OPTIMUM DESIGN OF ROTATING LAMINATE BLADE
WITH DYNAMIC BEHAVIOR CONSTRAINTS

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ABSTRACT

The optimum design of rotating laminated blade subject
to dynamic behavior constraint is investigated. The dynamic
behavior constraints consist of restrictions on multiple natu-
ral frequencies as well as on maximum dynamic deflections
of rotating laminated blade. The harmonic excitations are con-
sidered to simulate the aerodynamic forces acting on the blade.
The optimization techniques of optimality criterion method and
modified method of feasible directions have been successfully
developed and applied to minimize the weight of rotating lam-
inated blade. The effect of setting angles and rotating speed
on the system dynamic behaviors as well as on the optimum
design are also studied. The dynamic analysis shows that most
of the bending modes can be significantly affected by rotating
speed, setting angle, and the radius of disc. The results also in-
dicate that the optimum weight will decrease with the increase
of rotating speed for the multiple frequency constraints. The
similar phenomena is also obtained for the dynamic response
constraints. Moreover, the optimum weight of the dynamic
response constraints case is higher than that of multiple fre-
cuency constraints case. This study presents that the weight
of rotating laminated blade can be greatly reduced at optimum
stage.

NOMENCLATURE

\( A_i \) : area of the i-th layer
\( a \) : length of the blade
\( b \) : width of the blade
\( \{c\} \) : corresponding mode shape of the blade
\( \{F\}_x, \{F\}_y \) : equivalent nodal and global nodal D.O.F. force
vector respectively
\( g_j(z) \) : j-th inequality constraint
\( h_k(z) \) : k-th equality constraint

\( \{K\}_g, \{M\}_g \) : stiffness and mass matrix of global nodal D.O.F.
respectively
\( N_d \) : number of design variables
\( \vec{P} \) : external force vector
\( \{Q\} \) : dynamic response vector due to forces with dif-
ferent frequencies
\( \{Q\}_n \) : dynamic response vector of the n-th external
force
\( \{\bar{Q}\} \) : transformed reduced stiffness matrix
\( \{q\} \) : displacement vector of local nodal D.O.F.
\( R \) : radius of the rotating disc
\( T \) : kinetic energy
\( t_i, t_i', t_i'' \) : the i-th design variable, lower bound and upper
bound of the i-th design variable respectively
\( U, U_b, U_p \) : total potential energy, bending and in-plane ini-
tial force strain energy respectively
\( u, v, w \) : x,y,z direction displacement components
\( u_i, u_i, w_i \) : x,y,z direction displacement components of the
i-th node
\( \vec{v} \) : velocity vector
\( z_1, y_1, z_1 \) : rotating reference frame
\( z_i, y_i, z_i \) : the i-th nodal coordinate
\( a \) : relaxation factor
\( a^* \) : search step
\( \{\delta\} \) : displacement vector of gloable nodal D.O.F.
\( \{e\} \) : stain vector of the blade
\( \lambda_j, \lambda_k, \lambda_l' \) : Lagrange multipliers
\( \rho_i \) : density of the i-th layer
\( \sigma \) : stress vector of the blade
\( \phi_i \) : the i-th shape function
\( \varphi \) : setting angle of the balde
\( \psi_{x_i}, \psi_{y_i} \) : rotation components of the i-th node
\( \Omega \) : rotating speed of the disc
\( \Omega_n \) : frequency of the n-th external force
\( \Omega_1, \Omega_2, \Omega_3 \) : components of the angular velocity of the disc
\( \Omega_1 \) : angular velocity of the disc

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\( \tilde{\Omega} \) : nondimensional rotating speed
\( \omega \) : natural frequency of the blade
\( \omega_0 \) : fundamental natural frequency of non-rotating plate frequency
\( \bar{\omega} \) : nondimensional natural

INTRODUCTION

The dynamic behavior of rotating blade is of particularly important in modern high speed engine design. The dynamic analysis of blade have been investigated by many researchers. The blade can be modelled as cantilever beams, plates, or shells. The beam model is the simplest one which is usually used for simulating the blade with high aspect ratio. Carnegie (1959) first considered the natural frequencies of a rotating blade based on the beam model and provided a theoretical expression of potential energy and applied Rayleigh’s method to investigate the natural frequencies. Many authors [ Putter and Monor, 1978; Kaza and Kielb, 1984; Ansari, 1986 ] have studied the natural frequencies of tapered blade, the effect of warping and pretwist angle on torsional vibration, and the vibrations with the effect of shear deformation, rotatory inertia and Coriolis effects.

