Single Degree of Freedom Shear-Mode Piezoelectric Energy Harvester

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1 Introduction

Considerable interest has been focused recently on developing a wide variety of harvesters to harness the energy from ambient vibrations by using the direct piezoelectric effect [1–3]. Most of these harvesters are intended for powering small electronic devices and remote sensors in order to eliminate their dependence on external power sources such as batteries or power grids. In this manner, these self-powered devices and sensors can operate autonomously in an uninterrupted fashion over extended periods of time.

In the majority of the exerted efforts, the emphasis is placed on utilizing energy harvesters operating in thickness-mode (TMH) and longitudinal-mode (LMH). These efforts aim at maximizing the power transferred from the TMH and LMH to the output load. Various innovative approaches have been considered. Distinct among these approaches is the use of the concept of impedance matching between the piezoelectric harvester and the electrical load as reported, for example, by Kong et al. [4], Liang and Liao [5], Stephen [6], and Chen et al. [7]. However, other concepts have also been adopted. For example, Wu et al. [8] developed a tunable resonant frequency power harvesting device to continuously match the time-varying frequency of the external vibration in real time. Also, Badel et al. [9] augmented the harvester with an electrical switching device in which the switch is triggered on the maxima or minima of the displacement and realizes a voltage inversion through an inductor to maximize the output voltage of the harvester. Other approaches include the work of duToit [10] and duToit et al. [11] where the optimal parameters of the harvesters are selected to maximize the extracted power when mechanical damping is neglected. Daqaq et al. [12] and Renno et al. [13] extended the work of duToit [10] and duToit et al. [11] to include the effects of damping and electromechanical coupling when optimizing the harvester output power. In 2010, Aldraihem and Baz [14] coupled a TMH with a dynamic magnifier system to amplify the power output of the harvester. The dynamic magnifier consists of a specifically tuned spring-mass system that is coupled to a conventional piezoelectric harvester. It aims at amplifying mechanically the strain experienced by the piezo-element and in turn magnifying the electrical power output of the harvester. The concept of dynamic magnification is extended and applied both theoretically [15] and experimentally [16] to LMH which are used to harness the energy of vibrating beams.

In all the above mentioned efforts, the harvester systems have been primarily linear systems. Recent efforts have concentrated on maximizing the harvested power, over broad frequency range, by using multiharvesters as reported by Xue et al. [17], or by exploiting different sources of nonlinearities as in the work of LeFeuvre et al. [18], Moehlis et al. [19], and Adhikari et al. [20].

Recently, special interest has been focused on using a shear-mode harvester as a viable means for enhancing the harnessed electrical power output. The enhancement is generated by capitalizing on the fact that the strain constant of the piezoelectric in shear is much higher than those due to thickness or longitudinal deflections. To achieve such an enhancement, the piezoelectric element is poled along a direction parallel to its electrodes and is sandwiched between a proof mass and oscillating base in a design similar to that of the TMH and the LMH. Sinusoidal excitation of the base, along the poling direction, makes the piezo-element experience mechanical shear strain which when converted into electrical power produces outputs that are larger than those of the TMH and the LMH. The theory governing the operation of this class of SMH is developed for simple resistive electrical loads. Numerical examples are presented to illustrate the optimal performance characteristics of the SMH in comparison with the TMH and LMH. The effect of the piezo-element material, excitation frequency and electrical load on the harvested power is presented. The obtained results demonstrate the feasibility of the SMH as a simple and effective means for enhancing the power output characteristics of conventional TMH and LMH. [DOI: 10.1115/1.4023950]

Keywords: energy harvesting, piezoelectric harvesters, shear-mode harvester, thickness-mode harvester, longitudinal-mode harvester
the use of SMH in harnessing vibration energy is still in its infancy unlike the application of shear-mode actuators which has been studied extensively; for example, by Benjeddou et al. [23], Aldraihem and Khdeir [24], and Al-Ajmi and Tawfik [25].

In the present work, a single degree of freedom SMH configuration is considered in order to outline its merits and gain insight in its operation as compared to the conventional LMH and TMH. Therefore, the paper is organized in five sections. In Sec. 1, a brief introduction has been presented. In Sec. 2, the concept of the shear-mode energy harvester is introduced and compared with that of the conventional thickness-mode harvesters. In Sec. 3, the mathematical model of the SMH is developed using the Newtonian dynamics approach. The performance characteristics of the SMH in comparison with the conventional thickness-mode energy harvesters (TMH) and longitudinal-mode energy harvesters (LMH) are presented in Sec. 4. The conclusions and the future work are summarized in Sec. 5.

