ABSTRACT

A method for calculating the quasi-3D unsteady transonic flowfield in oscillating cascades is presented.

The unsteady flow is assumed to be a small, harmonic perturbation of the non-linear steady flow, so that the steady flow problem is decoupled from the unsteady problem. As long as the vibration amplitudes remain moderate, the higher order terms in the governing equations derived under this assumption can be neglected and the describing unsteady flow equations become linear. Thus every frequency component can be calculated separately and the results be obtained by superposition.

For the calculation of the steady state flow, about which the unsteady part is linearized, a finite-volume time-stepping Euler solver is used.

Due to the similarity of the derived time-linear unsteady flow equations and the basic equations for the steady solver, the discretization is almost identical for both solvers. Thus it is possible to use much of the steady code with little modification for the time-linearized unsteady code.

The time-linear unsteady flow equations are solved on a moving grid. This leads to a considerable simplification of the flow tangency boundary condition on the surfaces of the airfoils.

Results obtained for various test cases compare favourably to flat plate theory and time-linearized potential methods as well as to experimental results from the Lausanne standard configurations.

The approach presented is computationally more efficient than nonlinear unsteady Euler time-stepping methods, thus permitting application in the standard design procedure.

NOMENCLATURE

A, B, C Jacobian matrices

c airfoil chord

cₚ unsteady pressure coefficient, p'/ρ₁w₁²

c₇ coefficient of aerodynamic work per cycle

F, G flux in φ and m direction, respectively

H source term

h streamsheet thickness

i imaginary unit

k reduced frequency, w₀/w₁

Mₐ Mach number

n unit vector normal to airfoil surface

p static pressure

r radius of streamsheet

s coordinate along airfoil surface

t time

u vector of conservation variables

V volume

v absolute flow velocity

w flow velocity vector

wᵣ relative flow velocity

wₓ position vector of airfoil surface

γ ratio of specific heats

ρ static density

φ interblade phase angle

T unit vector tangential to airfoil surface

φ circumferential coordinate

Ω angular frequency of oscillation

ωₙ angular frequency of rotation

Sub- and superscripts

\( \bar{ } \) time mean value

\( ' \) perturbation value

1 condition at cascade inlet

2 condition at cascade outlet

3 in meridional direction

n normal to airfoil surface

τ tangential to airfoil surface

φ in circumferential direction

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INTRODUCTION

In the design of modern axial turbomachinery, the trend towards thin, highly loaded bladings makes it difficult to prevent these bladings from destructive vibrations over the whole operating range. Resonances (forced response) and self induced vibrations (flutter) pose a serious threat to the structural integrity of the blades.

These problems necessitate the development of reliable and efficient prediction methods for the unsteady aerodynamic loads for use in an early stage of the design process. Currently, flat plate models or linearized potential methods are widely used for this purpose, while nonlinear time-stepping Euler- or Navier-Stokes-methods are available, but too expensive and too time-consuming for design purposes.

During the last few years, significant efforts were made in the development of time linearized Euler methods as an improvement over linearized potential methods for transonic flows. After a long pause following the pioneering work of Ni (1974), the idea was taken up by Hall and Crawley (1987). Hall and Clark (1991) and Hall and Lorence (1992) have since developed a 2D and a 3D solver with considerable success, as well as Holmes and Chuang (1991) with a 2D method for unstructured grids.

The recent work of Lindquist (1991) on the validity of the linearized method for flows with shocks has furthermore shown the capability of these methods to accurately model unsteady transonic flows.

In this paper a linearized Euler method for quasi-3D oscillating cascade flows is presented. It is based on the method described in a previous paper of the authors (Kahl and Klose, 1991). For this problem, a harmonically oscillating grid is used, as proposed by Hall and Clark (1991) as well as by Holmes and Chuang (1991). At the far field boundaries, analytically nonreflecting boundary conditions as derived by Hall and Crawley (1987) and Acton and Cargill (1987) are employed to eliminate spurious reflections.

The method is sufficiently fast to permit application in the standard design procedure. Results of the current method are presented for a flat plate cascade and for sub- and transonic turbine and compressor cascades. The results are compared to flat plate theory, linearized potential results and experimental data.

An extension of the method to forced response problems, i.e. the time-linearized computation of unsteady cascade flows due to periodic upstream or downstream perturbations is currently under investigation.

