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Transverse Vibrations and Stability of Axially Traveling Sandwich Beam With Soft Core

The transverse vibrations and stability of an axially moving sandwich beam are studied in this investigation. The face layers are assumed to be in the membrane state, which bears only axial loading but no bending. Only shear deformation is considered for the soft core layer. The governing partial equation is derived using Newton's second law and then transferred into a dimensionless form. The Galerkin method and the complex mode method are employed to study the natural frequencies. In comparison with the classical homogenous axially moving beam, the gyroscopic matrix is no longer skew-symmetric because of the introduction of the soft core. The critical speed for the divergence of the axially moving sandwich beam is analytically obtained. The contribution of the core layer shear modulus to the natural frequencies and critical speed is discussed.
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Keywords: axially moving sandwich beam, stability, critical speed, Galerkin method natural frequency

1 Introduction

Composite beams are widely used in aerospace constructions and in many other structural applications. The sandwich beam made of thin and stiff face sheets connected by a thick and soft core is one typical design in the field of composite materials. During vibrations, the viscoelastic soft core deforms in the transverse shear and transverse directions while the energy may be partially dissipated to the surrounding medium. On the contrary, the stiff face sheets provide overall flexural rigidity that high enough to resist the imposed loadings. Sandwich construction offers many attractive features, such as a high specific stiffness, easy reparability, and a dissipative mechanism. Thus, a thorough understanding of the dynamical behavior of sandwich structures is required. The analysis of such structures has been investigated for over half a century. However, the physical configuration and the complicated constitutive modeling made the study of sandwich structures a challenging and tough task. There is no doubt that there is considerable difficulty in obtaining an accurate mathematical model to describe the very nature of the sandwich beam. The difficulty in formulation arises when considering equilibrium and compatibility conditions in the interfaces between layers and the distribution rules of the stress and strain.

Di Taranto [1] and Mead and Marcus [2] have studied the free vibrations of sandwich beams using the classical method. They obtained the governing differential equations, based on the assumption that the top and bottom faces deform according to the Bernoulli–Euler beam theory, whereas the core deforms only in shear. This model has been extended by considering the core to be viscoelastic or to have the ability to bear normal stress [3–8]. The readers can refer to the review papers [9,10] for detailed comments on recent developments of the sandwich structure analysis.

Axially traveling materials, such as power transmission belts and polymer sheets in publishing industries, are commonly used in industry. Vibration problems can arise because of the axially transporting motion, where excessive vibration could cause low quality, fatigue, or even failure. Many outstanding investigations have been performed in the field of the dynamics of axially traveling materials [11–18]. The introduction of the gyroscopic term due to the axially traveling speed made dynamical behavior very interesting. With the increase of the axial speed, the system behaves as the overall stiffness is lowered with a decrease of the natural frequencies. Beyond a critical speed, the axially traveling material becomes instable while the first natural frequency vanishes. Wickert and Mote [14] developed a complex modal analysis for such continuous systems. Parker [15] studied the eigenvalues of gyroscopic continua in the vicinity of the critical speeds by the perturbation method. Marynowski [16] studied the free vibrations of an axially moving plate via the analytical method. Ding and Chen [17] introduced the fast Fourier transform method in the study of an axially traveling material to determine the natural frequencies. Banichuk et al. [18] discussed the discrepancy between the 2D and the classical 1D model of an axially moving material by studying the instability phenomena. Lee and Oh [19] derived a spectral element model for the dynamics and stability of axially moving viscoelastic beams subjected to axial tension. Chen and Tang [20] studied the parametric stability of an axially moving beam with periodic varying tension. Sandilo and van Horssen [21] investigated the contribution of boundary damping due to the dashpot on one end of the axially moving beam.

Homogeneous material has been assumed in numerous earlier studies to model axially traveling systems. However, traveling beams and plates in the real engineering field are usually made of composite materials. In this investigation, we study the dynamics of an axially translating sandwich beam. Hatami et al. [22] studied axially moving symmetric laminated plates by the finite strip method. Ghayesh et al. [23] discussed the stability of axially traveling laminated plates by the analytical method. Yang et al. [24] studied the free and parametric vibrations of an axially moving plate of a composite material. They found the natural frequency on the longitudinal direction is much higher than that of the transverse direction, which ensures the validity of the beam model. Gaith and Müftü [25] used the double beam model interconnected

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by a Winkler foundation to study the dynamics of axially moving laminated materials. In 2005, Yang et al. [26] first performed an investigation on the vibrations of a traveling sandwich beam by the finite element method.

