COUPLED BLADE BENDING AND TORSIONAL SHAFT VIBRATION IN TURBOMACHINERY

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Abstract
The forced vibration of turbomachinery blading induced by torsional vibration of the rotor shaft is investigated. Torsional shaft vibrations, caused for example by disturbances in the electrical network, jeopardize the long blades in low pressure stages of steam and gas turbine generator rotors. A simple finite element model consisting of beam elements is used to calculate free and forced vibration. A parameter study has been performed to show the influence of design parameters like mass relation and eigenfrequency relation of the uncoupled system parts. The vibration analysis of a large steam turbine generator rotor is presented.

Nomenclature

A  cross-section area
a, b, c  polynomial coefficients
D, d  shaft diameter
D  damping matrix
f  frequency
f  vector of nodal forces
h  vector of transfer functions
l  length
K  stiffness matrix
M  mass matrix
n  rotor speed
q  vector of nodal displacements
r  radius
u, v, w  displacements
x, y, z  coordinates
a  blade twist angle
φ  rotation
ξ  modal damping ratio

1 Introduction
The vibrational behavior of shaft and blading as turbomachinery components have to be carefully taken into account in the design process of the machine. Although the rotor vibrations are always coupled with the blade vibrations, a separate analysis is performed in most cases.
A few authors indicate the necessity of a coupled treatment of shaft and blade vibration. Klompas (1981), Khader and Masoud (1990) and Crawley et al. (1986) for example investigated the dynamic excitation of the blades by shaft bending vibrations for jet engines. The problem of coupled vibration of blade bending and shaft torsion is given in turbine generator rotors. Disturbances in the electrical network may cause high dynamic air gap torque at the generator, inducing shaft and blade vibrations. Steigleder and Kraemer (1989) as well as Matsushita et al. (1989) among others examined this problem and presented simulation models. However, a simple lumped mass model of the blade with one or two degrees of freedom is used in most cases, hence only the first blade vibration mode in one or two directions can be described. A prediction of the dynamic blade stress is not possible. In this paper a finite element model will be presented to calculate the coupled shaft torsional and blade bending vibrational behavior.

2 Simulation Model
Considering only torsional and/or axial vibration of the shaft and excluding any mistuning effects, a reduction of the rotor to a sector model with one blade for each blade row can be performed (fig. 1). For the addressed shaft vibration modes all blade root points of a blade row vibrate in phase. Excitation of the blade system is only given for modes without nodal diameters. All other modes of the blade system have no coupling effect with the torsional or axial vibration of the shaft. Using the finite element method, the rotor shaft can be modelled by volume elements and the blades can be idealized by shell elements.
of shafts with sections of different diameters by the use of beam elements may cause considerable deviations from the actual eigen-solutions and response. In order to overcome this disadvantage, a correction of the stiffness for shafts with changes of cross-sectional area is performed, following Nestorides (1958).

![Diagram of a turbine generator rotor](image)

**Figure 1: Rotor and simulation model**

However, this sector model is not very suitable due to its complex assembly corresponding with a large number of degrees of freedom. This model was only used for verification of the final computational model composed of beam elements (fig. 1). Sub-structuring was used to evaluate the stiffness and mass matrices of the shaft and the blades separately, allowing a Guyan reduction of the substructure matrices.

### 2.1 Shaft elements

The rotor shaft is constructed of cylindrical or conical beam elements with two or three nodes per element, respectively. Each node contains two degrees of freedom: axial displacement and rotation about the longitudinal axis. The displacement function of the conical element bases on a quadratic polynomial expression with an exact representation of the axial variation of the area \( A(z) \) and the polar moment of inertia \( J(z) \).