GOVERNING EQUATIONS

The unsteady flow is described by the two-dimensional inviscid and compressible Euler-equations in conservation form on a streamsheet of revolution (blade-to-blade, SI) with varying radius \( r \) and varying streamsheet thickness \( h \). The Euler-equations are given in integral form for a moving control volume \( V \) which is instantaneously coincident with the control volume attached to the grid:

\[
\left[ \frac{\partial u}{\partial t} - \frac{\partial \boldsymbol{m}}{\partial t} - \frac{\partial \boldsymbol{u}}{\partial \phi} \right] dV + \frac{\partial}{\partial t} \left[ \rho \right] dV = \frac{\partial}{\partial \phi} \left[ \rho G \phi \right] dV - \frac{\partial}{\partial \phi} \left[ \rho F \phi \right] dV (1)
\]

with

\[
\begin{align*}
\boldsymbol{u} &= \begin{bmatrix} \rho \\ \rho \varepsilon_m \\ \rho \varepsilon_e \\ \rho \varepsilon_h \end{bmatrix} \\
\boldsymbol{F} &= \begin{bmatrix} \rho \varepsilon_m \\ \rho \varepsilon_m v_x \\ \rho \varepsilon_m v_y \\ \rho \varepsilon_h \end{bmatrix} \\
\boldsymbol{G} &= \begin{bmatrix} \rho \varepsilon_m + p \\ \rho \varepsilon_m v_x + p \\ \rho \varepsilon_m v_y + p \\ \rho \varepsilon_h \end{bmatrix} \\
\boldsymbol{H} &= \begin{bmatrix} 0 \\ \rho \varepsilon_m v_x + p \\ \rho \varepsilon_m v_y + p \\ 0 \end{bmatrix}
\end{align*}
\]

\[v_x = w_x + \Omega r\]

\[v_y = \frac{1}{\gamma - 1} p^2 + \frac{1}{\gamma} \left( w_x^2 + w_y^2 - (\Omega r)^2 \right)\]

\[h = \frac{1}{\gamma - 1} p^2 + \frac{1}{\gamma} \left( w_x^2 + w_y^2 - (\Omega r)^2 \right)\]

Here, \( u \) is the vector of the conserved quantities (mass, momentum in the meridional direction, momentum in the axial direction and total internal energy), \( F \) and \( G \) are the fluxes in the circumferential and meridional direction and \( H \) is a source term that accounts for the variable radius and variable streamsheet thickness.

The fluid under consideration is assumed to be a perfect gas, so that

\[
P = \rho RT (3)
\]

TIME LINEARIZATION APPROACH

The basic idea of the linearization is to treat the unsteady flow as the sum of the underlying steady flow plus a small harmonically varying perturbation:

\[
\begin{align*}
\boldsymbol{u}(m,\phi,t) &= \tilde{\boldsymbol{u}}(m,\phi) + \tilde{\boldsymbol{u}}(m,\phi,t) e^{i\omega t}
\end{align*}
\]

Similarly, the computational grid is assumed to oscillate harmonically with the same frequency \( \omega \) and at a small perturbation amplitude \( (\tilde{\phi},\tilde{m}) \). The streamsheet radius and streamsheet thickness are assumed to remain constant at their steady state values.

The assumption of small, harmonic perturbations is then substituted into eq. (1). Collection of zeroth order terms yields the steady state Euler equations and the first order terms constitute the time linearized Euler equations. Second and higher order terms (products of perturbation quantities) are neglected because of the assumption of small perturbation amplitudes.
After some algebra and elimination of the \( e^{i\omega t} \) terms, the resulting basic equation for time linearized Euler analysis becomes:

\[
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} + \nabla p - \frac{1}{\rho} \mathbf{F} = 0
\]

\[
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} + \nabla p = \mathbf{F}
\]

where, \( \mathbf{F} \) represents the body forces and \( \mathbf{u} \) is the velocity vector.