2 Concept of Shear-Mode Harvester

Figure 1(a) shows a conventional thickness-mode energy harvester (TMH). Generally, the TMH consists of a piezo-element which is sandwiched between a proof mass and a vibrating base structure. The piezo-element is poled along direction 3 which is perpendicular to its electrodes. When the base is subjected to a sinusoidal excitation, along direction 3, a relative motion (x-xb) is generated between the proof mass and the base producing across-the-thickness mechanical strain in the piezoelectric element. The resulting strain is converted into electrical power, flowing along direction 3, by virtue of the direct piezoelectric effect.

In Fig. 1(b), a longitudinal-mode harvester (LMH) has a design similar to that of the TMH with the piezo-element poled along direction 3 which is perpendicular to the electrodes. However, the base excitation and the resulting strain in the piezoelectric element occur along the longitudinal direction 1 while the electrical power is harnessed along direction 3.

Figure 1(c) shows a shear-mode harvester (SMH) which is considered, in this study, as a viable alternative to the TMH in order to enhance the harvested output power. The enhancement is generated by capitalizing on the fact that the strain constant of the piezoelectric in shear is much higher than that in compression/tension. To achieve the enhancement, the piezoelectric element is poled along direction 3 which is parallel to the electrodes and is sandwiched between a proof mass and oscillating base in a design similar to that of the TMH. Sinusoidal excitation of the base, along the poling direction 3, makes the piezo-element experience a shear strain (x-xb)/tp, which, in effect, is an angular rotation γ along direction 5. This strain is converted into electrical power that is collected by the electrodes along direction 1.

3 Mathematical Modeling of Shear-Mode Harvester

The theory governing the operation of this class of SMH is developed for simple resistive electrical loads.

The constitutive equations of the piezo-element are given by [26]

\[ T_S = c_{55}^E (S_S - d_{15} E_1) \]  

\[ D_1 = q_1 A = d_{15} T_S + c_{11}^E T_1 \]  

where \( S_S \) = shear strain, \( D_1 \) = electrical displacement, \( q_1 \) = electrical charge, \( T_S \) = shear stress, \( E_1 \) = electrical field, \( c_{55}^E \) = Young’s modulus, \( d_{15} \) = piezoelectric strain coefficient, \( c_{11}^E \) = permittivity, and \( A \) = surface area of piezo-patch.

Substituting Eqs. (1) into (2) gives

\[ q_1 = A d_{15} c_{55}^E S_S + C_{p_e} v \]  

\[ C_{p_e} = \left(1 - k_{15}^2\right) c_{p_e} \]  

with \( k_{15}^2 = d_{15}^2 c_{55}^E / c_{11}^E \) = electromechanical coupling factor and \( \psi_e = A c_{11}^E / t_p \).

Note that \( c_{p_e} \) and \( C_{p_e} \) denote the unblocked and blocked capacitances of the piezo-element respectively. Also, \( v \) denotes the voltage across the piezo-element = \( E_1 \ t_p \) where \( t_p \) is the thickness of the piezo-element.

Let \( z_1 = x - x_b \), hence the shear strain \( S_S \) can be written as

\[ S_S = z_1 / t_p \]  

Differentiating Eq. (3) with respect to the time, gives

\[ k_p d_{15} z_1 + C_{p_e} v + \frac{1}{R_{L}} v = 0 \]  

where \( k_p = (A c_{55}^E / t_p) \) = stiffness of piezo-element. Note that in the development of Eq. (6), the output voltage \( v \) and the current \( q_1 \) are related by

\[ v = -R_L q_1 \]  

where \( R_L \) is the resistance of the load.