In this paper, we study the vibrations of an axially traveling sandwich beam which is composed of face layers bearing the axial loading and a soft core layer bearing only the shear deformation. The Galerkin method and the complex mode method are used to study the natural frequencies with different axial velocities. The critical speed is analytically derived, accounting for the ratio of the elastic modulus of the face layer and the shear modulus of the core layer.

2 Mathematical Formulations

A sandwich beam is traveling at a constant velocity c between two simple supports, which are separated by a distance L. The configuration of the three layers is shown in Fig. 1.

The present analysis has following assumptions:

(1) No slipping occurs at the interfaces between the three layers of the plate.
(2) The constitutive materials of the beam layers are homogeneous and isotropic.
(3) The top and bottom elastic layers have the same Young's modulus, the same thickness, and the same mass density. Both layers bear only longitudinal stress. The shear and bending effects are neglected with the assumption that the face layers are thin enough.
(4) The core layer is of a soft material and, hence, only shear deformation is considered and the distribution of normal stress is zero.
(5) All points normal to the beam undergo the same transverse deflection.

Let u and w denote the displacements along the x and y directions at arbitrary points, respectively, ψ denote the rotational angle of the overall sandwich beam.

First, we consider the core layer based on the relation of the shear force and deformation. The equilibrium on the longitudinal direction can be satisfied by

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \qquad (1)$$

Based on assumption (4), we obtain the conclusion that τ_{xy} is constant after integrating both sides of Eq. (1). Because of the uniform distribution of the shear force, the shear stress in the core layer can be expressed as

$$\tau_{xy} = \frac{Q}{h+d} \qquad (2)$$

Substituting Eq. (2) and the strain-displacement relation

$$\gamma_{xy} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial y} \qquad (3)$$

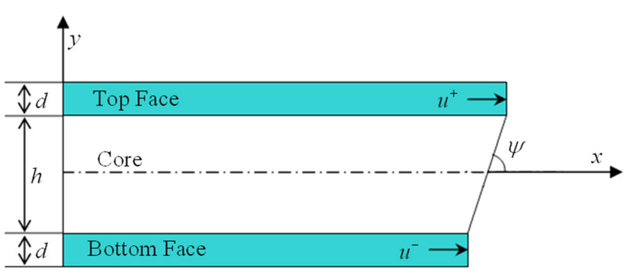

Fig. 1 Configuration of the sandwich beam

into the constitutive equation

$$\tau = G\gamma \qquad (4)$$

yields

$$\frac{\partial w}{\partial x} + \frac{\partial u}{\partial y} = \frac{Q}{G(h+d)} \qquad (5)$$

Integrating both sides of Eq. (5) leads to

$$u = -y\psi \qquad (6)$$

where the rotational angle ψ is

$$\psi = \frac{\partial w}{\partial x} - \frac{Q}{G(h+d)} \qquad (7)$$

Second, we consider the face layers. Let u^+ and u^- denote the longitudinal displacements of the top and bottom faces, respectively. Based on assumption (3), we can calculate the displacements of the face layers, accounting for the rotational angle of the core layer, as

$$u^{\pm} = \mp \frac{h+d}{2}\psi \qquad (8)$$

When we substitute the first derivative of Eq. (8) into the strain-displacement relation

$$\varepsilon = \frac{\partial u}{\partial x} \qquad (9)$$

and substitute the result into the constitutive equation

$$\sigma = E\varepsilon \qquad (10)$$

we obtain

$$\sigma^{\pm} = \mp E \frac{h+d}{2}\frac{\partial \psi}{\partial x} \qquad (11)$$

The moment of the beam, with respect to the neutral line due to the face layers' tension is

$$M = \frac{1}{2}(h+d)d(\sigma^+ - \sigma^-) \qquad (12)$$

Substituting Eqs. (11) into (12), we obtain the overall moment of the sandwich beam

$$M = -\frac{1}{2}E(h+d)^2 d\frac{\partial \psi}{\partial x} \qquad (13)$$

Finally, we study the overall equilibriums of the finite length of the sandwich beam. According to Newton's second law, the equilibriums of the moment and transverse force can be expressed as

$$\begin{aligned} &\frac{\partial M}{\partial x} - Q = 0 \\ &\frac{\partial Q}{\partial x} + q = 0 \end{aligned} \qquad (14)$$

