortex radial, peripheral and axial velocity components</td>
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<tr>
<td>W</td>
<td>free stream axial velocity</td>
</tr>
<tr>
<td>w</td>
<td>( w = W \cdot w' )</td>
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<tr>
<td>z</td>
<td>distance along the vortex axis</td>
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Greek symbols

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<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>( \delta )</td>
<td>boundary layer thickness</td>
</tr>
<tr>
<td>( \Gamma )</td>
<td>leakage vortex circulation</td>
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<tr>
<td>( \Delta z )</td>
<td>finite length along vortex axis</td>
</tr>
<tr>
<td>( \theta )</td>
<td>peripheral direction of the orthogonal curvilinear coordinate system</td>
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<tr>
<td>( \mu )</td>
<td>dynamic viscosity</td>
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<tr>
<td>( \nu )</td>
<td>kinematic viscosity</td>
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<tr>
<td>( \rho )</td>
<td>density</td>
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<tr>
<td>( \tau )</td>
<td>stresses</td>
</tr>
<tr>
<td>( \Psi )</td>
<td>Stream Function</td>
</tr>
<tr>
<td>( \omega )</td>
<td>vorticity</td>
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Subscripts

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<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>H</td>
<td>hub</td>
</tr>
<tr>
<td>m,n,u</td>
<td>meridional, normal, peripheral directions</td>
</tr>
<tr>
<td>p</td>
<td>pressure side</td>
</tr>
<tr>
<td>s</td>
<td>suction side</td>
</tr>
<tr>
<td>T</td>
<td>tip</td>
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INTRODUCTION
The theoretical models for the simulation of the basic tip clearance flow phenomena were presented in Part I of the paper. The first model, presented in Part I, concerns the flow inside the tip clearance gap. It was seen that applying the proper static pressure difference between the inlet and exit of
the gap flow, it is possible to obtain the correct mass flow rate and a reasonable prediction of the gap losses, which influence the mass flow rate.

The second model concerns the rolling up of the gap exit jet flow and the formation of the tip clearance vortex. A simple process for the vortex formation was assumed first, in which a diffusion mechanism was subsequently introduced. This model was proved to be capable of predicting the correct value of the shed vorticity into the leakage vortex, as well as the distribution of the total pressure loss due to the vortex.

In order to develop a calculation method for the prediction of the tip clearance effects, it is assumed that those effects are considered as a perturbation with respect to the rest of the flow. The principle of superposition is applied, for the tip clearance flow and the rest of the flow. The procedure is an iterative one due to the mutual interdependence of the boundary conditions of the two respective problems. In this part of the paper the complete modelling of the tip clearance flow will be presented first, which utilizes the two models discussed in Part I of the paper. Then, the secondary flow calculation, used in the context of the present work, will be presented very briefly, as it has already been presented elsewhere. Lastly, the complete computational procedure and the corresponding algorithm will be described, which permits the numerical calculation of the flow, when tip clearance effects are present. The present calculation procedure concerns the prediction of the meridional flow in both compressors and turbines.

THE COMPLETE TIP CLEARANCE MODEL

The already described in Part I model of the tip clearance gap, provides us with the mean jet velocity magnitude and

FIG.(1): SCHEMATIC REPRESENTATION OF THE PERIODIC AND MIRROR VORTICES, FOR THE PLANES DOWNSTREAM THE TRAILING EDGE.

FIG.(2): VORTICITY DISTRIBUTION INSIDE THE CALCULATION DOMAIN, FOR A PLANE DOWNSTREAM THE TRAILING EDGE.

FIG.(3): TYPICAL MESH USED FOR THE COMPUTATION OF THE INDUCED KINEMATIC FIELD.
direction at the gap exit, as well as the total pressure loss occurring inside the gap. For this calculation, the static pressure difference between pressure and suction sides at the tip level is needed, as this is modified by the tip clearance vortex presence. As the jet velocity is the key element for the vorticity shed from the gap inside the leakage vortex and its computation must be done accurately, the complete calculation must necessarily be iterative.