Applying Newton’s second law, yields the equation of motion of the proof mass under the influence of the shear force developed by the piezo-element as follows:
\[ m_\alpha \ddot{x} + c_d (\dot{x} - \dot{x}_h) = -T_3 A \]  

where \( m_\alpha = (m + (1/3)m_p) \) with \( m_p \) = effective mass of the piezoelement as outlined in the Appendix, and \( c_d \) = viscous damping coefficient. Eq. (6) can be combined with Eq. (1) to yield the following equation:

\[ m_\alpha \ddot{z}_s + c_d \dot{z}_s + k_d z_s - d_{15} k_p v_s = -m_\alpha \dot{x}_b \]  

Applying the Laplace transform to Eqs. (6) and (9) gives

\[ (s^2 + 2 \zeta_s \omega_n^2 + \omega_n^2)Z_s - d_{15} \omega_n^2 V_s = -s^2 X_b \]  

and

\[ k_p d_{15} Z_s = -\left( C_p s + \frac{1}{R_L} \right) V_s \]  

where \( s \) is the Laplace complex number, \( \omega_n^2 = k_p/m_\alpha \) = harvester natural frequency, and \( \zeta_s = c_d/(2m_p \omega_n) \) = harvester damping ratio. Also, \( Z_s, V_s, \) and \( X_b \) are the Laplace transforms of \( z_s, v_s, \) and \( x_b \), respectively. Eliminating \( V_s \) between Eqs. (10) and (11) or \( Z_s \) gives

\[ \frac{Z_s}{-s^2 X_b} = \frac{R_L C_p s + 1}{(s^2 + 2 \zeta_s \omega_n s + \omega_n^2) \left( R_L C_p s + 1 \right) + d_{15} R_L k_p \omega_n^2 s} \]  

and

\[ \frac{V_s}{-s^2 X_b} = \frac{-k_p d_{15} R_L s}{(s^2 + 2 \zeta_s \omega_n s + \omega_n^2) \left( R_L C_p s + 1 \right) + d_{15} R_L k_p \omega_n^2 s} \]  

For sinusoidal base excitation at a frequency \( \omega, \) \( s \) is replaced by \( j\omega, \) then the magnitude of the relative displacement \( Z \) and the voltage \( V \) can be written as

**Table 1** Main parameters of the numerical energy harvesters (duToit [10]; Daqqa et al. [12])

<table>
<thead>
<tr>
<th>Property</th>
<th>( m ) (kg)</th>
<th>( t_p - l_p ) (m)</th>
<th>( A ) (m²)</th>
<th>( \zeta_{15} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.01</td>
<td>0.01</td>
<td>0.0001</td>
<td>0.02</td>
</tr>
</tbody>
</table>

**Table 2** Physical properties of common piezoelectric materials

<table>
<thead>
<tr>
<th>Property</th>
<th>PZT-5A(^a)</th>
<th>PZT-5H(^b)</th>
<th>PMN-0.345PT(^b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta_{11} ) (pm²/N)</td>
<td>16.4</td>
<td>16.5</td>
<td>13.5</td>
</tr>
<tr>
<td>( \delta_{33} ) (pm²/N)</td>
<td>18.8</td>
<td>20.7</td>
<td>14.6</td>
</tr>
<tr>
<td>( \delta_{55} ) (pm²/N)</td>
<td>47.5</td>
<td>43.5</td>
<td>31</td>
</tr>
<tr>
<td>( d_{31} ) (pm/V)</td>
<td>-171</td>
<td>274</td>
<td>219</td>
</tr>
<tr>
<td>( d_{33} ) (pm/V)</td>
<td>684</td>
<td>741</td>
<td>554</td>
</tr>
<tr>
<td>( c_{11} ) (m²/N)</td>
<td>1700</td>
<td>3400</td>
<td>4057</td>
</tr>
<tr>
<td>( k_{11} ) (GPa)</td>
<td>1730</td>
<td>3130</td>
<td>3740</td>
</tr>
<tr>
<td>( k_{33} ) (GPa)</td>
<td>0.496</td>
<td>0.566</td>
<td>0.455</td>
</tr>
<tr>
<td>( k_{13} ) (GPa)</td>
<td>0.119</td>
<td>0.152</td>
<td>0.0993</td>
</tr>
<tr>
<td>( k_{31} ) (GPa)</td>
<td>0.478</td>
<td>0.420</td>
<td>0.299</td>
</tr>
</tbody>
</table>