<table>
<thead>
<tr>
<th>shape</th>
<th>general</th>
<th>geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>element 1</td>
<td>linear displacement function</td>
<td>( A(z) = \text{const.} ) &lt;br&gt; ( I_p(z) = \text{const.} )</td>
</tr>
<tr>
<td>2 nodal points</td>
<td></td>
<td></td>
</tr>
<tr>
<td>element 2</td>
<td>quadratic displacement function</td>
<td>( A(z) = a_0 + a_1 z + a_2 z^2 ) &lt;br&gt; ( I_p(z) = b_0 + b_1 z + b_2 z^2 + b_3 z^3 + b_4 z^4 )</td>
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<tr>
<td>3 nodal points</td>
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<td></td>
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</table>

**Table 1: Shaft beam elements**

Normally, the computation of torsional and axial vibrations of a shaft by the aid of beam elements is noncritical. A sufficient number of elements leads to good results. However, the treatment of changes of cross-section affects the results of a forced response analysis using beam elements. The response amplitude is given by the deflection angle of the two shaft ends. The ordinate in fig. 2 represents the errors of the first natural frequency and the corresponding response amplitude of the un-corrected beam model in relation to the volume element model outlined above. The diagram on the left hand side shows the influence of the axial position of a single collar. On the right hand side the effect of different numbers of changes of cross-section equally spaced on the rotor is depicted. Including the correction according to Nestorides results in a substantial reduction of the errors. The maximum frequency deviation in the first natural mode is now 0.9%, the max. response error reduces to 4.0% (corrected results not shown for clarity of figure).

**Figure 2: Calculation errors on shafts with changes of cross-section**

Fig. 2 shows how changes of cross-section affect the results of a forced response analysis using beam elements. The response amplitude is given by the deflection angle of the two shaft ends.

### 2.2 Blade elements

The beam elements for the blading (table 2) are based on Euler's beam-theory. Taper and twist are taken into account in elements with a displacement function of fifth order. More complex beam models, as presented e.g. by Akella and Craggs (1986) or Joshi and Suryanarayan (1984) have not been studied because they need additional parameters like warping constants, shear center coordinates, shear factors, etc., that are difficult to determine in most cases.

At each node displacements \( v \) and \( w \) in \( y \)- and \( z \)-direction as well as rotations about \( y \)- and \( z \)-axis are allowed. Longitudinal and torsional components are not of interest in this case and therefore not taken into account. A diminuation of the matrices is achieved by a Guyan reduction, eliminating the rotational degrees of freedom at each node with exception of the node at the blade root.

Convergence studies were carried out in order to determine the minimum number of degrees of freedom that give acceptable results for the first two or three eigen-solutions. Figure 3 shows the error of the computed natural frequencies of a tapered and twisted model blade (length: 200 mm, twist: 60°) with rectangular cross-section (root: 40×6 mm², tip: 30×2 mm²) idealized...
Table 2: Blade beam elements

<table>
<thead>
<tr>
<th>element 1</th>
<th>cubic displacement function</th>
<th>2 nodal points</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(x) = const.</td>
<td>I_1(x) = const.</td>
<td></td>
</tr>
<tr>
<td>I_2(x) = const.</td>
<td>a(x) = const.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>element 2</th>
<th>fifth order displacement function</th>
<th>3 nodal points</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(x) = const.</td>
<td>I_1(x) = const.</td>
<td></td>
</tr>
<tr>
<td>I_2(x) = b_0 + b_1x + b_2x^2</td>
<td>a(x) = a_0 + a_1x</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>element 3</th>
<th>fifth order displacement function</th>
<th>3 nodal points</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(x) = A_0 + A_1x + A_2x^2</td>
<td>I_1(x) = a_0 + a_1x + a_2x^2</td>
<td></td>
</tr>
<tr>
<td>I_2(x) = c_0 + c_1x + c_2x^2</td>
<td>a(x) = a_0 + a_1x</td>
<td></td>
</tr>
</tbody>
</table>

Larger deviations occur when real low pressure rotor blades are investigated. In that case a coincidence of the mass axis and the elastic axis is not given, resulting in coupling effects between torsional and bending vibration modes. Moreover, several modes show typical shell characteristics. The applied beam elements are not able to grasp such effects, resulting in a substantial deviation from the reference value (fig. 4). A finite element model with shell elements (775 nodes) was used to determine the reference frequencies. In the rotating case the deviations get smaller – in the shown case of a steam turbine last-stage blade, at the first natural eigenfrequency the reference value is matched. The geometric stiffness resulting from the centrifugal force field dominates the elastic stiffness, hence the torsion–bending coupling phenomena is covered. This, of course, is not typical for all blades and all eigenmodes, but there is a tendency towards improved results in the rotating case.