The second model described in Part I of the paper provides the necessary elements for the leakage vortex formation, its strength, its evolution characteristics and total pressure losses, but it does not provide its position in space. In the present calculation procedure Owen's (1970) model for the calculation of the eddy viscosity has been incorporated.

For the calculation of the position of the leakage vortex a similar approach with that of Chen et al (1990) is utilized, who employs a slender body approximation of the leakage vortex, decomposing the velocity field into independent throughflow and crossflow parts, in order to provide a generalized description of the clearance vortex evolution. In Chen's et al (1990) work the three dimensional steady flow is treated as a two dimensional unsteady one, using a vortex method for the reconstruction of the vortex shape in planes normal to the blade camber.

In the present work the formation of the leakage vortex and the calculation of the induced kinematic field is treated also in successive planes adopting the decomposition assumption of Chen et al (1990). In order to provide a better cooperation between a meridional flow calculation procedure, which will be described later, and the tip clearance model, the successive calculation planes are normal to the axis of the machine. In this way, each plane corresponds to a specific axial position and the peripherally mean values can be easily calculated.

The computation of the shed out vorticity is performed between two successive planes using the model described in part I of the paper. A linear variation of the leakage mass flow rate is assumed between the two planes. The resulting vortex strength corresponds to a vorticity distribution in the second of the two successive planes. Lamb's (1932) model of the diffusion of a line vortex is used for the distribution of the vorticity in the current calculation plane, which is expressed as

$$\omega (x, R) = \frac{\Gamma}{\pi R^2} \exp \left( -\frac{x^2}{R^2} \right)$$

where $R$ is the radius of the leakage vortex, calculated using the
mass balance equation and taking into account the diffusion process. However, the axis of the leakage vortex is not

FIG.(5): PRESSURE DIFFERENCE AT THE TIP CLEARANCE LEVEL.
perpendicular to the calculation planes, so that the angle of the vortex trace must be taken into account. In that way the component of vorticity normal to the calculation planes is used and the axisymmetric distribution of equation (1) is transformed into an elliptic one, by projecting it on the normal to the axis calculation planes.

The vortex trace and the peripheral position of the vortex centre at the current calculation plane are computed using the self induced velocity field in the previous plane. The time step resulting from the flow velocity at the vortex centre, considered in the absence of tip clearance effects, and the distance between the calculation planes are used in order to compute the peripheral displacement of the vortex centre. The computation of the vortex position based on the induced velocity field, as described above, can provide both the peripheral and the radial position of its centre. However, comparison between theory and experiment revealed an important disagreement for the radial position of the leakage vortex. Thus, the computed radial distance is ignored. This is estimated by assuming that the vortex core edge lies between the endwall and the blade tip.

The computation of the induced velocity field necessitates the solution of a Poisson equation of the following form

$$\Delta \Psi = -\omega$$

(2)

For calculation planes inside the blade passage a zero normal to the wall velocity is used as boundary condition, for all the boundary nodes except those at the tip clearance exit, where the leakage mass flow rate determines the boundary values of the stream function.

The zero normal to the wall velocity condition is used at planes downstream of the trailing edge, only for the hub and tip boundaries. At the other two boundaries, along which the flow is periodic, the stream function distribution is calculated using the induced velocities from the leakage vortex, taking into account its mirror image vortex and the two corresponding periodic vortices (fig. (1)). The periodic vortex arrangement is used also in the calculation of the vorticity distribution at planes downstream of the trailing edge, as corresponding calculation results presented in fig. (2) demonstrate. Typical stretched grids and corresponding induced velocity profiles are presented in figures (3) and (4a,b) respectively for the case presented in fig. (2).

Having completed the description of the tip clearance model, the secondary flow calculation method, in which this model will be implemented, will be briefly outlined.

SECONDARY FLOW CALCULATION METHOD

The development, the analytic derivation of the governing equations, the detailed consideration of the secondary flow calculation method and its application on axial and radial flow compressors and cascades, have been presented by Douvikas et al (1989) and Kaldellis et al (1988,1990). The basic principles and the formulation of the method will be described briefly in this section.

