CHAOTIC BEHAVIOR OF ROTOR/STATOR SYSTEMS WITH RUBS

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ABSTRACT

This paper outlines the dynamic behavior of externally excited rotor/stator systems with occasional, partial rubbing conditions. The observed phenomenon have one major source of a strong nonlinearity: transition from no contact to contact state between mechanical elements, one of which is rotating. This results in variable stiffness and damping, impacting, and intermittent involvement of friction. A new model for such a transition (impact) is developed. In case of the contact between rotating and stationary elements, it correlates the local radial and tangential ("super ball") effects with global behavior of the system. The results of numerical simulations of a simple rotor/stator system based on that model are presented in the form of bifurcation diagrams, rotor lateral vibration time—base waves, and orbits. The vibrational behavior of the considered system is characterized by orderly harmonic and subharmonic responses, as well as by chaotic vibrations. A new result (additional subharmonic regime of vibration) is obtained for the case of heavy rub of an anisotropically supported rotor. The correspondence between numerical simulation and previously obtained experimental data supports the adequacy of the new model of impact.

INTRODUCTION

During the last 30 years researchers have documented irregular, and unpredictable dynamic behavior of various physical systems under external excitation. Nonlinear phenomena occurring in these systems are responsible for these irregular and unpredictable effects that lead to chaos.

One of the mechanical systems which exhibits chaotic behavior is the rotating shaft with its stator and supporting structure. The normal conditions of rotor operation are determined by appropriate clearances at the joints and boundaries. There are possible divergences from the normal operational conditions, such as:

(i) Looseness in the stationary joints; for example, between the rotor supporting pedestal and foundation.

(ii) Oversize, poorly lubricated bearing: a looseness between the stationary and rotating elements. (The bearing clearance may increase from "normal" to "oversize" due to wear.)

(iii) Rubbing rotor: occasional physical contact between the rotating and stationary elements which, during normal operational conditions, should not take place.

In these cases the dynamic phenomena revealed in the rotor motion occur as secondary effects of the primary cause. The latter is most often a trade between the imbalance—related rotating exciting force, and the unilateral, radial force applied to the rotor. For instance, the looseness in the pedestal joint, or oversize bearing, would not be discovered until the lifting imbalance—related centrifugal force would exceed the gravity force that presses the joints into close contact. In the rotor—to—stator rub case, the unwelcome rotor/stator contact occurs when the rotor is moved to the side due to the radial force, and/or amplitudes of its lateral precessional motion, such as excited by imbalance, exceed allowable clearances.

There are several physical phenomena which take place in the discussed cases:

• Occasional, time—variable increase/decrease of the system stiffness. In the case of looseness, the average stiffness level decreases; in the case of rubs, it increases. Similarly to stiffness, the system damping varies.

• Impacts. The sudden physical contact between elements is accompanied by characteristic local phenomena, followed by global motion changes of the system.

• Friction. In cases of the oversize bearing and rub malfunctions, the rotating shaft enters into contact with a stationary part. This contact is accompanied by friction, as an effect of relative motion of the rotating and stationary parts.

All these phenomena introduce nonlinearities into the system, and individually or interactively, they can contribute to chaotic behavior of the system.

Even though the chaotic phenomena in similar rotor/stator mechanical systems were investigated by several researchers (Thomson et al (1982), Evan–Iwanowski et al (1991), Szczypinski (1987), Ehrich (1991), Gonzalves et al (1992)), their results have mostly qualitative character, since they employ oversimplified impact model. This paper presents a new, more adequate model of the impact. This model is further employed in numerical simulation of rotor/stator systems with various rubbing conditions. The results exhibit orderly and chaotic behavior pattern of rotors.

MODEL OF ELASTIC IMPACT AT ROTOR MASS LOCATION

Accuracy in the description of the looseness and/or rub—related phenomena in mechanical structures with intermittent inter—element contacts depends mainly on the adequacy of the impact model. The local/global effect of the impact introduces a nonlinearity to the system, and generates specific vibrational responses of the system.
There are three approaches employed in the description of impacts between rotational and stationary parts of the mechanical system. The first is based on the classical restitution coefficient model in which an impact is considered instantaneous and elastic (Szczygielski, 1987). The second approach considers inelastic impact with a zero restitution coefficient; the impact is followed by a sliding stage (Muszynska, 1989). In this model, changes of the rotor precessional speed during the impact also occur instantaneously. In both these models, the other forces applied to the rotor do not practically affect the result of the impact, because they are relatively small. Neither model takes into account changes in the rotor displacement which may occur during the impact. The third approach considers the mechanical system as having discontinuous piece-wise characteristics with additional stiffness and damping of the stator at the contact period (Ehrich, 1988). This model seems more accurate, but it creates certain numerical problems, since two different time scales should be considered: one during the free motion of the shaft when there is no contact (global motion), and the other during the contact period which is characterized by much higher stiffness (local motion). This causes some difficulties in the description of the behavior of the system.