If the steady flow is homogeneous, the Jacobians \( \Lambda, \Phi, \Gamma, \) and \( \Pi \) are independent of \( \phi \) and \( \mathbf{m} \) and the source terms in \( \Phi \) and \( \Pi \) vanish. If furthermore, the amplitudes of the grid motion are zero at the far field boundaries, eq. (5) becomes a linear homogeneous PDE with constant coefficients. This PDE can be cast into the form of an eigenvalue problem and can then be solved analytically. The result is a set of four independent perturbation waves for each circumferential wave number. These waves can be identified as up- and downstream travelling pressure waves (two downstream travelling waves for axially supersonic flow), one entropy and one vorticity wave, both of which are convected downstream.

As these perturbation waves are independent of each other, it is possible to prescribe at the far field boundaries only the values of the waves entering the computational domain while leaving the outgoing waves completely unaffected. Thus these waves will not be reflected at the boundaries. For the problem of vibrating airfoils, the value of the waves entering the domain will be set to zero.

**Flow Tangency Condition**

On the oscillating airfoil, the inviscid flow remains tangent to the airfoil at all times. If we assume an arbitrary motion of the airfoil, the boundary condition on the airfoil surface is

\[
\mathbf{w} \cdot \mathbf{n} = \frac{\partial \mathbf{u}}{\partial t} \cdot \mathbf{n} = 0
\]

where \( \mathbf{w} \) is the flow velocity vector, \( \mathbf{x} \) is the position vector of the airfoil surface and \( \mathbf{n} \) is the unit vector normal to the airfoil surface. Furthermore, \( \mathbf{t} \) is the unit vector tangential to the surface and \( \mathbf{s} \) is the coordinate along the surface.

We now assume that \( \mathbf{x}, \mathbf{n}, \) and \( \mathbf{t} \) are, just like the flow variables, composed of a mean part plus a small harmonic perturbation, i.e.:

\[
\mathbf{x}(s,t) = \mathbf{x}(s) + \mathbf{x}'(s)e^{i\omega t}
\]

\[
\mathbf{n}(s,t) = \mathbf{n}(s) + \mathbf{n}'(s)e^{i\omega t}
\]

\[
\mathbf{t}(s,t) = \mathbf{t}(s) + \mathbf{t}'(s)e^{i\omega t}
\]

Defining

\[
\mathbf{x}' = \mathbf{x}'(s)\mathbf{n}
\]

\[
\mathbf{w}' = \mathbf{w}'(s)\mathbf{n}
\]

\[
\mathbf{w}' = \mathbf{w}'(s)\mathbf{t}
\]

we find for the unit vector normal to the surface that

\[
\mathbf{n}' = -\mathbf{t} \frac{\partial \mathbf{n}}{\partial s}
\]

The linearized flow tangency condition then becomes

\[
\frac{\partial \mathbf{n}}{\partial s} = \mathbf{w}' + \mathbf{w}'
\]

\[
\frac{\partial \mathbf{n}}{\partial s} = \mathbf{w}' + \mathbf{w}'
\]

where \( \mathbf{w}' \) is the time derivative of the perturbation vector and \( \mathbf{w}' \) is the spatial derivative of the perturbation vector.
Here the first term is due to the velocity of the airfoil surface whereas the second term is due to the rotation of the airfoil. Eq. (11) is valid at the instantaneous position of the airfoil. On a fixed grid, an additional extrapolation to the instantaneous position of the airfoil must be included. This extrapolation introduces large numerical errors into the calculation, especially in regions of large mean flow gradients. By utilizing a moving grid, the linearized unsteady motion of the grid conforming to the motion of the airfoils on the surfaces, eq. (11) can be applied without any extrapolation. This approach, suggested by Hall and Clark (1991) as well as by Holmes and Chuang (1991), makes it possible to achieve accurate unsteady solutions even on fairly coarse grids.

Periodicity Condition

For aerelastic purposes, it can be assumed that the blades of the cascade are vibrating in a travelling wave mode. For the airfoils located inside the modeshape with a constant time lag between the motions of adjacent blades (= constant interblade phase angle). In cases where this assumption is not immediately valid, (i.e.: clustered blades), the modeshapes can always be decomposed into the sum of travelling wave modes due to the linearity of the system (Gerolymos, 1990, Nixon, 1982). The condition of all airfoils vibrating in the same modeshape with a constant interblade phase angle imposes a similar condition on the complex amplitudes of the flow variables. All perturbation amplitudes at locations being one spacing apart in the circumferential direction will be equal in magnitude, with a phase shift equal to the interblade phase angle. The application of this condition to the periodic boundary allows the computational domain to be restricted to a single blade passage.