The secondary flow calculation is an integro-differential one,

FIG (6): MEAN JET VELOCITY COMPONENT NORMAL TO THE BLADE CAMBER LINE.
which makes use of the governing equations of the flow in their circumferentially mean form, in order to calculate the development of the hub and tip wall shear layers of the machine simultaneously. The method is a parabolic one and incorporates a coherent two-zone model. According to this model, the influence of the hub and tip wall shear layers on the "inviscid" core of the flow is accounted by using two striction (blockage) parameters in the peripheral and meridional directions.

In this formulation special attention is paid to the flow analysis inside various geometrical configurations (unbounded, annulus and bladed spaces), as well as in the procedure of passing from one space type to another.

The method uses a set of independent variables, which are the boundary layer thicknesses for hub and tip shear layers \(6_H,6_T\), the corresponding friction velocities \((u_{tH},u_{tT})\) and the striction parameter in the meridional direction. The method solves the problem in an inverse way.

The equations which are solved in differential form are the following:

a) The meridional vorticity transport equation in order to calculate the radial distributions of the meridional vorticity and the resulting from it peripheral velocity, in conjunction with the striction parameter in the peripheral direction.

b) The total temperature conservation equation for the calculation of the flow field energy level.

c) The mass conservation equation, in order to calculate the normal velocity component.

d) The peripheral component of the momentum equation for the calculation of the peripheral defect force distribution. The other two components of the defect force acting on the blade, are calculated assuming that the defect force vector is perpendicular to the blade surface.

e) The normal component of the momentum equation, for the calculation of the static pressure distribution.

The equations solved in an integral form are the following:

a) The integral momentum equations in the meridional direction for both hub and tip shear layers.

b) The integral total kinetic energy equations for both hub and tip wall shear layers.

c) The global mass equation.

The solution of the above equations, in combination with the necessary semi-empirical frame for turbulent flow and conditions resulting from the space configuration, provide the kinematic and energy flow fields.

THE COMPLETE CALCULATION PROCEDURE

The complete calculation procedure will be described in this section, with the remark that the tip clearance flow will be introduced as a modification to the basic flow existing in the absence of tip clearance effects. Consequently, the tip clearance flow is introduced as a third zone in the already existing two-zone model, which is used for the calculation of the secondary flow effects.

The flow field without tip clearance effects is established
through the meridional flow calculation procedure, including secondary flow effects, as briefly described in the previous section. Peripherally mean flow quantities are calculated and, it is possible to compute, through momentum considerations, the forces acting on the blade surface (per unit surface), defined in the general case as

\[ F_{v} = \frac{N}{2\pi} \left[ (P_a - P_s) + R (\tau_{uu} - \tau_{ww}) - R(\partial P_a/\partial m - \partial P_s/\partial m) \right] \]  \hspace{1cm} (3)

\[ F_{n} = \frac{N}{2\pi} \left[ (\tau_{nu} - \tau_{nw}) + R (\partial P_a/\partial n - \partial P_s/\partial n) - R(\partial \tau_{uu}/\partial m - \partial \tau_{ww}/\partial m) \right] \]  \hspace{1cm} (4)

\[ F_{s} = \frac{N}{2\pi} \left[ (\tau_{nu} - \tau_{nw}) + R (\partial P_a/\partial n - \partial P_s/\partial n) - R(\partial \tau_{uu}/\partial m - \partial \tau_{ww}/\partial m) \right] \]  \hspace{1cm} (5)