The analytical model of the impact discussed in this paper is based on the third approach. It presents the analysis of a piece-wise system with an assumption that the local stiffness of the stator or bearing at the intermittent contact location is much higher than the stiffness of the rotor. A similar approach was used in the description of "normal-loose" phenomena occurring due to an untightened rotor pedestal (Goldman et al, 1991). The impact model will be derived with the assumption that the impact occurs at the rotor modal mass location. The first lateral mode of the rotor is taken into consideration. This approach can be easily modified for other cases.

A simple isotropic rotor with an anisotropic support is considered. During its lateral motion, the rotor occasionally rubs against the stator (Fig. 1). The mathematical model of the rotor lateral motion \( x(t), y(t) \) at the mode uncoupled from the stator is as follows:

\[
M\ddot{x} + D\dot{x} + K_{xx} = mr\omega^2 \cos(\omega t + \alpha) + S \cos \gamma
\]

\[
M\ddot{y} + D\dot{y} + K_{yy} = mr\omega^2 \sin(\omega t + \alpha) + S \sin \gamma
\]

\[
\text{where } \gamma = \frac{d}{dt}
\]

which can be transformed using polar coordinates \( R(t) \) and \( \psi(t) \):

\[
x = R \cos \psi, \quad y = R \sin \psi
\]

In Eqs. (1), (2), \( M, D, K_x, K_y \) are modal mass, damping and anisotropic stiffnesses of the rotor system, \( K = (K_x + K_y)/2 \), \( \alpha = (K_x - K_y)/2 \), \( \psi = \alpha + \alpha \) is the total phase of the unbalance, \( \omega \) is rotative speed of the shaft, \( S \) is radial force, \( \gamma \) is angle between this force and horizontal direction, \( m, r, \alpha \) are the mass, radius, and angular orientation of the rotor respectively. The rotor/stator contact occurs when \( R \geq c \) where \( c \) is a radial clearance between the rotor and the stator. It is assumed that, during the contact, the stator acts as a spring with stiffness \( K_r \) and damping \( D_r \) in the radial direction, and creates a tangential friction force \( F_r \). Therefore, the system equations of motion at the rotor/stator contact stage written in polar coordinates are as follows:

\[
M\ddot{R} + D\dot{R} + K\ddot{\psi} - (D+D_r)\dot{R} + MR\dot{\psi}^2 + mr\omega^2 \cos(\varphi - \psi) + S \cos(\gamma - \psi)
\]

\[
M\ddot{\psi} - (D+D_r)\dot{\psi}^2 - 2M\dot{\psi} + K\ddot{\psi} + mr\omega^2 \sin(\varphi - \psi) + S \sin(\gamma - \psi) + F_r(\omega a/R)
\]

\[
\text{where } R \text{ is radius of the shaft at the contact cross-section. The friction force } F_r \text{ is attached to the shaft at the point of contact (Fig. 1). It is assumed that it has a mixed viscous/dry pattern, thus following relation holds true:}
\]

\[
F_r = -D_{cont}(\dot{\psi}R + \omega a) - f[D\dot{R} + K_r(\ddot{R} - \psi)\text{[sign}(\dot{\psi}R + \omega a)]}
\]

\[
\text{where } \text{sign}(\dot{\psi}R + \omega a) \text{ is the relative velocity of the rotor at the stator rubbing surface. } D_{cont} \text{ and } f \text{ are local contact viscous and dry friction coefficients respectively.}
\]

Eq. (4) is true only until \( \dot{\psi}R + \omega a \neq 0 \). If the sliding velocity becomes zero, the friction force creates a moment which is equal to the absolute value of the sum of all the other moments, \( M_\psi \) applied to the rotor, and has an opposite direction to it. It means that in the absence of sliding, there is \( \ddot{\psi} = 0, \dot{\psi} = -\omega a/R \). The friction can start again if \( M_\psi \) satisfies the following inequality: \( |M_\psi| < f[D\dot{R} + K_r(\ddot{R} - \psi)](a/R) \). In this case, the friction force at the sliding period can be expressed as follows:

\[
F_r = f[D\dot{R} + K_r(\ddot{R} - \psi)] \text{sign}(M_\psi)
\]

It is further assumed that if the sliding stopped, it would not start again during that particular contact period. This assumption means that the direction of sliding cannot be changed.

For further analytical considerations a small parameter, \( \epsilon \), is introduced. An assumption has been made that \( \epsilon = \omega a/\sqrt{M/K_r} \) is small. It means that considered range of rotative speeds is limited, and that the stator local stiffness is comparably high, therefore, the radial displacement of the stator is small. It is also assumed that, during the rotor/stator contact stage, the rotor radial motion is performed close to the stator surface, so that \( R - c = \epsilon z \) where \( \epsilon \) is again the small parameter, and \( z \) is a new variable.

