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THE INVESTIGATION OF THE TRAVELLING WAVE RESONANCE OF DRIVEN HELICAL GEAR

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ABSTRACT

The main vibration type gearing in modern aeroengine transmission is the travelling wave vibration which is divided into two kinds: the forward travelling wave vibration and the backward travelling wave vibration. If the frequency of vibration exciting force, which comes from the meshing of a set of helical gears, is the same as the frequency of travelling wave resonance, the dangerous resonant vibration will take place. However both of forward and backward travelling wave resonance can not be excited as easily. The results of an aeroengine transmission test indicate that forward travelling wave resonant vibrations are more easily excited to the driven helical gear and more dangerous.

The work done by the component force, which is generated because of the angle of travelling wave vibration, on the driven helical gear, work done on the forward travelling wave vibration is positive and the backward travelling wave vibration is negative. In other words, for backward travelling wave vibration the induced force acts as a damping force, but for forward travelling wave vibration the induced force acts as a self-exciting force. Therefore it is more dangerous to driven helical gear when forward travelling wave vibration appears.

INTRODUCTION

There are many kinds of defects and faults of gear, of which the tooth faults are common. These defects apears as wear and spalling of the tooth, tooth plastic deformation, and even tooth cracking and breaking, etc. Many articles of papers deal with these kinds of faults including the analysis techniques for diagnosis of gear. It must be point out that sometimes another kind of fault, that is lumped fracture fault of helical gear, can occur and is usually related to the lateral vibration of the gear.

In modern machinery the use high-speed, light-weight high-load gearing is common, especially in aero-engine transmissions. Gear resonance is one of the most insiduous. and destructive of all gear failure modes. It generally occurs quite suddenly with catastrophic results. Since the gear often fails by the seperation of large fragments from the blank, the consequential damage is usually extensive, especially when high rotational speeds are involved. The mechanism of the gearing sudden catastrophic resonant. response is still ont very clear. To prevent and reduce catastrophic resonance of gearing, it's necessary to research this mechanism. Through a experimental and theoretical investigation some aeroengine transmission's resonant response mechanisms have been revealed.

RESONANT RESPONES OF GEARING

Figure 1 shows the structure of a certain real aero-engine transmission.

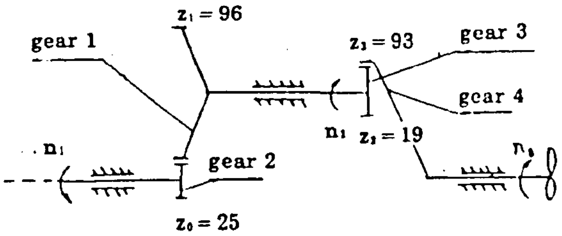

Fig.1 The structure of an aero-engine transmission

The highest input rotational speed of the transmission reaches $n_1 = 40000$rev/min and output rotational

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speed reaches $n_0 = 20000$rev/min. The transmitted power of the transmission reaches 750 horse power. Thin – shell and lighweight structure of gear is adopted to meet the demand of hegh ratio of thrust to weight of aeroengine. So the transmission is a high – speed, lightweight, highly loaded gearing system. The vibration problems emerged in transmission testing.

In aero – engine transmission testing, the sudden resonant response appeared at output rotating speed of n = 1600rev/min. An acceleration transducer mounted on the middle bearing housing indicated 200g vibration accelerating. When the output rotating speed was increased to 2000rev/min, the sudden resonant response appeared again, acceleration transducer reached 300g. Both vibration amplitudes of the two sudden resonances exceeded the permit operating range. If the sudden resonant response occure in the application of a real aero – engine transmission, catastrophic results would have appeared.