The pressure differences \(F_{v}, F_{n}, F_{s}\) can be computed from equation (3), if it is assumed that the shear stresses at the blade surfaces can be neglected. This pressure difference provided by the secondary flow calculation procedure, does not include the contribution of the leakage vortex. This last contribution may be computed integrating the following equation, developed in Part I of the paper

\[ \frac{\partial P}{\partial z} = \frac{\rho t^2}{2\pi} \left[ 1 - 2 \exp \left(-\frac{z}{R} \right) - \exp \left(-\frac{2z}{R} \right) \right] \]  \hspace{1cm} (6)

using the undisturbed flow value for the static pressure at the boundary. The vortex strength is required for this computation, so that an iterative procedure must be set up in order to provide the various flow quantities, which need for their calculation the modified pressure difference at the blade tip. In the present work the complete calculation procedure, which was set up, consists of determining the vortex trace at the previous calculated position, providing the amount of vorticity at the considered axial position. The vortex trace at the previous calculated position is used in order to determine the direction of the vortex axis. The calculation of the induced kinematic field is then carried out.

The total pressure loss profile is then estimated, using the formulation described in the Appendix of Part I of the paper. In particular, equation (21) of Part I is used to provide the total pressure difference between the cases with and without tip clearance effects, which reads

\[ \Delta R^2 = 4v \frac{\Delta z}{W} \]  \hspace{1cm} (7)

where \(\Delta z\) is measured along the vortex axis, and \(W\) is the component of the free stream velocity at the same direction.

The vortex model, described in Part I, is then applied providing the amount of vorticity at the considered axial position. The vortex trace at the previous calculated position is used in order to determine the direction of the vortex axis. The calculation of the induced kinematic field is then carried out.

The total pressure loss profile is then estimated, using the formulation described in the Appendix of Part I of the paper. In particular, equation (21) of Part I is used to provide the total pressure difference between the cases with and without tip clearance effects, which reads

\[ \Delta P_t = \Delta P_s + \frac{1}{2} \rho \left[ 2\nu w - w^2 - v^2 - u^2 \right] \]  \hspace{1cm} (8)

RESULTS AND DISCUSSION

The secondary flow calculation code with the tip clearance model implemented in it, was used in order to compute the tip clearance effects for certain cases for which experimental data are available. Two linear cascades cases (one compressor and one turbine) and a single compressor rotor were considered. Storer's (1991) experiment provided the data for the first case, while Yaras et al (1989) experiment was used for the second one. The single rotor case was the one documented by Inoue et al (1985). Measurements for various tip clearances are available and concern the kinematic and pressure field at the tip clearance region and for downstream of the trailing edge.

In figures (5b) to (5f) calculation results of the pressure difference at the tip gap height are compared with experimental data, for various tip clearances (Storer's (1991) case). The agreement is quite good and only in the case of \(e/c=4\%), there is a discrepancy. These calculations were performed, using the pressure difference obtained by the secondary flow calculation.
FIG. (8): PERIPHERALLY AVERAGED INDUCED PERIPHERAL VELOCITY PROFILES FOR VARIOUS TIP CLEARANCE HEIGHTS.

method (fig. (5a)). From these results it may be concluded that the formulation presented in the Appendix of the Part I of the paper may be used for predicting reasonably accurately the pressure disturbance caused by the leakage vortex.

The calculation results for the mean jet velocity component normal to the blade's camber line, for the three available tip clearances, are compared with the experimental data in figures (6a) to (6c). The corresponding mean flow angles of the leakage jet are presented in figures (7a) to (7c). The good agreement with the experimental data indicates the validity of the assumptions adopted for the gap flow model. The satisfactory computational results for the discharge coefficient shows that the proposed model is suitable for engineering calculations. Also the assumed linear variation of the pressure difference, for the first thirty percent of the chord, provides satisfactory results for the corresponding area. The validity of the Rains' assumption for the conservation of the longitudinal

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component of momentum through the gap is verified by the results of the mean jet angle.

The previous satisfactory comparisons between calculated and experimental results show that the present calculation procedure is capable of computing the leakage flow rate correctly. It is emphasized that this quantity is very crucial for the proper function of the tip clearance model as a whole.