DISCRETIZATION AND TIME MARCHING ALGORITHM

The scheme employed to solve the linearized Euler equations is a cell-vertex five-step Runge-Kutta finite-volume scheme with adaptive artificial viscosity (Jameson et al., 1981) on a H-type structured grid.

The discretization of the integrals employs two types of finite volumes: the "basic elements" consisting of four nodes and the "Euler elements" consisting of four basic elements that are adjacent to the center node of the Euler element. The Euler element thus contains nine nodes.

Each integral is formed by summing up the contributions of all four basic elements belonging to the Euler element. Accordingly, all integrals are first evaluated for the basic elements. To evaluate the volume integrals, the averaged values of the values at the nodes along the circumferential gridlines are multiplied by the appropriate fraction of the control volume and the results are summed up.

The surface integrals are evaluated by averaging the fluxes of the adjacent nodes for each cell face. The average is then multiplied by the appropriate cell face length and the contributions of all four cell faces are summed up. The flux balance is then set up for the basic element. Finally, the flux balances of all four elements belonging to one Euler element are summed up to yield the flux balance of the Euler element.

The artificial damping is calculated in the way described by Jameson et al. (1981). In its original form, it is a blend of second- and fourth difference operators with strongly nonlinear coefficients. The linearization of these coefficients would be very complicated, if at all feasible. In the current method, the nonlinear coefficients are frozen at their steady state values, as proposed by Holmes and Chuang (1991).

RESULTS

In this section, sample results of the linearized Euler method are shown. All calculations were done on an IBM RS6000 workstation. Three different mesh sizes were used: the "coarse" grid containing 41x11 nodes, the "medium" grid containing 81x21 nodes and the "fine" grid containing 161x41 nodes. CPU requirements per iteration were approx. 0.09 sec. on the coarse grid, 0.25 sec on the medium grid and 0.84 sec on the fine grid. Typically, the calculations needed 400 - 600 iterations to converge.

Subsonic Flat Plate Cascade

The first test case is a flat plate cascade in subsonic flow. The stagger angle is 30 deg., the space/chord ratio is 1.0, the Mach number is Ma=0.5 and the reduced frequency is k=1.0.

Figure 1 shows the real and imaginary parts of the unsteady pressure difference coefficients ΔCP for a torsional oscillation about midchord at a phase angle of α=90 deg. Also shown as symbols are the results obtained by the LINSUB program (Whitehead, 1987). The calculation was done for all three mesh sizes. Even for the coarsest grid (dashed lines), the agreement in the amplitude is quite good. It can be seen, however, that the leading edge singularity is much better resolved by the medium (dotted lines) and fine mesh (solid lines).

Gostelow Cascade (Low Speed Compressor Cascade)

The second test case is the cascade described by Gostelow (1984), consisting of Joukowski-like airfoils. The space/chord ratio is 0.99, stagger angle is 37.5 deg., inlet Mach number is Ma=0.25 and inlet flow angle is 53.5 deg. The blades are vibrating in pitch around their midchords at a reduced frequency of k=0.4 with an interblade phase angle of 180 deg.

Fig. 2 and 3 compare the calculated unsteady pressure distribution to the results of Whitehead’s (1982) FINSUP program, which is a linearized unsteady potential solver. For the coarse grid, the solution shows some deviations from the FINSUP results near the leading- and trailing edges, while for the finer meshes, the agreement is seen to be quite good. This was to be expected, as for subsonic flow there should be no difference between Euler and potential methods. In fig. 4 the real and imaginary parts of the unsteady moment coefficients for different interblade phase angles are shown. Again, the results compare favourably to the coefficients calculated using FINSUP.

Standard Configuration 4 (High Speed Turbine)

Additional results have been obtained for the high speed turbine cascade, which has been tested by Bölcs and Fransson (1986). It consists of blades with 45 deg. of camber set at a stagger angle of 55 deg. The cascade was tested over a wide range of exit Mach numbers (Ma=0.58 to 1.19) with the blades oscillating in a...
plunging motion. Figures 5 and 6 show the amplitude and phase of the unsteady pressure coefficient $c_{\text{w}}$ on the blade surface for an exit Mach number of $M_{\infty2} = 0.9$. Figures 5 and 6 also show an interblade phase angle of $\sigma = 90$ deg. (Note that this condition corresponds to $\alpha = -90$ deg in the definition of Bölcs and Fransson (1986).) The results of the current method (fine mesh) are drawn as solid lines, the dotted lines are calculated by the FINSUP program and the symbols represent the experimental results. The results of FINSUP and the current method agree quite well, apart from the pressure amplitude maximum on the suction side, which is overpredicted by the linearized Euler code. Compared to the measured values, both methods show correct trends, although the agreement is far from perfect.