In all practical cases, the shaft radius \( a \) is much larger than the clearance \( c \). This means that the arm of the friction force can be approximated by \( a \). Using these assumptions, and neglecting the terms of higher powers of the small parameter \( \epsilon \), Eqs. (3) can be transformed to the following:
$z'' + 2\zeta_c z' + z = \epsilon \left[ \Phi + \epsilon (\beta^2 - \frac{1}{\beta^2}) - 2\zeta z' \right] + \epsilon^2 \left[ \ldots \right] + \ldots$

$\beta' + \zeta_{\text{cont}}(\beta + \frac{\beta}{c}) + \frac{\beta}{c} \text{sign}(\beta + \frac{\beta}{c}) (z'' + z) = \epsilon \Phi + \epsilon \zeta_{\text{cont}}(\beta + \frac{\beta}{c}) - \frac{2\beta}{c} \left( \frac{\beta}{c} + \frac{\beta}{c} \right) \text{sign} \left( \beta + \frac{\beta}{c} \right) + \epsilon^2 \left[ \ldots \right] + \ldots \quad (6)$

where the following notation was introduced (the rest of notations is in Nomenclature):

$' = \frac{d}{dt}, \quad \zeta_{\text{cont}} = \frac{\alpha}{c}, \quad \Phi = \frac{S}{M \omega^2} \cos(\gamma - \psi) + \frac{m R}{M} \cos(\psi - \psi) - \frac{\Delta K}{M \omega^2} \cos 2\psi$

$F = \frac{m R}{M} \sin(\psi - \psi) + \frac{S}{M \omega^2} \sin(\gamma - \psi) + \frac{\Delta K}{M \omega^2} \sin 2\psi$

The solution of Eqs. (6) is sought in the form of a power series of the small parameter $\epsilon$:

$z = z_0 + \epsilon z_1 + \epsilon^2 z_2 \ldots, \quad \beta = \beta_0 + \epsilon \beta_1 + \epsilon^2 \beta_2 \ldots, \quad \psi = \psi_0 + \epsilon \psi_1 + \epsilon^2 \psi_2 \ldots \quad (7)$

with the following initial conditions:

$x(0) = 0, \quad z(0) = z_-, \quad \beta(0) = \beta_- \quad (8)$

where the subscript "-" means the value of the corresponding variable just before rotor/stator contact.

From Eqs. (6) and (7), the equations for zero approximation (all terms standing in front of $\epsilon^2$) are as follows:

$z''_0 + 2\zeta \zeta z_0 + z_0 = 0$

$\beta'_0 + \zeta_{\text{cont}}(\beta_0 + \frac{\beta_0}{c}) + \frac{\beta_0}{c} (2z''_0 + \frac{\beta_0}{c}) \text{sign} (\beta_0 + \frac{\beta_0}{c}) = 0 \quad (9)$

where $z_0$ and $\beta_0$ represent the zero approximation of the corresponding variables. Eqs. (8) are considered as initial conditions for Eqs. (9). All higher approximations would have then zero initial conditions.

The integration of the first Eq. (9) yields the following solution:

$z_0(\tau) = \begin{cases} 
\frac{(z_- e^{-\tau})}{\sqrt{1 - \zeta^2 \tau}} \sin \left[ \sqrt{1 - \zeta^2 \tau} \right] & \text{if } \zeta_1 \leq 1 \\
\frac{(z_- e^{-\tau})}{\sqrt{\zeta^2 \tau^2 - 1}} \sinh \left[ \sqrt{\zeta^2 \tau^2 - 1} \right] & \text{if } \zeta_1 > 1 
\end{cases} \quad (10)$

As it was noted earlier, the condition for the beginning of the contact stage is geometrical equality $z = 0$, that is $R = R_c$. Once occurred, the contact is maintained if the rotor/stator normal force is positive. The zero approximation for the normal force is equal to $2z'_0 z_0 + z_0$, or according to the first Eq. (9), is equal to $(-z'_0)'$. Therefore, the contact lasts until the radial acceleration is negative.