For a blade with low aspect ratio, the beam model is not accurate enough. Then the shell model or plate model may be used. Rawtani and Dokainish (1971) determined the influence of rotation on plane plates by using finite element method. Petricone and Sisto (1971) used Rayleigh-Ritz method based on thin shell theory to calculate the influence of pretwist and skew angle of nonrotating blades. Henry and Lalanne (1974) determined the modes and frequencies by using plane triangular elements. Sreenivasamurthy and Ramamurti (1981) considered the Coriolis effect by using triangular shell elements. It was found that the Coriolis effect would increase with the increase of rotating speed and it would decrease the bending and the first torsional natural frequency. Assuming the displacement functions to be algebraic polynomials, some studies have been done by Leissa et al. (1982, 1985). They have analyzed the effect of setting angles, variable thickness, and curvature by using Ritz method in conjunction with shallow shell theory.

The works stated above focused on the isotropic materials. Crawley (1979) determined the natural frequencies and mode shapes of a number of Graphite/Epoxy and Graphite/Epoxy-Aluminum stationary plates and shells by using quadrilateral finite elements and experimental method. Crawley and Dugundji (1980) used partial Ritz method to determine the nondimensional natural frequencies of stationary composite laminated plates. Wang et al. (1987) studied the vibration of symmetrically stacked rotating composite laminated plates. They chose complete sets of orthogonal polynomials in conjunction with Galerkin method.

The optimum weight design of rotating blade with satisfaction of the constraints on structural frequencies and dynamic deflections as well as the constraints on the design variables has been a very interesting topic in the field of turbomachinery blade design. The optimization techniques employed in this study are the optimality criteria method (OCM), and the method of modified feasible directions. The method of feasible directions (MFD) is studied by Zoutendijk (1960). Then Van-derplaats (1984) have described the modified method of feasible directions (MMFD) which combines the best features of MFD and the generalized reduced gradient method [Gabriele and Ragsdell, 1977].

For the optimality criteria method (OCM), Dobbs and Nelson (1976) developed a redesign formula based on Kuhn-Tucker condition. Kiusalaas and Shaw (1978) improved the numerical technique to multiple frequency constraints case. The optimum structural design with dynamic behaviors constraints have been studied by many authors [Sadek, 1989; Lin and Yu, 1991; Shiau and Chang, 1991].

In this study, the OCM is applied for solving the optimum design of rotating laminated blade with multiple frequencies and/or dynamic deflection constraints. Furthermore, the MFD and MMFD are employed for the comparison of optimum results.

DYNAMIC ANALYSIS

Consider a composite laminated blade rotating with a constant angular velocity \( \tilde{\Omega} \) about a fixed axis. The blade is fixed with setting angle \( \varphi \) on the rotating disc of radius \( R \), as shown in Fig. 1. Some basic assumptions are made as follows:

1. Mindlin plate theory is applied.
2. The blade is fixed on the rotating disc, and the disc is assumed to be rigid and rotating with a constant speed.
3. The composite laminate is symmetric stacking with no delamination and sliding between layers.
4. Neglect the damping and Coriolis effect. The pretwist, tapered effects are not taken into account.
5. The external forces are assumed to be periodic and are functions of time only.

The isoparametric plate element with eight nodal points of finite element method [Zienkiewicz, 1977; Bathe, 1982; Rao, 1985] is employed to approximate the system. The coordinate at each point of the element can be expressed as

\[
\begin{align*}
\mathbf{x} &= \sum_{i=1}^{8} \phi_i x_i \\
y &= \sum_{i=1}^{8} \phi_i y_i \\
z &= \sum_{i=1}^{8} \phi_i z_i
\end{align*}
\]

where \( x_i, y_i, z_i \) denote the \( i \)th nodal coordinate, and \( \phi_1, \phi_2, \ldots, \phi_8 \) denote shape functions. Rewriting these expressions into matrix form, one get

\[
\{ \mathbf{r} \} = \{ N \} \{ \mathbf{z} \}
\]

where \( \{ \mathbf{r} \} = (x, y, z)^T \) and \( \{ \mathbf{z} \} = (x_1, y_1, z_1, \ldots, x_8, y_8, z_8) \)

The displacement field \((u, v, w)\) of the element can be expressed in terms of nodal displacements \((u_i, v_i, w_i)\) and rotations \((\psi_{zi}, \psi_{vi})\):

\[
\begin{align*}
u &= \sum_{i=1}^{8} \phi_i u_i + z \sum_{i=1}^{8} \phi_i \psi_{zi}
\end{align*}
\]
\[ \begin{align*}
\nu &= \sum_{i=1}^{s} \phi_i v_i + z \sum_{i=1}^{s} \phi_i y_i, \\
\omega &= \sum_{i=1}^{s} \phi_i \omega_i
\end{align*} \]

Rewrite these expressions into matrix form, we get:
\[ \{d\} = [N_2]\{q\} \quad (1) \]

where \( \{q\} \) is the nodal displacement vector in local coordinate system. Substituting equation (1) into the strain-displacement relationship, it yields
\[ \{\varepsilon\} = [N_3]\{q\} \]
\[ \{\varepsilon_y\} = [N_4]\{q\} \]

where \( \{\varepsilon\} = \{\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}\}^T \) and \( \{\varepsilon_y\} = (\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y})^T \).