Circumferentially averaged distributions of the peripheral components of the vortex flow field, for various tip clearances, are presented in figures ( 8a ) to ( 8e ). They are compared with measured distributions of the mass averaged peripheral velocity components for every tip clearance height, minus the corresponding distribution for the case with zero clearance ( the computation is based on a secondary flow calculation, thus the peripheral variations of the flow quantities are not available). In this way the model itself, as well as the adopted in the present work superposition principle, are validated. Considering the simplicity of the models and the assumptions adopted, comparisons are quite good.

Similar results were obtained also for the third case ( Inoue et al. (1985)). The circumferentially mean meridional velocity component, as calculated by the secondary flow calculation method, is presented in fig. ( 9 ) for a plane at the rotor exit.

In fig. ( 10 ) the circumferentially averaged relative peripheral

FIG.(9): CIRCUMFERENTIALLY AVERAGED MERIDIONAL VELOCITY PROFILE.

FIG.(10): CIRCUMFERENTIALLY AVERAGED RELATIVE PERIPHERAL VELOCITY PROFILE, WITH AND WITHOUT THE TIP CLEARANCE EFFECT.

FIG.(11): CIRCUMFERENTIALLY AVERAGED INDUCED PERIPHERAL VELOCITY PROFILES.
FIG. (12): PRESSURE DIFFERENCE AT THE TIP CLEARANCE LEVEL, FOR VARIOUS GAP HEIGHTS.

velocity component, at the same plane, as calculated by the complete calculation procedure, is compared with experimental results for the case of 0.5 mm clearance. As can be seen, the disturbance caused by the presence of the leakage vortex is very small for this minimum clearance case, so that it was treated for comparison purposes as the zero clearance case (in order to calculate the leakage vortex effects for the larger clearance cases).

The circumferentially mean values of the peripheral component of the velocity, minus the one for the minimum clearance case, are presented in figures (11a,b) for the 3.0 and 5.0 mm gap heights respectively. Comparing them with the calculated values of the corresponding component of the induced velocity field, it can be seen that the agreement between calculations and experiment is good. The peripheral component of the induced velocity depends on the correct computation of the shed vorticity. A further evidence is, thus, provided for the validity of the vortex model presented in the Part I of the paper.

The complete calculation procedure was used also in order to predict the tip clearance flow effects in the Yaras et al (1989) turbine case. Three available gap heights were considered and the comparison between calculation results for the modified pressure difference at the tip level and the corresponding experimental data are presented in figures (12a) through (12c).

The present model predicts sufficiently well the modified pressure difference due to the leakage vortex except in the first part of the blading near the leading edge region. The strong curvature of the blade near the leading edge and the adopted linearization for the pressure difference of the first thirty percent of the chord can partially explain the observed discrepancies. However, the calculation results of the mean jet velocity, presented in figures (13a) through (13c), demonstrate that the corresponding discrepancies in pressure difference have a minor effect upon the calculation of the mass flow rate through the gap. The mechanism of the jet flow in the first thirty percent of the chord does not seem to depend on the existing pressure difference and the adoption of a linear variation seems to describe with an adequate accuracy the gap exit jet velocity distribution.

CONCLUSIONS

The implementation of the models presented in Part I of the paper into a theoretical calculation procedure for the computation of the tip clearance effects as a whole, was discussed in this paper.

A two - zone computational procedure was used as a basis, which could give a good description of the field (external flow plus secondary flow), without tip clearance effects.

Comparisons of the complete theoretical procedure with available experimental results were very satisfactory, so that the following conclusions may be reached:

(a) The superposition principle adopted was proved to be adequate for setting up the complete calculation procedure.

(b) The proposed tip clearance models can cooperate quite
FIG. (13): MEAN JET VELOCITY AT THE GAP EXIT FOR VARIOUS GAP HEIGHTS.

satisfactory with a sound external plus secondary flow calculation procedure.

Finally, at least for the compressor case, the present work demonstrates the ability of simple modelling to predict quite complicated phenomena, so that fast computational tools may be developed, particularly attractive for industrial purposes. For the turbine case it is concluded that more experimental data are necessary for testing the capabilities of models proposed.

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REFERENCES