The zero approximation $\tau_0$ for the contact stage duration is the first root of the equation $z''_0(\tau_0) = 0$, at which the radial acceleration has a positive derivative. This yields:

$$\tau_0 = \begin{cases} 
\pm \frac{\arccos \left( \frac{2}{1 - \zeta^2} \right)}{\sqrt{1 - \zeta^2}} & \text{if } \zeta_1 \leq 1 \\
\pm \frac{2 \ln \left[ \frac{\sqrt{\zeta_1^2 - 1} + \sqrt{\zeta_1}}{\zeta_1} \right]}{\sqrt{\zeta_1^2 - 1}} & \text{if } \zeta_1 > 1 
\end{cases} \quad (11)$$

The zero approximation $\zeta_0$, for the radial velocity $z'_0$ at the moment of the shaft departure from the contact is easily calculated using Eqs. (10) and (11):

$\zeta_0 = \frac{\arccos \left( \frac{2}{1 - \zeta^2} \right)}{\sqrt{1 - \zeta^2}} \quad (12)$

with

$$\kappa_0 = \begin{cases} 
\exp \left[ -\frac{\zeta_1}{\sqrt{1 - \zeta^2}} \arccos \left( \frac{2}{1 - \zeta^2} \right) \right] & \text{if } \zeta_1 \leq 1 \\
\exp \left[ -\frac{2 z'_0}{\sqrt{\zeta_1^2 - 1}} \ln \left[ \frac{\sqrt{\zeta_1^2 - 1} + \sqrt{\zeta_1}}{\zeta_1} \right] \right] & \text{if } \zeta_1 > 1 
\end{cases} \quad (13)$$

where the subscript "+" means the value of the variable at the instant of separation, $\kappa_0$ is defined as a zero approximation for the radial restitution coefficient. Eqs. (11) and (13) determine the zero approximation for the rotor/stator coupling duration, and the radial restitution coefficient in the entire range of $\zeta_1$. A graphical representation of these functions is shown in Figure 2. It can be seen that the dwelling time decreases with an increase of the stator damping factor. The physical explanation of it is based on the change of the rotor radial velocity sign from positive to negative (Fig. 3). With the negative sign, the damping force opposes the normal force direction. Therefore, a higher stator damping leads to an earlier nullification of the normal force.

**Figure 2** Zero approximation to the (a) radial restitution coefficient, (b) nondimensional dwelling time versus local damping factor of the stator.

**Figure 3** Forces in the radial direction during the impact (a) when radial velocity is positive, (b) when radial velocity is negative. Note that in the case (b), damping causes a decrease of the normal force.
Looking at the graph of \(\kappa_{20}\) versus \(\zeta_r\) (Fig. 2A), it is clear that if \(\zeta_r > 8\), then \(\kappa_{20} < 10^{-3}\), and \(\kappa_{20}\) can be considered small, even in comparison with the terms of the first order of the small parameter. This case needs a separate consideration, and further it is referred to as the case of non-elastic impact (Muszynska et al, 1993).

The integration of the second Eq. (9) yields the following expression:

\[
\beta_1(\tau) = (\beta_+ + \frac{a}{c}i) \zeta_r \zeta_{\text{cont}} \tau - \frac{a}{c} i \frac{f_{\beta}}{\tau} \left[ \beta_0(\tau) - \zeta_r \zeta_{\text{cont}} \beta_0(\tau) \right] - \frac{a}{c} i \zeta_r \zeta_{\text{cont}} + \frac{e^2}{c} \zeta_r \zeta_{\text{cont}} \int_0^\tau \beta_0(\tau) e^{\zeta_r \zeta_{\text{cont}} \tau} d\tau \]
\[
(14)
\]

where \(a = \zeta_r (\beta_+ + \frac{a}{c}) = \pm 1\).

Assuming that the contact tangential damping \(D_{\text{cont}}\) is relatively small so that the last term of Eq. (14) can be neglected, calculate now \(Q_0\) at the moment of departure \(T_0\):

\[
\beta_1(T_0) = (\beta_+ + \frac{a}{c}i) \kappa_{20} \frac{a}{c} \zeta_r \zeta_{\text{cont}} + \zeta_r \zeta_{\text{cont}} \beta_0(T_0) \]
\[
(15)
\]

where Eq. (12) was used, and, by definition, the zero approximation to the sign, and implies from Eq. (15) the following condition:

\[
\zeta_r \zeta_{\text{cont}} (\zeta_r \zeta_{\text{cont}}) = \zeta_r \zeta_{\text{cont}} (\zeta_r \zeta_{\text{cont}}) + \zeta_r \zeta_{\text{cont}} \beta_0(T_0)
\]

and can be further neglected.

As it follows from Eqs. (12) and (15), the zero approximation describes the impact in terms of the stator properties, as well as the normal and tangential velocities of the rotor during the contact period. In order to analyze the influence of the shaft unbalance and radial forces on the impact, it is necessary to calculate the first approximation of the solution (7). The balance of terms associated with the first power of the small parameter \(\epsilon\) in Eqs. (6) yields the following equations for the first approximation:

\[
x_1' + 2\zeta_r x_1 + x_1 = \Psi_1 + c(\beta_+ - \frac{1}{p^2}) - 2\zeta_r \sigma
\]

\[
\beta_+ + \zeta_r \zeta_{\text{cont}} \beta_1 = \frac{a}{c} \zeta_r \zeta_{\text{cont}} \left( (\zeta_r \zeta_{\text{cont}} + 2\zeta_r \zeta_{\text{cont}}) \right) = \frac{a}{c} \zeta_r \zeta_{\text{cont}} + \zeta_r \zeta_{\text{cont}} \beta_0(T_0) + \zeta_r \zeta_{\text{cont}} \beta_0(T_0)
\]