The total potential energy \( U \) is defined as the summation of strain energy due to plate bending (\( U_b \)) and that due to in-plane initial force (\( U_p \)) [Timothenko, 1959; Jones 1975] and they are of the following form:
\[ U = U_b + U_p \]
\[ U_b = \frac{1}{2} \iint_{V} \{q\}^T [N_2]^T [\Omega] [N_2] \{q\} \, dV \]
\[ U_p = \frac{1}{2} \iint_{V} \{q\}^T [N_4]^T [\sigma] [N_4] \{q\} \, dV \]

where \([\sigma]\) is initial stress matrix of the blade due to the effect of centrifugal force of the rotating system and \( [\Omega] \) is the transformed reduced stiffness matrix (see Appendix A).

To find the system kinetic energy one need define the position vector of any point of the blade which is of the form
\[ \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \{d\} = (u, v, w)^T \]
\[ \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \frac{d}{dt}(\overline{OP}) + \Omega \times \overline{OP} \]

where \( \Omega = (\Omega_1, \Omega_2, \Omega_3)^T \). The system kinetic energy of rotating blade \( T \) can be expressed as
\[ T = \frac{1}{2} \iint_{V} \rho \dot{v} \cdot \dot{v} \, dV \]
\[ = T_2 + T_1 + T_0 \]
\[ T_2 = \frac{1}{2} \iint_{V} \rho \{q\}^T [N_2]^T [A] [N_2] \{q\} \, dV \]
\[ T_1 = \iint_{V} \rho \{q\}^T [N_2]^T [A] [N_1] \{z_n\} \, dV \]
\[ + \iint_{V} \rho \{q\}^T [N_2]^T [A] [N_1] \{z_n\} \, dV \]
\[ T_0 = \frac{1}{2} \iint_{V} \rho \{q\}^T [N_2]^T [A] [N_2] \{q\} \, dV \]
\[ + \frac{1}{2} \iint_{V} \rho \{z_n\}^T [N_1]^T [A] [N_1] \{z_n\} \, dV \]
\[ + \iint_{V} \rho \{q\}^T [N_2]^T [A] [N_1] \{z_n\} \, dV \]

where the matrix
\[ [A] = \begin{bmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{bmatrix} \]

If the external forces are assumed to be periodic and uniformly distributed on the surface of the blade. They can be expressed as:
\[ \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \sum_{n=1}^{N_f} \{P_n\} e^{i\Omega_n t} \]

With knowing the system potential energy, kinetic energy, and the excitations, one can find the equivalent nodal force:
\[ \{F\}_e = \iint_{A} [N_2] \left( \sum_{n=1}^{N_f} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} e^{i\Omega_n t} \right) \, dA \]

where \( N_f \) denotes the number of frequencies of the excitation force. Similarly, one can find the equivalent nodal force:
\[ \{f^e\}_e = - \iint_{V} \rho \{q\}^T [N_2]^T [A] [N_2] \{q\} \, dV \]
\[ + \iint_{V} \rho \{z_n\}^T [N_1]^T [A] [N_1] \{z_n\} \, dV \]
\[ = [K_{f^e}]_e \]
\[ \{f^e\}_e = - \iint_{V} \rho \{q\}^T [A] [N_1] \{z_n\} \, dV \]

The matrix \([G]\) is due to the Coriolis effect which will be neglected, and \( \{f^e\}_e \) is the so called centrifugal force. \( [K_{f^e}]_e \) is called the rotary stiffness matrix, \( [K_e]\) is called the elastic stiffness matrix, and \( [K_g]\) is called the geometric stiffness matrix. The system equations of motion for the rotating blade in global coordinate system can be obtained as:
\[ [M]_y(\delta) + [K]_y(\delta) = \{F\}_y \]  
where \( \{\delta\} \) denotes the displacement vector including total degrees of freedom of the system.

1. Natural Frequency

Assume the solution to be of the form:

\[ \{\delta\} = \{C\} e^{j\omega t} \]  
where \( \omega \) is the natural frequency and \( \{C\} \) is the corresponding response amplitude. Substituting equation (13) into the homogeneous part of equation (12), one can obtain

\[ ([K]_y - \omega^2[M]_y)\{C\} = 0 \]  
(14)

Solving the eigenvalue problem, the natural frequencies and corresponding mode shapes can be obtained.