\(^a\)Heinonen et al. [27].  
\(^b\)Alguero et al. [28].
The power $P_s$ harvested by the shear-mode harvester is given by

$$P_s = V_s^2 / R_L \tag{16}$$

Hence, from Eq. (15), it can be written as

$$\left| \frac{P_s}{(\omega X_b)^2} \right| = \frac{k_p x_a k_s^2 \Omega_s^2}{\omega_n \left(1 - \left(1 + 2\zeta_x \Omega_s \right) \Omega_s^2 \right) + \left(\{2\zeta_s + (1 + k_2^s) x_s\} \Omega_s - x_s \Omega_s^3 \right)^2} \tag{17}$$

The power attains a maximum when $dP_s/dx_s = 0$, yielding an optimum electrical load $x_{op}$ given by

$$x_{op}^2 = \frac{1}{\Omega_s^2 \left(1 - \Omega_s^2 + (2 \zeta_s \Omega_s)^2 \right)} \tag{18}$$

For the thickness-mode harvester, the corresponding expressions are obtained by Daqaq et al. [12], giving

$$\left| \frac{P_t}{(\omega X_b)^2} \right| = \frac{k_p x_a k_t^2 \Omega_t^2}{\omega_n \left(1 - \left(1 + 2\zeta_t \Omega_t \right) \Omega_t^2 \right) + \left(\{2\zeta_t + (1 + k_2^t) x_t\} \Omega_t - x_t \Omega_t^3 \right)^2} \tag{19}$$

with the optimum value attained at an optimum electrical load $x_{op}$ given by

$$x_{op}^2 = \frac{1}{\Omega_t^2 \left(1 - \Omega_t^2 + (2 \zeta_t \Omega_t)^2 \right)} \tag{20}$$

where $k_p = (AcT_i / t_p) \Omega_p = \omega_1 / \omega_m, x_t = R_t C_{pf} \omega_n, k_2^t = (k_p d_31) / (k_p C_{pf}), \omega_n = k_p / m_t, \zeta_t = c_d / (2m_t \omega_n), C_p = (1 - k_3^t) C_{pf}$, $c_p = A_{pf} / t_p$, and $k_3^t = d_33^2 / e_{33}^2$.

Similar expressions can be derived for the longitudinal-mode harvester, yielding:

$$\begin{array}{c|c|c|c}
\text{Property} & \text{Piezoelectric material} & \text{PZT-5A} & \text{PZT-5H} & \text{PMN-0.345PT} \\
\hline
\text{Power} & \frac{P_s}{(\omega X_b)^2} & 4.478E-7 & 4.283E-7 & 3.611E-7 \\
\hline
\text{Short-circuit resonant frequency} & SMH & 1.000 & 1.000 & 1.000 \\
\text{TMH} & 1.000 & 1.000 & 1.000 & 1.000 \\
\text{LMH} & 1.000 & 1.000 & 1.004 & 1.044 \\
\text{Open-circuit resonant frequency} & SMH & 1.384 & 1.312 & 1.196 \\
\text{TMH} & 1.408 & 1.512 & 1.356 & 1.365 \\
\text{LMH} & 1.096 & 1.080 & 1.044 & 1.044 \\
\text{Optimal electrical load (x)} & SMH & 0.044 & 0.056 & 0.093 \\
\text{TMH} & 0.041 & 0.029 & 0.047 & 0.047 \\
\text{LMH} & 0.205 & 0.234 & 0.422 & 0.422 \\
\end{array}$$

4 Numerical Examples

Numerical examples are presented, in this section, to illustrate the performance characteristics of the SMH in comparison with those of TMH and LMH. The effect of the piezo-element material, excitation frequency and electrical load on the harvested power is presented.

Tables 1 and 2 list the main geometrical and physical parameters of the piezoelectric elements considered in this study.

Figure 2 shows comparisons between the frequency response of the maximum power output of thickness, longitudinal, and shear-mode harvesters made of different piezoelectric materials. The considered materials are PZT-5A, PZT-5H, and PMN-0.345PT.

The corresponding frequency response characteristics of these three materials are shown in Figs. 2(a), 2(b), and 2(c), respectively. All the figures are plotted such that at each frequency ($\Omega_p, \Omega_t$, and $\Omega_s$), the TMH, LMH, and SMH produce their maximum power by operating against the optimal electrical loads ($x_t, x_s$, and $x_0$), as predicted by Eqs. (18), (20), and (22) for the TMH, LMH, and SMH, respectively.