2.3 Coupled system

For the blade substructure a coordinate transformation

\[ \mathbf{v}_i^b = r_i^b \varphi_{z_i} \]

is performed in order to change the tangential displacements to rotational degrees of freedom. \( r_i^b \) denotes the radius of the \( i \)-th nodal point of the blade. The blade–shaft coupling conditions are illustrated in figure 5.

For the equation of motion of the coupled system

\[ \mathbf{M}(t) \ddot{\mathbf{q}}(t) + \mathbf{D}(t) \dot{\mathbf{q}}(t) + \mathbf{K}(t) \mathbf{q}(t) = \mathbf{f}(t) \]

the frequency response \( q(f) \) and the transfer function

\[ h(f) = \frac{q(f)}{f(f)} \]

are computed. Rayleigh and/or modal damping is assumed.
3 Parameter Studies

To get a better insight in the phenomena of coupled rotor–blade vibrations some case studies have been investigated. It is obvious, that there are many parameters, properties of the individual components as well as of the coupled system which effect the vibrational behavior, e.g.

- number of blade rows
- position of the blade rows on the shaft
- blade shape (incidence, twist)
- rotor shape (mass distribution)

The object of investigation was a simple rotor with constant diameter and two blade rows, each with 50 geometrically simple blades with rectangular cross-section at the endings of the shaft.

In this study two coupling parameters are varied:

- shaft to blade mass ratio,
  which means the ratio of the moment of inertia of the shaft to the moment of inertia of the blade rows. The variation is done between the limits $10^{-2} = -40 \text{ dB}$ (which means heavy blades on a light rotor) and $10^5 = 100 \text{ dB}$ (small blades on a heavy rotor). The total moment of inertia has been remained unchanged.

- shaft to blade frequency ratio,
  which means the ratio of the first eigenfrequency of the unbladed rotor to the first eigenfrequency of the cantilevered blade. The variation is done between -10 and 10 (-20 dB and 20 dB).

In order to achieve different mass and frequency ratios the radius and the elastic modulus of the shaft is adapted. The rotor was excited at one end by a harmonic torque amplitude of 1 Nm, hence the response amplitude function $q(f)$ is identical to the transfer function $h(f)$.

For each parameter combination a coupled and an uncoupled forced response computation has been carried out. Modal damping with a constant damping ratio $\xi = 0.01$ for all vibration modes is assumed. For the uncoupled case the response of the unbladed shaft considering only the mass effect of the blading is investigated in a first step. In a second step the blade response is calculated using the shaft response as a prescribed motion of the blade root. Hence the general vibrational behavior for different shaft–blade combinations and the differences between coupled and uncoupled computation can be examined.

For the coupled computation fig. 6 shows the maximum shaft response amplitude, given by the maximum deflection angle between the two shaft ends as a function of the coupling parameters. In a wide range the shaft response is independent of the mass ratio. For systems with mass ratios less than 1 (0 dB) an increase of the shaft response can be observed. This effect can be explained by the decrease of the eigenfrequencies due to the mass concentration at the shaft ends. A decoupled computation would result in a smooth surface, however in the coupled case some deviations in form of two valleys can be observed. The blade rows act as "elastic vibration absorbers", i.e. the total energy in the system now concentrates more on the blades.

In figure 7 the differences between a decoupled and a coupled computation is shown in detail. For each frequency ratio greater than 1 (0 dB) there is at least one optimum mass ratio where the absorber effect reaches a maximum. For frequency ratios over 6 (16 dB) a second maximum appears, which is caused by the absorbing effect of the second blade mode. For low mass ratios which means heavy blades on light rotors the blade frequencies rise, hence the absorber maxima shift to higher frequency ratios.