2. Steady State Response

The steady state response due to external periodic forces with frequency \( \Omega_p \) is assumed to be of the form:

\[ \{\delta\} = \{Q\}_n e^{j\Omega_p t} \]  
(15)

Substituting equation (15) into equation (12), it yields

\[ ([K]_y - \Omega^2[M]_y)\{Q\}_n = \{F\}_y \]  
(16)

Solving the above equation, one can obtain the dynamic response due to excitation with frequency \( \Omega_p \). If the excitation is of multiple frequencies, the total dynamic response can be obtained by superimposing all the response due to forces with different frequencies, i.e.

\[ \{\delta\} = \sum_{n=1}^{N_f} \{Q\}_n e^{j\Omega_p t} \]  
(17)

OPTIMIZATION ALGORITHM

Two optimization techniques are employed to study the optimum design of rotating laminated blade. One is the optimality criteria method, the other is the modified method of feasible directions of the mathematical programming method.

1. Optimality Criteria Method

The optimum design problem of present study can be stated as

\[ \text{Minimize : } F(t) = \sum_{i=1}^{N_f} \rho_i A_i t_i \]
\[ \text{Subject to : } g_j(t) = \omega_j^2 - \omega_j^2 \leq 0 \quad j = 1, J \quad (18) \]
\[ g_k(t) = |Q_{1i} - Q_{1i}^*| \leq 0 \quad k = 1, K \]
\[ g_l(t) = |Q_{yi} - Q_{yi}^*| \leq 0 \quad l = 1, L \]

where \( \omega_j^*, \ Q_{1i}^*, \ Q_{yi}^*, \) and \( \omega_j, \ |Q_{1i}|, \ |Q_{yi}|, \) are the specified and calculated natural frequency, components of dynamic response amplitude in \( z^- \) and \( y^- \) direction respectively. The side constraints is given as \( t_i^1 \leq t_i \leq t_i^* \). The Lagrangian functional is defined as:

\[ \Phi = \sum_{i=1}^{N_f} \rho_i A_i t_i + \sum_{j=1}^{J} \lambda_j g_j(t) + \sum_{k=1}^{K} \lambda_k' g_k(t) + \sum_{l=1}^{L} \lambda_l'' g_l(t) \]
(19)

The corresponding Kuhn-Tucker conditions which are the necessary conditions for minimizing \( F(t) \) [Dobbs and Nelson, 1976] are:

\[ \frac{\partial \Phi}{\partial t_i} = \rho_i A_i + \sum_{j=1}^{J} \lambda_j \frac{\partial g_j(t)}{\partial t_i} + \sum_{k=1}^{K} \lambda_k' \frac{\partial g_k(t)}{\partial t_i} + \sum_{l=1}^{L} \lambda_l'' \frac{\partial g_l(t)}{\partial t_i} \]
(20)

and

\[ \lambda_j \begin{cases} > 0 & \text{if } g_j(t) = 0; \\ = 0 & \text{if } g_j(t) < 0. \end{cases} \]
(21)

where \( \lambda_j, \lambda_k', \lambda_l'' \) are the Lagrange multipliers. Substituting equation (18) into equation (20) and multiplying both side by \( (1 - \alpha) \), with rearrangement, one obtain

\[ t_i - \left[ \alpha + \frac{1 - \alpha}{\rho_i A_i} \sum_{j=1}^{J} \lambda_j \frac{\partial (\omega_j^2)}{\partial t_i} - \frac{1 - \alpha}{\rho_i A_i} \sum_{k=1}^{K} \lambda_k' \frac{\partial (|Q_{1i}|)}{\partial t_i} \right] t_i \]
\[ - \frac{1 - \alpha}{\rho_i A_i} \sum_{l=1}^{L} \lambda_l'' \frac{\partial (|Q_{yi}|)}{\partial t_i} \]
\[ \{ = 0 & \text{if } t_i^1 < t_i < t_i^*; \\ \geq 0 & \text{if } t_i = t_i^1; \\ \leq 0 & \text{if } t_i = t_i^*. \}
(22)

or

\[ t_i - f_i t_i \begin{cases} = 0 & \text{if } t_i^1 < t_i < t_i^*; \\ \geq 0 & \text{if } t_i = t_i^1; \\ \leq 0 & \text{if } t_i = t_i^*. \}
(22)

where \( \alpha \) is the relaxation factor that determines the magnitude of the redesign vector, \( 0 < \alpha < 1 \). The optimum value of \( \alpha \) is case dependent. The recursive design formula is based on equation (22) and is given by

\[ t_i' = \begin{cases} f_i t_i & \text{if } t_i^1 < f_i t_i < t_i^*; \\ t_i^1 & \text{if } f_i t_i \leq t_i^1; \\ t_i^* & \text{if } f_i t_i \geq t_i^*. \}
(23)

where \( t_i' \) is the new design variable. Equation (23) simply implies that \( f_i \) is equal to one, when the optimum design is reached. Note that the speed of convergence depends not only on the initial design variables but also on relaxation factor \( \alpha \).

i) Calculation of Lagrange Multipliers
In order to process the redesign, the Lagrange multipliers \( \lambda_j, \lambda_k, \lambda_l \) must be calculated first. It is assumed that the change of the natural frequency can be approximated by the linear approximation as

\[
(\omega'_{j1})^2 - \omega^2_j = \sum_{i=1}^{N_k} \frac{\partial(\omega_j^2)}{\partial t_i} (t_i' - t_i)
\]  