In fig. 7, the coefficient of unsteady work per cycle $c_{\text{w}}$ is plotted for different phase angles at an exit Mach number of $M_{\infty2} = 0.9$. The results of FINSUP and the current method are again very close. Over the whole range of phase angles, the linearized Euler seems to predict slightly higher amplitudes than FINSUP. The reason for this discrepancy, which is found in many computed cases, is not clear yet. Compared to the experimental results, the linearized methods both correctly predict the region of aerodynamic instability ($c_{\text{w}}$ positive), although the magnitude of $c_{\text{w}}$ is strongly overpredicted for negative phase angles.

The reason for using the linearized Euler method instead of linearized potential methods is the expectation to obtain correct results in the transonic flow regime where the potential method can only be an approximation due to the isentropic assumption. Thus, the results of the current method for exit Mach numbers above unity are the most significant test cases.

Figures 8 and 9 show the amplitude and phase of the unsteady pressure distribution for the highest tested exit Mach number, $M_{\infty2} = 1.19$, at an interblade phase angle of $\sigma = 90$ deg. Along the pressure side, there are little differences between FINSUP and the current method, the agreement with the measured values is acceptable both for amplitude and phase angle.

Along the suction side, the differences are much more notable. The phase angles show similar trends for FINSUP, experiment and the current method, although the agreement is rather poor for the rear half of the airfoil.

The distribution of pressure amplitudes shows two distinct peaks in the experimental data. The first is at the point of the highest Mach number at about 20% chord. This peak is well predicted by FINSUP and again overpredicted by the current method. The second peak results from the trailing edge shock that impinges on the suction side (Bölcs and Fransson, 1986). The shock oscillates due to the airfoil vibration and, as it moves over the pressure transducer, creates an unsteady pressure signal with the amplitude of the steady pressure difference over the shock. This "shock impulse" contributes significantly to the overall unsteady aerodynamic work per cycle of the airfoil. The shock impulse is not seen in the FINSUP results, whereas the results of the linearized Euler method show it quite clearly.

The magnitude of the shock impulse depends on the pressure amplitude and on the amplitude of the shock motion. The results of the linearized method in conjunction with shock capturing yield no shock motion, but instead a region of high pressure amplitudes in the vicinity of the shock. Amplitude and width of this "impulse region" depend strongly on the way the shock is modeled. A crisp shock will yield a high amplitude in a small region whereas a highly smeared shock will result in moderate amplitudes over a wider region. For aeroelastic purposes, however, the most important result when trying to predict flutter is the total aerodynamic work of the airfoil over one period of oscillation, which is influenced by the product of amplitude and width of the impulse.

Fig. 10 shows the unsteady aerodynamic work coefficient $c_{\text{w}}$ for the whole range of tested Mach numbers and for an interblade phase angle of $\sigma = 90$ deg. The results of the current method show a close agreement with the experiments.

Standard Configuration 10 (Transonic NACA 0006)

As a final test case, results for a cascade of NACA 0006 airfoils oscillating in torsion in transonic flow are shown. The cascade is staggered at 45 deg, the space/chord ratio is unity, the inlet flow angle is 58 deg and the inlet Mach number is $M_{\infty1} = 0.8$.

This case has been studied by Verdon (1989) using a linearized potential method which includes the effects of oscillating shocks by the use of shock fitting. Therefore, Verdon's results are particularly useful to assess the capability of the current method to model these transonic unsteady effects.