(19)

The first approximation of the rotor-to-stator contact stage duration \(\tau_{dw}\) can be calculated based on Eqs. (7), (10), and (21) if \(\tau\) in the expression \(2\zeta_r x_1' + \zeta_r x_1\) is substituted by the series \(\tau_{dw} = T_0 + \epsilon T_1 + \ldots\). The balance of the first power terms gives the following equation for \(\tau_1\):

\[
\tau_1 = [\Phi_1 + c(\beta_+ - \frac{1}{p^2})] \frac{1 + \kappa_{20} \kappa_{20}}{\kappa_{20}}
\]

(23)

The first approximations of the radial \(s_1'\), and angular \(\beta_1\), velocities at the moment of rotor departure from the contact can be calculated using Eqs. (10), (12), (14), and (21) with the accuracy up to the second power of the small parameter \(\epsilon\):

\[
s_1' = -\kappa_{20} \left( s_1' \right) - 2c[\Phi_1 + c(\beta_+ - \frac{1}{p^2})] \zeta_r
\]

\[
\beta_1 = \kappa_{20} \beta_+ \frac{a}{c} (1 - \kappa_{20}) - \frac{a}{c} \left( \zeta_r \zeta_{\text{cont}} \kappa_{20}\right) - \frac{a}{c} \zeta_r \zeta_{\text{cont}} \beta_0(T_0) + \zeta_r \zeta_{\text{cont}} \beta_0(T_0)
\]

(18)
The first approximation of the rotor polar coordinates $R_1$, $\psi_1$ at the moment of departure from the contact are the sums of two components respectively. The first component reflects the influence of the first approximation $T_1$ of the dwelling duration on the zero approximation of the coordinates. The second one represents the result of the integration of the first approximation of the angular and radial velocities on the zero of the coordinates. Using Eqs. (23), (24), the resulting expressions for $R_1$, $\psi_1$ are as follows:

$$R_1 = \frac{d\psi_1}{dt} \left|_{\tau=T_0} \right. \psi_1 + \nu (\tau_0) = -4 \kappa_0 \sqrt{\frac{c^2}{\Omega_0}} \left[ \Phi_0 \omega^2 + c(\psi^2 - \nu^2) \right]$$

$$\psi_1 = \frac{\omega}{\Omega} \int_0^\tau \beta(\tau) d\tau = \frac{\omega}{\Omega} \left[ \Phi_0 \omega^2 + c(\psi^2 - \nu^2) \right] \frac{(1+\kappa_0)}{\kappa_0} \psi_1 + \frac{1}{\kappa_0} \omega^2 R_- + \frac{F \tau f}{2} - 2(\psi_1)(R_0)(1-\kappa_0(\psi_1 - 1))$$

(25)

Since the shaft motion in the radial and tangential directions at the contact stage are relatively small, this entire stage can be considered as a transformation mapping of the starting point to the ending point of the contact:

$$t_0 + t_1 = t + t_{dw}$$

$$R_0 \rightarrow R_+ \psi_0 \rightarrow \psi_+ \rightarrow \psi, R_0 \rightarrow R_+ \psi_0 \rightarrow \psi_+ \rightarrow \psi_+$$

(26)

where using previously obtained relationships for the first order approximation, and returning to the original polar variables:

$$t_{dw} = \frac{\sqrt{M}}{K_f} T_0 + \frac{1}{K_f} \kappa_{0n} \frac{[\omega^2 \Phi_1 + c(\psi^2 - K_M)]}{\frac{1}{\kappa_0}}$$

(27)

$$R_+ = \frac{\kappa_{0n}}{\left( \frac{1}{\nabla_1} - \frac{\kappa_{0n}}{\kappa_{0}} \right)} \left( \psi \right) = \frac{K_f}{\kappa_0} \kappa_{0n} \frac{[\omega^2 \Phi_1 + c(\psi^2 - K_M)]}{\frac{1}{\kappa_0}}$$

(28)

$$\psi_+ = \psi_0 \frac{\nabla}{c} \left( 1 - \frac{\kappa_{0n}}{\kappa_{0}} \right) \frac{-\nabla}{c} \frac{\kappa_{0n} (\nabla_1 + \kappa_{0n}) \text{sign}(\psi_+ - \frac{\nabla}{c})}{-2 \frac{\kappa_{0n}}{K_f} \Gamma} + \frac{\sqrt{M}}{K_f} \tau \omega \Phi_1$$

(29)

$$\text{if sign}(\psi_+ + \frac{\nabla}{c}) = \text{sign}(\psi_+ + \frac{\nabla}{c})$$