(24)

If the Lagrange multipliers are positive, the constraints \( g_j(t), j = 1, J \) must be zero. Replacing \( \omega'_j \) by \( \omega'_j \), and substituting equations (22) and (23) into equation (24), one can get

\[
- \sum_{j=1}^{J} \lambda_j \left( \frac{\partial(\omega^2_j)}{\partial t_i} \frac{\partial(\omega^2_j)}{\partial t_i} t_i \right) + \sum_{k=1}^{K} \lambda_k \left( \frac{\partial(\omega^2_j)}{\partial t_i} \frac{\partial(\omega^2_j)}{\partial t_i} t_i \right) \]

\[
+ \sum_{l=1}^{L} \lambda_l \left( \frac{\partial(\omega^2_j)}{\partial t_i} \frac{\partial(\omega^2_j)}{\partial t_i} t_i \right) = - \sum_{i \in \text{act}} \frac{\partial(Q_{yi})}{\partial t_i} t_i + \frac{1}{1 - \alpha} \sum_{i \in \text{pass}1} \frac{\partial(Q_{yi})}{\partial t_i} (t_i' - t_i)
\]

\[
+ \frac{1}{1 - \alpha} \sum_{i \in \text{pass}2} \frac{\partial(Q_{yi})}{\partial t_i} (t_i' - t_i)
\]

\[
+ \frac{1}{1 - \alpha} (Q_{yi} - Q_{yi}^*) = \sum_{j=1}^{J} \lambda_j \left( \frac{\partial(Q_{yi})}{\partial t_i} \frac{\partial(\omega^2_j)}{\partial t_i} t_i \right) + \sum_{k=1}^{K} \lambda_k \left( \frac{\partial(Q_{yi})}{\partial t_i} \frac{\partial(\omega^2_j)}{\partial t_i} t_i \right) + \sum_{l=1}^{L} \lambda_l \left( \frac{\partial(Q_{yi})}{\partial t_i} \frac{\partial(\omega^2_j)}{\partial t_i} t_i \right)
\]

(25)

The active design variables are those in the range of their lower and upper bound. The passive design variables are governed by their side constraints, i.e. \( t_i = t_i' \) or \( t_i = t_i^* \). Solving the equations (25), (28), and (29) simultaneously, the Lagrange multipliers will be obtained.

\section*{ii) Sensitivity Analysis}

Multiplying both side of equation (14) by \( \{C\}^T \) and then taking partial derivatives with respect to the design variables, one obtain:

\[
\frac{\partial \omega^2}{\partial t_i} = \frac{\{C\}^T \left( \frac{\partial[K]}{\partial t_i} - \omega^2 \frac{\partial[M]}{\partial t_i} \right) \{C\}}{\{C\}^T \{M\} \{C\}}
\]

(30)

Taking partial derivatives of equation (16) with respect to the design variables, it gives

\[
\frac{\partial(Q)}{\partial t_i} = (\{K\} - \Omega^2 \{M\})^{-1} \left( \frac{\partial[K]}{\partial t_i} - \Omega^2 \frac{\partial[M]}{\partial t_i} \right) \{Q\}
\]

(31)

Equations (30) and (31) express the sensitivities of system frequencies and dynamic response with respect to design variable, respectively.

\section*{2. Modified Method of Feasible Directions}

The technique of Modified Method of Feasible Directions [Vanderplaats, 1984] which combines the best features of the method of feasible directions [Zoutendijk, 1960] and the generalized reduced gradient method [Gabriele and Ragsdell, 1977] is employed. It includes two procedures: (1) to determine the searching direction and (2) to decide the searching step size. The details of the algorithm has been shown by Vanderplaats (1984).
NUMERICAL RESULTS

1. Dynamic Behavior

The nondimensional frequencies suggested by Crawley and Dugundji (1980) \(\omega^2 \sqrt{\rho h/D_{11}}\) for bending modes, \(\omega ab \sqrt{\rho h/48D_{66}}\) for torsional modes, \(\omega b^2 \sqrt{\rho h/D_{22}}\) for chordwise modes are employed. The bending stiffness \(D_{ij}\) is defined as

\[
\frac{1}{3} \sum_{k=1}^{N} (\bar{Q}_{ij}) (x_k^2 - x_{k-1}^2)
\]

where \(N\) is the total number of layers. The results show good agreement with those in the existing literature.

The effect of setting angles and rotating speeds on natural frequencies are shown in Tables 1-3 for the first four modes and plotted in Figs. 3-8 for first two modes of various laminates. The results indicate that the increase of rotating speed and/or disk radius will increase the system natural frequencies. The effect is more significant for the lower frequencies. Furthermore, the effect of the setting angle on the natural frequencies is increased as the rotational speed is increased. However, the effect is not significant for higher modes. In addition, the nondimensional natural frequencies will decrease as the setting angle is increased.