In all the considered case, the SMH is found to harvest the highest power followed by the TMH and then the LMH. For example, for harvesters using PZT-5A piezo-elements, the SMH generates a maximum power, normalized with respect to the square of
the base acceleration, of 4.478E-7 W/(m/s^2)^2, while the TMH produces 2.811E-7 W/(m/s^2)^2 and the LMH yields 2.627E-7 W/(m/s^2)^2. It is evident that the SMH generates a maximum power which is nearly 159.3% and 170.2% the power of the TMH and LMH, respectively. Such significant enhancement resulting from using the SMH as compared to the TMH and LMH becomes less pronounced when the piezo-element is manufactured from PZT-5H and much less pronounced when using PMN-0.34PT as is clear from Figs. 2(b) and 2(c) respectively.

Table 3 lists a more comprehensive summary of the power enhancement obtained by using the SMH as compared to the TMH and LMH for all the considered piezoelectric materials.

The plots displayed in Fig. 2 indicate that the frequency response characteristics of all the harvesters have two distinct peaks. These peaks occur at the short and open-circuit resonant frequencies of each harvester. Physically, these frequencies correspond to the resonant and antiresonant natural frequencies of the harvester (Daqaq et al., [12]).
A complete list of the short and open-circuit resonant frequencies of the different harvester configurations and materials is given in Table 3.

Because the short and open-circuit conditions produce equal harvested power, the remainder of this study will focus only on the short-circuit condition.

Figure 3 shows the frequency response of the three harvesters as obtained at the short-circuit conditions with each harvester operating against the optimal electrical load as indicated in Table 3. This load is maintained constant over the considered frequency range. Under such a condition, the frequency response characteristics of the harvester exhibits a single resonance occurring at the short-circuit resonant frequency as listed in Table 3.

Figure 4 displays the effect of the load resistance on the output power of each harvester while operating at the short-circuit resonant frequency. It can be seen that the power output attains a maximum at a particular load resistance. The values of these load
Table 4 Effective mass of the piezoelectric elements used in the TMH, LMH, and SMH

<table>
<thead>
<tr>
<th>Property</th>
<th>TMH</th>
<th>LMH</th>
<th>SMH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Configuration and degrees of freedom</td>
<td><img src="image" alt="Piezo-element" /></td>
<td><img src="image" alt="Piezo-element" /></td>
<td><img src="image" alt="Piezo-element" /></td>
</tr>
<tr>
<td>Interpolation equation</td>
<td>$u = [(1 - x/l) x/b] u_1$</td>
<td>$u = [(1 - x/l) x/b] u_1$</td>
<td>$u = [(1 - x/l) x/b] u_1$</td>
</tr>
<tr>
<td></td>
<td>$= [N] {\delta_l}$</td>
<td>$= [N] {\delta_l}$</td>
<td>$= [N] {\delta_l}$</td>
</tr>
<tr>
<td>Element kinetic energy</td>
<td>$T_i = \frac{1}{2} {\delta_i}^T \rho A \int_0^l [N]^T [N] dx {\delta_i}$</td>
<td>$T_i = \frac{1}{2} {\delta_i}^T \rho A \int_0^l [N]^T [N] dx {\delta_i}$</td>
<td>$T_i = \frac{1}{2} {\delta_i}^T \rho A \int_0^l [N]^T [N] dx {\delta_i}$</td>
</tr>
<tr>
<td></td>
<td>$= \frac{1}{2} {\delta_i}^T [M] {\delta_i}$</td>
<td>$= \frac{1}{2} {\delta_i}^T [M] {\delta_i}$</td>
<td>$= \frac{1}{2} {\delta_i}^T [M] {\delta_i}$</td>
</tr>
<tr>
<td>Element mass matrix</td>
<td>$[M] = \frac{1}{6} \rho Al \begin{bmatrix} 2 &amp; 1 \ 1 &amp; 2 \end{bmatrix}$</td>
<td>$[M] = \frac{1}{6} \rho Al \begin{bmatrix} 2 &amp; 1 \ 1 &amp; 2 \end{bmatrix}$</td>
<td>$[M] = \frac{1}{6} \rho Al \begin{bmatrix} 2 &amp; 1 \ 1 &amp; 2 \end{bmatrix}$</td>
</tr>
<tr>
<td>Effective mass with node 1 fixed (eliminating first row and column of $[M]$)</td>
<td>$m_{n_1} = \frac{1}{3} \rho Al = \frac{1}{3} m_p$</td>
<td>$m_{n_1} = \frac{1}{3} \rho Al = \frac{1}{3} m_p$</td>
<td>$m_{n_1} = \frac{1}{3} \rho Al = \frac{1}{3} m_p$</td>
</tr>
</tbody>
</table>