Two different conditions are analyzed. In the first, the airfoils oscillate in phase ($\alpha = 0$ deg) at a reduced frequency of unity. The calculated work coefficients are $c_{\text{w}} = -0.77$ for the coarse mesh, $c_{\text{w}} = -0.88$ for the medium mesh and $c_{\text{w}} = -0.96$ for the fine mesh, while Verdon gives a value of $c_{\text{w}} = -1.02$, of which -0.54 are due to the oscillating shock. This shows that the linearized Euler method needs sufficient grid resolution to accurately model the unsteady transonic effects. The distribution of $8c_{\text{w}} \delta s$ over the airfoil, called pressure-displacement function PDF by Verdon, is shown in fig. 11 for the fine grid, along with the data given by Verdon. Along the pressure side, the difference is minimal. Along the suction side, the discontinuity at the shock seen in Verdon's results is smeared to a "shock region", thus accounting for the unsteady shock loads. The shock loads are not seen in Verdon's PDF but calculated separately.

For the second condition, the airfoils oscillate out of phase ($\alpha = 180$ deg) at a reduced frequency of $k = 1.5$. The calculated work coefficients are $c_{\text{w}} = -1.54$ for the coarse mesh, $c_{\text{w}} = -1.40$ for the medium mesh and $c_{\text{w}} = -1.32$ for the fine mesh, while Verdon gives a value of $c_{\text{w}} = -1.24$, of which +0.16 are due to the oscillating shock. The PDF for this case is shown in fig. 12. In this case, the agreement both along the pressure side and the suction side is quite acceptable.

These results support the conclusion that the linearized Euler method is able to correctly model the main features of transonic flows including the effect of moving shock waves, although the reliability needs to be confirmed by additional test cases.

CONCLUDING REMARKS

A method for calculating the unsteady airloads on airfoils in oscillating cascades has been presented. The governing equations are linearized in time with the perturbations assumed to vary harmonically in time. This restricts the applicability of the method to small
disturbances while at the same time reducing the computational effort required to solve the problem.

The equations are solved on a moving grid, which conforms to the motion of the airfoils. Nonreflecting boundary conditions are employed to accurately model isolated cascades.

The solution algorithm uses a cell-vertex formulation, the steady state of the complex amplitudes is reached by time marching using a Runge-Kutta integration scheme with Jameson-type artificial damping employed.

The results presented have shown that the accuracy of the current method is similar to that of linearized potential methods for subsonic flows.

First results for transonic flows demonstrate the ability to model the effects of shocks in unsteady flow.

Future development will concentrate on the further validation of the transonic results and on the development of a fully 3D time-linearized solver.

ACKNOWLEDGEMENTS

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REFERENCES


Figure 1 Real and imaginary part of unsteady pressure difference coefficient $\Delta c_p$ on flat plate cascade, $Ma=0.5$, space/chord=1, stagger angle=30 deg, $k=1$, $\sigma=90$ deg.

Figure 2 Gostelow's cascade - real part of $c_p$ for torsional oscillation about midchord, $k=0.4$, $\sigma=180$ deg.

Figure 3 Gostelow's cascade - imaginary part of $c_p$ for torsional oscillation about midchord, $k=0.4$, $\sigma=180$ deg.

Figure 4 Gostelow's cascade - real and imaginary part of unsteady moment coefficient $c_m$ for torsional oscillation about midchord, $k=0.4$, deg.
Figure 5 Standard Configuration 4 - amplitude of unsteady pressure coefficient $c_p$, $Ma_2=0.76$, $\alpha=90$ deg.

Figure 6 Standard Configuration 4 - phase of unsteady pressure coefficient $c_p$, $Ma_2=0.76$, $\alpha=90$ deg.

Figure 7 Standard Configuration 4 - coefficient of aerodynamic work per cycle $c_w$, $Ma_2=0.9$.

Figure 8 Standard Configuration 4 - amplitude of unsteady pressure coefficient $c_p$, $Ma_2=1.19$, $\alpha=90$ deg.
Figure 9 Standard Configuration 4 - phase of unsteady pressure coefficient $c_p$, $M_2=1.19$, $\sigma=90$ deg.

Figure 11 Standard Configuration 10 - pressure displacement function PDF, $M_1=0.8$, $k=1$, $\sigma=0$ deg.

Figure 10 Standard Configuration 4 - coefficient of aerodynamic work per cycle $c_w$, $\sigma=90$ deg.

Figure 12 Standard Configuration 10 - pressure displacement function PDF, $M_1=0.8$, $k=1.5$, $\sigma=180$ deg.