2. Optimum Results

The primary purpose of this study is to minimize the weight of rotating laminated blade subject to multiple frequency constraints and/or dynamic behavior constraints. The design variables are the thicknesses of each layer. The material properties are listed in Appendix B. Two examples are used to demonstrate the performance of present optimization procedures.

Example 1:

The configuration of this example is a [0/±45/90], composite laminate rotating blade with \(a = 0.0762m\), \(b = 0.0381m\) fixed on a disc with \(R = 0.0762m\). The aspect ratio of the blade is 2, the rotating speed (\(\Omega\)) is 600\(\pi\) rad/sec, and the setting angle (\(\phi\)) is 45\(^\circ\). The flow-induced aerodynamic forces for both lift and drag are expressed as

\[
\text{Lift} = 1.5 \times 10^6 (e^{i\Omega_1 t} + e^{i\Omega_2 t})
\]
\[
\text{Drag} = 5.0 \times 10^3 (e^{i\Omega_1 t} + e^{i\Omega_2 t})
\]

where \(\Omega_1 = 1080\pi\) rad/sec, and \(\Omega_2 = 3240\pi\) rad/sec. All the constraints are listed as follows:

- side constraints: \(t_i \geq 0.00013 m\), \(i = 1,4\)
- natural frequency constraints: \(\omega_1 \geq 1620\pi\) rad/sec
  \(\omega_2 \geq 4000\pi\) rad/sec
- dynamic response constraints: \(\|Q_1\|_{2p} \leq 0.001 m\)
  \(\|Q_2\|_{2p} \leq 0.00001 m\)

The dynamic analysis in the optimization process is performed by using finite element method (FEM) with four elements. Eight design variables (ply thicknesses) are considered in this problem. Four of them are associated with the four layers of elements 1 and 2 and are expressed as \(t_1, t_2, t_3\) and \(t_4\). The others are associated with elements 3 and 4 and are expressed as \(t_5, t_6, t_7\) and \(t_8\). The schematic plot of these elements of the blade is shown in Fig. 2.

The optimum weight results by using MFD, MMFD, and OCM are shown in Table 4 for the multiple frequency and side constraints. The constraints on the natural frequencies are chosen such that the fundamental natural frequency is higher than 1.5 \(\Omega_n\), and the second natural frequency is approximately higher than 1.25 \(\Omega_n\). The design history for various algorithms with multiple frequency constraints is shown in Fig. 9. For the dynamic response constraint case, the design history is shown in Table 5 and Fig. 10. For the constraints on both multiple frequency and dynamic response, the design history is shown in Table 6 and Fig. 11. The choice of the relaxation factor (\(\alpha\)) for OCM will influence the convergent speed. It is recommended to carefully chose \(\alpha\). A more efficient scheme, which has been incorporated in the computer program suggested by Sabek (1989), is to initially select a smaller \(\alpha\) and check the results after each cycle. From Tables 4-6 and Figs. 9-11, one can find that the optimum weight for the multiple frequency constraint case is much smaller than that of dynamic response constraint case. This is because, for the multiple frequency constraint case, it allows more dynamic response. The optimum design weights are shown in Table 5 and Fig. 10 for the dynamic response constraints as well as in Table 6 and Fig. 11 for both the multiple frequency and dynamic response constraints. The results are very close since the optimum design weight is dominated by the dynamic response constraint.

Example 2:

The blade considered in this example is the same as the previous example, but only four design variables which describe the ply thickness of the four layers of blade are used for all the four elements. The results of optimum design weight under the influence of rotating speed (\(\Omega\)) and setting angle (\(\phi\)) are shown in Figs. 12 and 13 for the multiple frequency constraints and the dynamic response constraints respectively. One can find that the optimum weight decrease with the increase of rotating speed. Moreover, the increase of setting angle will increase the optimum weight. It can also be found that the optimum design weight is higher than those obtained in Example 1 since only four design variables are considered. In addition, the optimum design weight of multiple frequency constraints is much smaller than that of dynamic response constraints. The optimum weight obtained with the consideration of both multiple frequency and dynamic response constraints is close to that of dynamic response constraints only. The reason for these phenomena is the same as that discussed in Example 1.
CONCLUSIONS

In this study, the isoparametric elements are employed to model the rotating laminated blade. The natural frequencies and dynamic response of the system are investigated. Three optimization algorithms are applied to study the minimum weight design with constraints on system dynamic behaviors. The conclusions can be summarized as follows:

1. For the dynamic behavior analysis, the results show that the natural frequencies will decrease with the increase of setting angle. Moreover, the increase of rotating speed and/or disk radius will increase the system natural frequencies.
2. The optimum design weight will decrease as the increase of rotating speed. However it will increase as the setting angle is increased.
3. The optimum design weights using OCM are well compared to those using MFD and MMFD. And the weight of rotating laminated blade can be greatly reduced when the optimum is reached.
4. The optimum weight with dynamic response constraints is higher than that with multiple frequency constraints. The optimum weight with both multiple frequency and dynamic response constraints are very close to those with dynamic response constraints only. This is because the design satisfies the dynamic response constraints will satisfy the multiple frequency constraints simultaneously. Moreover, the dynamic response constraints are found to be dominant for the weight optimization.