5 Conclusions

This paper has presented a basic study of three configurations of single degree of freedom energy harvesters. The considered configurations included thickness-mode (TMH), longitudinal-mode (LMH) and shear-mode harvester (SMH). The SMH is considered as a viable alternative to the TMH and LMH in order to enhance the harvested output power. The enhancement is generated by capitalizing on the fact that the strain constant of the piezoelectric in shear is much higher than those due to thickness or longitudinal deformations. The theory governing the operation of this class of harvesters has been introduced using Newtonian dynamics. Numerical examples are presented to illustrate the merits of the SMH in comparison with the conventional energy harvesters TMH and LMH. It is shown that with proper selection of the material of the piezoelectric element of the harvester, the harvested power of the SMH can be 145.3 to 159.3% higher than that produced by the TMH and 150.6 to 170.2% higher than that generated by the LMH depending on the piezoelectric element material.

The obtained results demonstrate the theoretical favorability of the SMH as a simple and viable means for enhancing the magnitude of the harvested power. Currently, an experimental effort is being conducted to validate the findings of this study with particular emphasis on comparing the performance of the SMH with that of the conventional TMH.

Acknowledgment

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Nomenclature

- $A =$ area, $m^2$
- $C_p =$ blocked capacitance of piezoelectric rod, $F$
- $c_p =$ capacitance of piezoelectric rod, $F$
- $c =$ mechanical modal damping coefficient, $N\cdot s/m$
- $c_{11,33,55} =$ elasticity coefficient of the piezoelectric layer in directions 1, 3, and 5, $N/m^2$
- $D_{13,13} =$ electrical displacement collected in directions 1 and 3, coloumb/m$^2$
- $D_{i,s,0} =$ Rayleigh dissipation functions for the $x, x_{s,o}$, and $Q$ degrees of freedom
- $d_{13,15,33} =$ piezoelectric strain constants, m/volt
- $E_{3,3} =$ electrical field, volt/m
- $I =$ current, $A$
- $k_e =$ stiffness of the piezoelectric rod, N/m
- $k_i =$ dimensionless stiffness ($k_i = k_0 (d_i^2/C_{p0})$ where $i = l,s,t$
- $k_{13,15,33}^2 =$ electromechanical coupling factors
- $m =$ proof mass, kg
- $m_{p,e} =$ mass of piezo-element and total mass, kg
- $P =$ output power of harvester, Watt
- $P_i =$ power per the squared acceleration of harvester $i$ ($i = l,s,t$
- $Q_{s,t} =$ charge, coloumb
- $R_L =$ electrical resistance of load, Ohm
- $S_{1,3,5} =$ strain
- $\xi =$ Laplace complex number
- $T_{i,1,3,5} =$ stress
- $t_p =$ thickness of piezoelectric element, m
- $v =$ output voltage, volt
- $V =$ output voltage in Laplace domain
- $W =$ natural frequency ratio of dynamic magnifier and plain harvester ($o_{mph}/o_{ph}$)
- $x =$ displacement of proof mass, $m$
- $X =$ displacement in the Laplace domain
- $x_{0,0,0} =$ base displacement, $m$
- $z =$ relative displacement of piezoelectric element ($z = z_{s,o}$)
- $Z =$ relative displacement in the Laplace domain

Greek Symbols

- $\alpha =$ dimensionless time constant ($\alpha = \omega_0 C p R_L$)
- $\varepsilon_{11,33,55} =$ permittivity of piezoelectric layer, $F/m$
- $\mu =$ mass ratio ($\mu = m_{ph}/m$)
- $\omega =$ excitation frequency, rad/s
Appendix: Effective Mass of the Piezoelectric Element

The effective mass of the piezoelectric elements used in the TMH, LMH, and SMH are determined using the theory of finite elements as outline in Table 4. It can be seen that the mass matrix results in an effective mass of the piezoelement, treated as one finite element, is the same for all the three harvesters. If node 1 is fixed, then imposing this boundary condition on the mass matrix results in an effective mass of $m_e = (1/3)pA = (1/3)m_p$ for all the three harvesters [29].

References


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