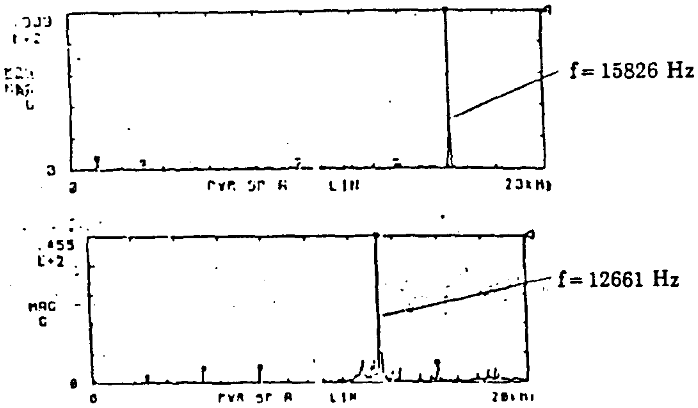


Fig.2 Spectrum analysis of the transmission

The further spectrum analysis (figure 2) shows that the sudden resonant response were caused by the travelling wave vibration of the middle gear (gear 2).

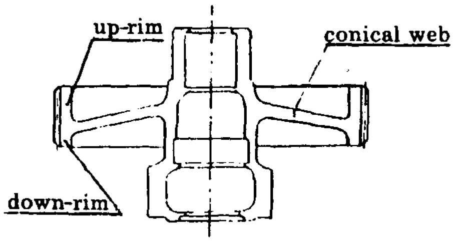
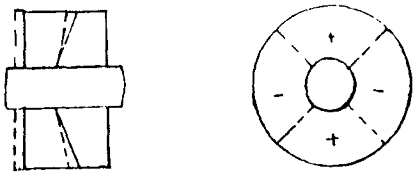
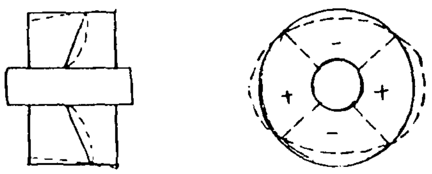


Fig.3 The blank cross – section of gear 2.

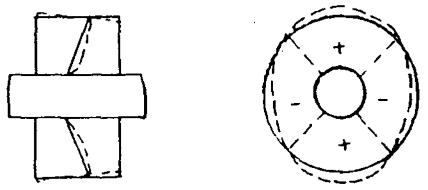
As shown in Fig.3, gear 2 is built with a web – rim construction, a conical web and a thin rim like a hollow cylinder, which sometimes is called disc – shell shaped of gear. Becanse of its complicated shape, the mode shapes are more complex. Except nodal diameter vibration and nodal circle vibration, there are still another two kinds of mode shapes, appeared in the modal test, which are often misregarded as nodal diameter vibration modes, we name this two kinds of mode shapes as "up – sway vibration" and "down – sway vibration". Usually, it's difficult to classify nodal diameter vibration, up – sway and down – sway vibration in application, for their mode shapes look similar. It's must be notod that with the same number of nodal diameters, the frequency of up – sway vibration is higher than that of nodal diameter vibration, and the freguency of down – sway vibration is higher than that of up – sway vibration. In faet, the nodal diameter vibration is due to vibration of the conical web, but the up – sway or down – sway vibtation is due to vibration of up – rim or down – rim, as shown in Fig.4. As the rim shell is directly contacted with excited force, the up – sway or down – sway resonance is more dangerous in operating especially at high retating speed.



a. nodal diameter vobration mode



b. up – sway vibration mode



c. down – sway vibration mode

Fig.4 Mode shapes of gear 2

The natural frequencies of the middle gear are shown in table 1. In operation, the meshing of gear 1 and gear 2 produce high frequency exciting force to gear 2, and the exciting frequency is 15826 Hz at 20000rev/min. Therefore, it's found both one down – sway vibration mode (12500 Hz) and four up – sway vibration

(14850 Hz) are dangerous vibration modes.

tabble 1. Natural frequencies of gear 2

2-nod	2 up	3 up	4-up	1 down	2 down
2053	8699	11306	14850	12575	13632

2-nod: 2 nodal diameter vibration
n-up: n up-sway vibration
n-down: n down-sway vibration