REFERENCES


APPENDIX A

TRANSFORMED REDUCED STIFFNESS MATRIX

The transformed reduced stiffness matrix is a 6 by 6 symmetric matrix, and is defined as:

\[
\begin{bmatrix}
\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\
\bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\
\bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\
0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\
0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55}
\end{bmatrix}
\]

where

\[
\bar{Q}_{11} = Q_{11} \cos^4 \theta + 2(Q_{12} + 2Q_{66}) \cos^2 \theta \sin^2 \theta + Q_{22} \sin^4 \theta
\]

\[
\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66}) \cos^2 \theta \sin^2 \theta + Q_{12}(\cos^4 \theta + \sin^4 \theta) + (Q_{12} - Q_{22} + 2Q_{66}) \cos \theta \sin^3 \theta
\]

\[
\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66}) \cos^3 \theta \sin \theta
\]

\[
\bar{Q}_{22} = Q_{11} \sin^4 \theta + 2(Q_{12} + 2Q_{66}) \cos^2 \theta \sin^2 \theta + Q_{22} \cos^4 \theta
\]

\[
\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66}) \cos \theta \sin^3 \theta + (Q_{12} - Q_{22} + 2Q_{66}) \cos \theta \sin^3 \theta
\]

\[
\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) \cos \theta \sin \theta
\]

\[
\bar{Q}_{44} = Q_{44} \cos^2 \theta + Q_{55} \sin^2 \theta
\]

\[
\bar{Q}_{45} = (Q_{55} - Q_{44}) \cos \theta \sin \theta
\]

\[
\bar{Q}_{55} = Q_{44} \sin^2 \theta + Q_{55} \cos^2 \theta
\]

with

\[
Q_{11} = \frac{E_1}{1 - \nu_{12} \nu_{21}} \\
Q_{12} = \frac{E_1 \nu_{21}}{1 - \nu_{12} \nu_{21}} \\
Q_{22} = \frac{E_2}{1 - \nu_{12} \nu_{21}}
\]

\[
Q_{44} = G_{12} \\
Q_{45} = G_{13} \\
Q_{55} = G_{23}
\]

APPENDIX B

MATERIAL PROPERTIES OF GRAPHITE/EPOXY

Young's Modulus:

- \( E_1 \): 128 GPa
- \( E_2 \): 11 GPa

Shear modulus:

- \( G_{12} \): 4.48 GPa
- \( G_{13} \): 4.48 GPa
- \( G_{23} \): 1.53 GPa

Poison ratio:

- \( \nu_{12} \): 0.35

Density:

- \( \rho \): 1.5 \( \times \) 10^3 kg/m^3

Thickness:

- \( t \): 0.00013 m

Table 1 The effect of nondimensional rotating speed (\( \Omega \)) and setting angle on nondimensional natural frequencies for [0°/±45°/90°], with a/b = 2, R/a = 1; mesh size: 4 x 4.

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Fig. 2 Eight-node element mesh of the laminated blade.

Table 2 The effect of nondimensional rotating speed ($\Omega$) and setting angle on nondimensional natural frequencies for $[0^\circ \pm 30^\circ]$, with $a/b = 2$, $R/a = 1$; mesh size: $4 \times 4$.

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Fig. 3 Effect of setting angle and rotating speed on 1st bending and torsional mode frequency for $[0^\circ / \pm 45/90]$, with $a/b = 2$, $R/a = 1$.

Table 3 The effect of nondimensional rotating speed ($\Omega$) and setting angle on nondimensional natural frequencies for $[\pm 45/\pm 45]$, with $a/b = 2$, $R/a = 1$; mesh size: $4 \times 4$.

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</table>
Fig. 4 Effect of setting angle and rotating speed on 1st bending and torsional mode frequency for \([0^\circ/\pm 30^\circ]\), with \(a/b = 2, R/a = 1\).

Fig. 5 Effect of setting angle and rotating speed on 1st and 2nd bending mode frequency for \([\pm 45^\circ/45^\circ]\), with \(a/b = 2, R/a = 1\).

Table 4 Initial and final design of various optimization algorithms with constraints on natural frequencies and \(a/b = 2, R/a = 1, \varphi = 45^\circ, \Omega = 600\pi \text{ rad/sec}; \) mesh size: \(2 \times 2\).