(30)

$$\psi_+ = \frac{\nabla}{c} \text{if sign}(\psi_+ - \frac{\nabla}{c}) \neq \text{sign}(\psi_+ - \frac{\nabla}{c})$$

(31)

$$R_+ = c + \frac{\Delta t}{\kappa_{0n}} \frac{\Gamma}{\sqrt{K_M}} \left( \psi_+ \right) = \frac{\Delta t}{\kappa_{0n}} \frac{\Gamma}{\sqrt{K_M}} \left( (\psi_+ - \frac{\nabla}{c}) + \frac{\sigma a f}{K_f} \kappa_{0n}(\psi_+) + \frac{\sigma a f}{K_f} \frac{K_f}{\kappa_{0n}} R_- + \frac{\Delta t \kappa_{0n}^2}{2} \omega^2 R_- - 2(\psi_+)(R_+)(1+\kappa_{0n}(\psi_+ - 1)) \right)$$

(32)

The rotor system presented in Figure 1 has two stages of motion: free uncoupled stage without contact described by Eq. (2), and impacting stage, when the contact is maintained between rotor and stator. The last stage is described by Eqs. (26) to (32). The switch from the free stage to the impact occurs when $R = c$, $R > 0$.

The expression (32), which describes the impact angular position with the accuracy up to the second power of the small parameter defined as

$$\varepsilon \equiv \frac{\omega}{\Omega} = \frac{M}{\sqrt{K_f}}$$

depends on the angles between the normal line at the point of contact and the radial load force $(\gamma - \nu)$, as well as between the normal line and the unbalance force $(\Phi_0 - \psi_0)$. For example, Eq. (27) yields that the maximum dwelling time occurs when $\psi_0 = \gamma + 2\pi i$, $\Phi_0 = \gamma + 2\pi f i$ (i, j are integers). It means that, in that case, unbalance and radial forces have the direction opposite to the direction of the normal line. Therefore, the radial motion of the rotor at the moment of departure from the contact is opposed by the unbalance and radial forces. This case corresponds to the minimum absolute value of the radial velocity $R$, of the rotor departing from the contact. The angular velocity of precession $\psi_0$ in this case, does not depend on the external forces since they have perpendicular direction to the tangential component of velocity.

**RESULTS OF THE NUMERICAL SIMULATIONS**

The model of rotor-to-stator contact impact developed in previous section is used for numerical simulation. The numerical values of the parameters chosen for the numerical simulation are given in Table 1, and the results are illustrated in Figures 4 to 8. Case R.1 represents lightly radially loaded isotropic rotor with isotropic supports; Cases R.2 and R.3 show the behavior of heavily radially loaded rotors with isotropic and anisotropic supports respectively. In all cases the parameters were chosen so that partial rub was maintained in the wide range of rotative speeds. The rotative speed was incremented with 30 rpm. At every rotative speed, the data started to be sampled after 200 rotations in order to eliminate transient regimes. When the steady-state regime was established, 100 consecutive once-per-rotative speed (Keyphasor) marks which appear on the vibration response waveforms were recorded. If the vibrational regime (response) was periodic, the Keyphasor marks were at constant positions for any number of rotations. For a synchronous (1x) regime, there will be one constant position, that is one mark. For a subsynchronous regime of 1/2, there will be two constant Keyphasor positions, etc. Chaotic regimes result in multiple positions, sampled through 100 consecutive rotations. These results are presented in form of bifurcation diagrams versus frequency ratio, where the observed variables are vertical y and horizontal x positions of the Keyphasor (Figs. 4, 5, 7). The bifurcation diagrams obtained by the described technique are known as brute-force bifurcation diagrams (Parker et al, 1989). Some bifurcation diagrams are accompanied by sequences of typical orbits of the rotor precessional motion. The rotor orbits represent the actual path of the rotor centerline in the $xy$ plane. As it follows from observation of the rotor waveforms and contact data (Figs. 5, 8), the change from one regime to the other occurs when inside one oscillation period the number of contact occurrences changes.

In all considered cases there exist frequency ratio ranges exhibiting orderly harmonic (1x) and subharmonic (1/2x, 1/3x ...), as well as chaotic patterns of responses. The latter not only occur as transitions from one to another harmonic regime, but also appear inside the same harmonic regime, such as 1x, 1x+ (Figs. 4, 6, 7).