TRAVELLING WAVE VIBRATION

In the case of non-rotating, the vibration displacement of gear can be written as follow:

$$X = A(r)\cos m\theta \cos pt \tag{1}$$

where A(r) is amplitude at radius r: θ is position angle in polar coordinates; p is angular frequency. The term A(r) $\cos m\theta$ represents the amplitude at any point of the gear. If $m\theta = (2j+1)n/2$ $(j = 1,2,\cdots)$, there X is equal to zero. Therefore position angle of nodal lines are at $\theta = (2j+1)n/2m$. In order to investigate resonance of rotating gear, the nature frequencies of nodal diameter vibration of gear observed from coordinated fixed to space should be derived. Equation (1) can be divided into two parts as follow:

$$X = \frac{A(r)}{2}\cos m\theta(e^{ipt} + e^{-ipt}) \tag{2}$$

or:

$$X = \frac{A(r)}{2}(\cos(m\theta + pt) + \cos(m\theta - pt)) \tag{3}$$

Equation (2) represents two types of travelling wave vibration with the same magnitudes but the rotating directions, that are oposite to each other. They are called forward and bachward travelling wave vibration respectively.

The position angle of nodal line is

$$\theta = [(2j+1)\pi/2 \pm pt]/\pi \qquad (j = 0,1,2,\cdots) \tag{4}$$

The rotating angular velocity of nodal lines is

$$\theta = \pm p/m \tag{5}$$

for rotating gear, if the angular velocity is w, the angular velocity of travelling wave in relation to the fixed coordinates is:

$$\theta = \pm p_d/m + \omega \tag{6}$$

where p_d is dynamic angullas frequency of the gear. Because each vibration has m waves, the vibration frequencies of forward and backward travelling waves observing from fixed coordinates will be

$$f_f = f_d + mN/60 \tag{7}$$
$$f_b = f_d - mN/60$$

where f is dynamic frequency (Hz): N is rotating speed of gear (rps).

A convenient method of displaying the relationship between gear natural frequency and transmission system excitation is with a frequency-versus-speed diagram.The frequercy of oscillating of the mesh forces and their harmonics is:

$$F_d = k(ZN)/60 \qquad (k = 1,2,\cdots) \tag{8}$$

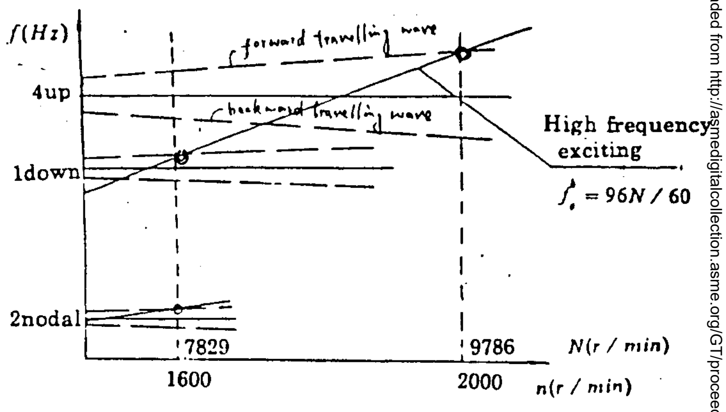


Fig.5 Campbell Diagram for the middle gear of the transmission

In figure 5, it is shown that the resonant response occured at rotating speed of 2000rev/min high frequency exciting producing 4 up-sway forward travelling wave resonance. The resonant response occur at rotating speed of 1600rev/min due to the one down-sway forward travelling wave reaonance caused by high frequency exciting.

It can be noted that all of these resonant responses are forward travelling wave resonances. Though some backward travelling wave vibration is within the range of the operating speed, they were not excited.