<table>
<thead>
<tr>
<th>Initial Design</th>
<th>Optimum Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t_1)</td>
<td>(1.50)</td>
</tr>
<tr>
<td>(t_2)</td>
<td>(1.50)</td>
</tr>
<tr>
<td>(t_3)</td>
<td>(1.50)</td>
</tr>
<tr>
<td>(t_4)</td>
<td>(1.50)</td>
</tr>
<tr>
<td>(t_5)</td>
<td>(1.50)</td>
</tr>
<tr>
<td>(t_6)</td>
<td>(1.50)</td>
</tr>
<tr>
<td>(t_7)</td>
<td>(1.50)</td>
</tr>
<tr>
<td>(t_8)</td>
<td>(1.50)</td>
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<td>18</td>
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<tr>
<td>Function Evaluations</td>
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</tr>
<tr>
<td>Maximum Dynamic Response of the Tip ((10^{-3} m))</td>
<td></td>
</tr>
<tr>
<td>(y)-direction</td>
<td>(0.00761)</td>
</tr>
<tr>
<td>(z)-direction</td>
<td>(11.395)</td>
</tr>
</tbody>
</table>

* Weight unit is \(10^{-3} kg\)
* Design variable \(t_i\) unit is \(10^{-3} m\)

Table 5 Initial and final design of various optimization algorithms with constraints on dynamic responses and \(a/b = 2, R/a = 1, \varphi = 45^\circ, \Omega = 600\pi \text{ rad/sec}; \) mesh size: \(2 \times 2\).

<table>
<thead>
<tr>
<th>Initial Design</th>
<th>Optimum Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t_1)</td>
<td>(1.50)</td>
</tr>
<tr>
<td>(t_2)</td>
<td>(1.50)</td>
</tr>
<tr>
<td>(t_3)</td>
<td>(1.50)</td>
</tr>
<tr>
<td>(t_4)</td>
<td>(1.50)</td>
</tr>
<tr>
<td>(t_5)</td>
<td>(1.50)</td>
</tr>
<tr>
<td>(t_6)</td>
<td>(1.50)</td>
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<tr>
<td>(t_7)</td>
<td>(1.50)</td>
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<tr>
<td>(t_8)</td>
<td>(1.50)</td>
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<td>Weight</td>
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<tr>
<td>Iteration Number</td>
<td>11</td>
</tr>
<tr>
<td>Function Evaluations</td>
<td>98</td>
</tr>
<tr>
<td>Maximum Dynamic Response of the Tip ((10^{-3} m))</td>
<td></td>
</tr>
<tr>
<td>(y)-direction</td>
<td>(0.00857)</td>
</tr>
<tr>
<td>(z)-direction</td>
<td>(0.9953)</td>
</tr>
</tbody>
</table>

* Weight unit is \(10^{-3} kg\)
* Design variable \(t_i\) unit is \(10^{-3} m\)
Fig. 6 Effect of setting angle and disk radius on 1st bending and torsional mode frequency for $[0/\pm45/90]$, with $a/b = 2$, $\bar{\Omega} = 1$.

Table 6 Initial and final design of various optimization algorithms with constraints on both natural frequencies and responses and $a/b = 2$, $R/a = 1$, $\varphi = 45^\circ$, $\Omega = 600\pi$ rad/sec; mesh size: $2 \times 2$.

<table>
<thead>
<tr>
<th>Initial Design</th>
<th>Optimum Design</th>
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</thead>
<tbody>
<tr>
<td>$t_1$</td>
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<td></td>
<td>1.31480</td>
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<td></td>
<td>1.18250</td>
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<td></td>
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<td>$t_2$</td>
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<td></td>
<td>0.96637</td>
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<tr>
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<tr>
<td></td>
<td>0.13000</td>
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<td>5</td>
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<tr>
<td>Function Evaluations</td>
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<td></td>
<td>0.8791</td>
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* Weight unit is $10^{-3}$ kg
* Design variable $t_i$ unit is $10^{-3}$ m

Fig. 7 Effect of setting angle and disk radius on 1st bending and torsional mode frequency for $[0/\pm30]$, with $a/b = 2$, $\bar{\Omega} = 1$.

Fig. 8 Effect of setting angle and disk radius on 1st and 2nd bending mode frequency for $[\pm45/\pm45]$, with $a/b = 2$, $\bar{\Omega} = 1$. 
Fig. 9 Comparison of design history various algorithms, constraints: natural frequency constraint.

Fig. 10 Comparison of design history various algorithms, constraints: dynamic response constraint.

Fig. 11 Comparison of design history various algorithms, constraints: both the natural frequency and the dynamic response constraint.

Fig. 12 The optimum weight under the influence of rotating speed ($\Omega$) and setting angle ($\varphi$), constraints: natural frequency constraint.

Fig. 13 The optimum weight under the influence of rotating speed ($\Omega$) and setting angle ($\varphi$), constraints: dynamic response constraint.