The case R.3 differs from R.2 (Table 1) by taking into account the rotor support stiffness anisotropy. The anisotropy results in an additional 1/2x regime (named "butterfly rub," associated with the specific orbit patterns (Bently et al, 1992)) occurring in the range between 2.3 and 2.6 frequency ratio. The orbits obtained by numerical simulation for that case correspond very well to those obtained experimentally (Bently et al, 1992))

**FINAL REMARKS**

This paper presented the analytical and numerical simulation of unbalanced rotor rubbing against the stator. The rotor vibrational behavior is characterized by orderly harmonic and subharmonic responses,
as well as by chaotic pattern of vibrations. The most important result presented in this paper is the development of the more adequate impact model. The impact dynamic effects in similar systems discussed in the available references were usually modeled in very simplified forms. The model developed here takes into account the local stator stiffness and radial/tangential damping parameters, and correlates them with the globally considered impacting body, that is, with rotor parameters. During the impact of a rotating shaft against a stationary element, not only the radial (straight impact) effects are taken into consideration, but also, due to shaft rotation, the tangential effects. The latter are sometimes called "super ball" effects (Szczygielski, 1987). The correlation of the local contact conditions, and global dynamic behavior of the rotor presented in this paper, throws a new light on the problem of adequate modeling of the mechanical systems with intermittent, partial rubbing. In the wide frequency range, the numerical results showed the sequence of subharmonic and chaotic regimes strongly correlated with the number of impacts per oscillation. The obtained orbits are typical for the real rubbing rotor responses (Muszynska et al, 1990, Ehrich, 1991). For the lower orders of harmonic and subharmonic vibrations (1x and 1/2x), the dominant, with the widest frequency ranges, are one-impact-per-oscillation periodic regimes. All other regimes represent bifurcation and transition between the latter. The same is true for 1/3x oscillation, but

FIGURE 4 BIFURCATION DIAGRAMS: ROTOR VERTICAL (a) AND HORIZONTAL (b) DISPLACEMENTS VERSUS ROTATIVE FREQUENCY-TO-NATURAL FREQUENCY \( \sqrt{K/M} \) RATIO WITH A SEQUENCE OF ORDERLY REGIME ORBITS (a), AND CHAOTIC REGIME ORBITS (b) FOR THE CASE R.1 (Table 1) OF ISOTROPIC LIGHTLY RADIIALLY LOADED ROTOR. SEMI-ELASTIC IMPACT, \( K_{2m} = 0.5 \).

FIGURE 5 SEQUENCE OF ROTOR TIME-BASE WAVEFORMS AND ORBITS FOR THE CASE R.1 OF ISOTROPIC LIGHTLY RADIIALLY LOADED ROTOR (TABLE 1). AMPLITUDE SCALE = 3.2 MIL/DIV. THE PRESENTED ORBITS ARE TYPICAL IN THE INDICATED BANDS OF THE FREQUENCY RATIO.

### TABLE 1. ROTOR-TO-STATOR RUB. PARAMETERS USED IN NUMERICAL SIMULATION.

<table>
<thead>
<tr>
<th>Case No.</th>
<th>R.1</th>
<th>R.2</th>
<th>R.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_x )</td>
<td>1</td>
<td>1</td>
<td>0.65</td>
</tr>
<tr>
<td>( K_y )</td>
<td>0.0816</td>
<td>0.0816</td>
<td>0.073</td>
</tr>
<tr>
<td>( \sqrt{K_x / K_f} )</td>
<td>1/s</td>
<td>3000</td>
<td>3000</td>
</tr>
<tr>
<td>( D_{2/K_fM} )</td>
<td>0.015</td>
<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td>( D_{2/K_fM} )</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>( \omega )</td>
<td>0.0857</td>
<td>0.0857</td>
<td>0.0857</td>
</tr>
<tr>
<td>( S_{K_fC} )</td>
<td>6.76 ( \times 10^{-4} )</td>
<td>6.69 ( \times 10^{-4} )</td>
<td>6.69 ( \times 10^{-4} )</td>
</tr>
<tr>
<td>( f_a )</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>( D_{cont} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>270</td>
<td>270</td>
<td>270</td>
</tr>
</tbody>
</table>
**FIGURE 6** BIFURCATION DIAGRAMS: ROTOR VERTICAL AND HORIZONTAL DISPLACEMENTS VERSUS ROTATIVE FREQUENCY-TO-ROTOR NATURAL FREQUENCY RATIO FOR THE CASE OF ISOTROPIC HIGHLY RADIALLY LOADED ROTOR, CASE R.2 (TABLE 1). SEMI-ELASTIC IMPACT, $\kappa_{\text{Dm}} = 0.5$.