SELF-INDUCE VIBRATION OF PASSIVE GEAR

From above equation (3), it's known that for forward travelling wave the displacement of vibration is:

$$X_1 = \frac{A(r)}{2}\cos(m\theta + \omega t) \tag{9}$$

and for backward travelling wave the displacement is:

$$X_2 = \frac{A(r)}{2}\cos(m\theta - \omega t) \tag{10}$$

When a travelling wave resonance was excited by the meshing force, there were small turning angle δ of gear tooth appeared in driven gear shown in fig.6

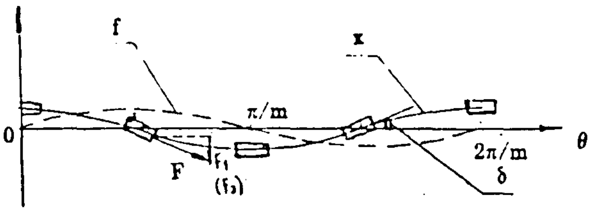


Fig.6 The driven gear's meshing force analysis in travelling resonance

For forward travelling wave resonance, the turning angle of driven gear is

$$\delta_1 = \frac{\partial X_1}{r\partial\theta} = \frac{mA(r)}{2r}\sin(m\theta + wt) \tag{11}$$

the angle δ_1 will induce a lateral component force F_1 in same direction as the travelling wave vibration.

$$F_1 = f\sin\delta_1 \tag{12}$$

Here F represents meshing force of a set of gears. Because δ_1 is very small, it's can be simplified as

$$\delta_1 \doteq \sin\delta_1 \tag{13}$$

then
$$F_1 \doteq F\delta_1 = -\frac{mA(r)F}{2r}\sin(m\theta + wt) \tag{14}$$

The work done by the induced component force F_1 to driven gear in forward travelling wave vibration is

$$W_1 = \int_0^{2\pi}\int_0^T F_1\dot{X}_1 dtd\theta \tag{15}$$

as:

$$\dot{X}_1 = \frac{dX_1}{dt} = -\frac{\omega A(r)}{2}\sin(m\theta + \omega t) \tag{16}$$

so:

$$w_1 = \frac{\omega mFA^2(r)}{4r}\int_0^{2\pi}\int_0^T \sin^2(m\theta + \omega t)dtd\theta \tag{17}$$

Obviously, $W_1 > 0$

Similarly, for backward travelling wave resonance, the turning angle of driven gear is

$$\delta_2 = \frac{\partial X_2}{r\partial\theta} = -\frac{mA(r)}{2r}\sin(m\theta - \omega t) \tag{18}$$

the angle δ_2, will induce a lateral component force F_2 in same direction as travelling, wave vibration.

$$F_2 = F\sin\delta_2 \tag{19}$$

Here F represents meshing force of a set of gears. Because δ_2 is very small, it's can be simplified as shown above

$$\delta_2 \doteq \sin\delta_2 \tag{20}$$

then

$$F_2 \doteq F\delta_2 = -\frac{mA(r)F}{2r}\sin(m\theta - \omega t) \tag{21}$$

The work done by the induced component force F_2 to driven gear in forward travelling wave vibration is

$$W_2 = \int_0^{2\pi}\int_0^T F_2\dot{X}dtd\theta \tag{22}$$

as:

$$\dot{X}_2 = \frac{dX_2}{dt} = \frac{mA(r)}{2}\sin(m\theta - \omega t) \tag{23}$$

so:

$$W_2 = -\frac{\omega mFA^2(r)}{4r}\int_0^{2\pi}\int_0^T \sin^2(m\theta - \omega t)dtd\theta \tag{24}$$

$$w_2 < 0$$

Obviously,

From above analysis, the induced component force does negative work on basckward travelling wave vibration, which will be an energy loss and attenuate the vibration, Therefore the induce component force possesses damping. Forward travelling wave vibration (equation (17)), shows that the induced component force does positive work on the forward travelling wave vibration, the vibration will be amplified. Therefore the force is an exciting force. Equation (14) shows that the amplitude of the induced component force is proportion to that of the vibration. so the induced component force should be considered as a exciting force and forward travelling wave vibration of driven gear must be self-exciting, which would be unstable if sufficient damping is not presented.

CONCLUSION

Resonant response is a serious consideration in the design of high-speed, high-load, light-weight gearing. The failures induced by resonance are usually catastrophic and can cause great secondary damage. In the case of helical gear rotation, the forward travelling wave vibration is a damaging factor, which will produce an induced component force that is very likely to lead to a self-induce vibration, which often occur Sudden catastrophic failure. therefore, in practce, it is necessary to avoid this kind of self-induce vibration.

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