More interesting, however, is the maximum blade response. Figure 8 shows the maximum amplitude of the blade tip relative to the blade base motion for each coupling parameter (coupled). For some discrete frequency ratios lower than 1 (0 dB) an interference between blade and shaft modes occurs. That results in
higher blade amplitudes. Decreasing the mass ratio reduces this amplification. For most parameter combinations the maximum blade response is constant at a low level (about 5 • 10^-8 m/Nm).

In the decoupled case a different result is obtained (fig. 9). A strong resonance occurs, when a shaft eigenfrequency comes close to the blade's first natural frequency. These regions of maximum resonance amplitudes form typical branches. The irregular occurance of resonance peaks on the branches can be explained by the accidental match of a shaft eigenfrequency with the blade base frequency. A very fine discretisation in both axes – frequency ratio and mass ratio – would be necessary to achieve a smoother shape. A decoupled computation, however, may cause large deviations from the actual values, because the dynamic coupling is not taken into account. Acceptable results can be achieved for very high mass ratios and for frequency ratios greater than 1, presuming mass ratios greater than 20 dB (factor 10). In all other cases a coupled computation should be preferred.

A very important aspect which occurs in coupled blade-shaft systems is the shifting of the eigenfrequencies and thereby the resonance frequencies which cannot be grasped in the decoupled calculation. In figure 10 the resonance frequency of the maximum blade response for the coupled analysis is shown. Generally, the blade resonance occurs at higher frequencies for a decreasing mass ratio. This was expected, because only for high mass ratios the blade root boundary condition can be treated as cantilevered. Surprising, however, is the effect, that for some parameter combinations the resonance occurs at lower frequencies, even below the first natural frequency of the cantilevered blade.

4 Example

In an example the vibrational behavior simulation of a real rotor is demonstrated. A 50 Hz, 720 MW turbine generator rotor was chosen. The rotor blades of the last three stages of each of the four low-pressure-parts were taken into account in the analysis. Each blade is modelled by eight beam elements yielding twenty degrees of freedom after the Guyan reduction. The rotor shaft is composed of 80 beam elements with altogether 126 degrees of freedom after reduction. The excitation is given by a harmonic torque amplitude of 1 kNm acting on the generator.

In fig. 11 the first eight eigenvectors of the rotor are shown. The blade deflections are plotted in axial direction, whereas the rotor amplitudes are presented on the ordinate. Axial deflection and rotation about the z-axis are distinguished by symbols. Typical blade vibration modes appear multiply (corresponding frequencies splitting over a small range) following the number of identical blade rows.
Figure 11: Eigensolutions of a turbine generator rotor

Figure 12: Blade transfer function
In figure 12 the transfer function of the four last-stage blades are shown (amplitude of blade tip relative to blade base deflection). Following the previous section, the forced response calculation is done in a coupled and in an uncoupled manner. Response differences between the two calculation models are observed up to the factor of two. Also a splitting of eigenmodes occurs, which expresses in multiple resonance peaks, while the decoupled calculation yields only a single peak. The maximum blade amplitude can be determined to about $4 \times 10^{-2}$ mm/kNm. This seems to be negligible, but it has to be considered that electrical short circuits could cause dynamic torques which have three or four times the magnitude of the static torque under normal load. Hence blade amplitudes in the range of several millimeters may occur under the circumstances of a short circuit on the generator side.

5 Conclusions

A simple finite element beam model was presented to calculate the vibrational behavior of coupled rotor-blade systems. First, the performance of the used elements was shown. A parametric study with a symmetrically designed bladed rotor was done to find out the main vibrational characteristics of rotor-blade systems. It was shown, that a decoupled forced response calculation may cause large errors especially concerning the blade amplitudes. Less deviation has been obtained for the rotor amplitudes. Regarding only this aspect, a decoupled calculation may be tolerated. A calculation of a 720 MW turbine generator rotor was performed to demonstrate the analysis of a real rotor.

References