- $\omega = 2000$ rpm
  - $\frac{V_m}{V_A} = 0.65$
  - 5 impacts per rot
  - Period = 1 rot
  - $0.6 < \frac{V_m}{V_A} < 1.0$
  - Orbit scale = 6.0 mils/div

- $\omega = 2400$ rpm
  - $\frac{V_m}{V_A} = 1.14$
  - 2 impacts per rot
  - Period = 1 rot
  - $1.11 < \frac{V_m}{V_A} < 1.25$
  - Orbit scale = 2.4 mils/div

- $\omega = 2700$ rpm
  - $\frac{V_m}{V_A} = 1.31$
  - 1 impact per rot
  - Period = 1 rot
  - $1.25 < \frac{V_m}{V_A} < 1.42$
  - Orbit scale = 2.4 mils/div

- $\omega = 2900$ rpm
  - $\frac{V_m}{V_A} = 1.36$
  - 1 impact per rot
  - Period = 1 rot
  - $1.42 < \frac{V_m}{V_A} < 1.64$
  - Orbit scale = 2.4 mils/div

- $\omega = 3500$ rpm
  - $\frac{V_m}{V_A} = 1.64$
  - 1 impact per rot
  - Period = 1 rot
  - $1.64 < \frac{V_m}{V_A} < 2.33$
  - Orbit scale = 2.4 mils/div

- $\omega = 4900$ rpm
  - $\frac{V_m}{V_A} = 2.33$
  - 1 impact per rot
  - Period = 1 rot
  - $2.33 < \frac{V_m}{V_A} < 3.18$
  - Orbit scale = 2.4 mils/div

**FIGURE 7** BIFURCATION DIAGRAMS: ROTOR VERTICAL AND HORIZONTAL DISPLACEMENTS VERSUS ROTATIVE FREQUENCY-TO-ROTOR NATURAL FREQUENCY $\sqrt{\frac{V_m}{W_B}}$ RATIO FOR THE CASE OF ANISOTROPIC, HIGHLY RADIALLY LOADED ROTOR, CASE R.3 (Table 1) ACCOMPANIED BY SOME ORBITS. NOTE DIFFERENT ORBIT SCALES SPECIFIED IN FIGURE 8. SEMI-ELASTIC IMPACT, $\kappa_{\text{Dm}} = 0.5$.

- $\omega = 3500$ rpm
  - $\frac{V_m}{V_A} = 1.64$
  - 1 impact per rot
  - Period = 1 rot
  - $1.64 < \frac{V_m}{V_A} < 2.33$
  - Orbit scale = 2.4 mils/div

- $\omega = 4900$ rpm
  - $\frac{V_m}{V_A} = 2.33$
  - 1 impact per rot
  - Period = 1 rot
  - $2.33 < \frac{V_m}{V_A} < 3.18$
  - Orbit scale = 2.4 mils/div

- $\omega = 5200$ rpm
  - $\frac{V_m}{V_A} = 2.49$
  - 1 impact per rot
  - Period = 2 rot
  - $2.49 < \frac{V_m}{V_A} < 3.18$
  - Orbit scale = 2.4 mils/div

- $\omega = 5600$ rpm
  - $\frac{V_m}{V_A} = 2.67$
  - 1 impact per rot
  - Period = 2 rot
  - $2.67 < \frac{V_m}{V_A} < 3.41$
  - Orbit scale = 2.4 mils/div

- $\omega = 5900$ rpm
  - $\frac{V_m}{V_A} = 2.88$
  - 1 impact per rot
  - Period = 2 rot
  - $2.88 < \frac{V_m}{V_A} < 3.69$
  - Orbit scale = 2.4 mils/div

**FIGURE 8** SEQUENCE OF ROTOR TIME-BASE WAVEFORMS AND ORBITS FOR THE CASE R.3 OF ANISOTROPIC HIGHLY RADIALLY LOADED ROTOR (TABLE 1). THE PRESENTED ORBITS ARE TYPICAL IN THE INDICATED BANDS OF THE FREQUENCY RATIO.

- $\omega = 3500$ rpm
  - $\frac{V_m}{V_A} = 1.64$
  - 1 impact per rot
  - Period = 1 rot
  - $1.64 < \frac{V_m}{V_A} < 2.33$
  - Orbit scale = 2.4 mils/div

- $\omega = 4900$ rpm
  - $\frac{V_m}{V_A} = 2.33$
  - 1 impact per rot
  - Period = 1 rot
  - $2.33 < \frac{V_m}{V_A} < 3.18$
  - Orbit scale = 2.4 mils/div

- $\omega = 5200$ rpm
  - $\frac{V_m}{V_A} = 2.49$
  - 1 impact per rot
  - Period = 2 rot
  - $2.49 < \frac{V_m}{V_A} < 3.18$
  - Orbit scale = 2.4 mils/div

- $\omega = 5600$ rpm
  - $\frac{V_m}{V_A} = 2.67$
  - 1 impact per rot
  - Period = 2 rot
  - $2.67 < \frac{V_m}{V_A} < 3.41$
  - Orbit scale = 2.4 mils/div
the tendency of the multi—impacting and chaotic regimes to widen their frequency band was noted. This indicates an increasing intensity of the chaotic motion at higher frequencies.

Another worth—mentioning result presented in this paper is an insight into the influence of mixed viscous/dry friction forces at the contact, as well as the influence of rotor anisotropy on the rub—related responses.

REFERENCES