Substituting Eqs. (6), (7), and (13) into Eq. (14) yields

$$\frac{1}{2}E(h+d)^2 d\frac{\partial^2 \psi}{\partial x^2} + G(h+d)\left(\frac{\partial w}{\partial x} - \psi\right) = 0 \qquad (15)$$

$$\frac{\partial^2 w}{\partial x^2}+\frac{1}{G(h+d)}q=\frac{\partial \psi}{\partial x} \tag{16}$$

When we differentiate Eq. (16) twice, with respect to x, and substitute the result and Eq. (16) itself into the first derivative of Eq. (15), we obtain

$$\frac{1}{2}E(h+d)^2 d\frac{\partial^4 w}{\partial x^4}+\frac{1}{2}\frac{E(h+d)d}{G}\frac{\partial^2 q}{\partial x^2}-q=0 \tag{17}$$

If we consider only the inertia force due to the vibration as loadings, q can be written as

$$q=-\rho\left(\frac{\partial^2 w}{\partial t^2}+2c\frac{\partial^2 w}{\partial x\partial t}+c^2\frac{\partial^2 w}{\partial x^2}\right) \tag{18}$$

Substituting Eq. (18) into Eq. (17) yields the governing equation

$$\frac{1}{2}E(h+d)^2 d\frac{\partial^4 w}{\partial x^4}+\rho\left(1-\frac{1}{2}\frac{E(h+d)d}{G}\frac{\partial^2}{\partial x^2}\right)\times\left(\frac{\partial^2 w}{\partial t^2}+2c\frac{\partial^2 w}{\partial x\partial t}+c^2\frac{\partial^2 w}{\partial x^2}\right)=0 \tag{19}$$

Now we introduce the dimensionless variables and parameters as follows:

$$\bar{x}=\frac{x}{L},\quad \bar{w}=\frac{w}{L},\quad \bar{t}=t\sqrt{\frac{E(h+d)^2 d}{2\rho L^4}},\quad \bar{c}=c\sqrt{\frac{2\rho L^2}{E(h+d)^2 d}},\quad Y=\frac{1}{2}\frac{E(h+d)d}{GL^2} \tag{20}$$

The governing equation (19) can be cast into a dimensionless form

$$\frac{\partial^4 w}{\partial x^4}+\left(1-Y\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial t^2}+2c\frac{\partial^2 w}{\partial x\partial t}+c^2\frac{\partial^2 w}{\partial x^2}\right)=0 \tag{21}$$

where the overbars of all of the dimensionless symbols have been omitted, without causing any further confusion.

For simply supported boundaries, we have

$$w_{x=0}=w_{x=1}=\left.\frac{\partial^2 w}{\partial x^2}\right|_{x=0}=\left.\frac{\partial^2 w}{\partial x^2}\right|_{x=1}=0 \tag{22}$$

If we let the ratio of the elasticity and shear modulus $Y=0$, Eq. (21) recovers the governing equation for an axially moving beam of the homogeneous material [12,14]

$$\frac{\partial^4 w}{\partial x^4}+\frac{\partial^2 w}{\partial t^2}+2c\frac{\partial^2 w}{\partial x\partial t}+c^2\frac{\partial^2 w}{\partial x^2}=0 \tag{23}$$

3 Natural Frequency Analyses

3.1 Galerkin Discretization Approach. Without a dissipative mechanism, the motion of the system should be periodic under the initial disturbance. Hence, according to the boundary conditions in Eq. (22), we assume that the solution to Eq. (21) is

$$w=e^{\lambda t}\sum_{k=1}^{N}C_k\sin(k\pi x) \tag{24}$$

Substituting Eq. (24) into Eq. (21) and multiplying the results with $\sin(n\pi x)$ and integrating every term of the final equation, we may obtain a set of algebraic equations

$$\left[(n\pi)^4+\lambda^2-c^2(n\pi)^2+\lambda^2 Y(n\pi)^2-c^2 Y(n\pi)^4\right]C_n+\sum_{k=1}^{N}8c\lambda\frac{nk(1+Yk^2\pi^2)}{(n^2-k^2)}\left[1-(-1)^{k+n}\right]C_k=0,\quad n=1,2,\ldots,N \tag{25}$$

The matrix-vector form of Eq. (25) can be expressed in matrix form as

$$\lambda^2\mathbf{M}+\lambda\mathbf{G}+\mathbf{K}=0 \tag{26}$$

where $\mathbf{M}$, $\mathbf{G}$, and $\mathbf{K}$ are, respectively, the mass, gyroscopic, and stiffness matrix.

In former studies of axially moving homogeneous material, the mass matrix $\mathbf{M}$ is diagonal, the gyroscopic $\mathbf{G}$ is skew-symmetric, and the stiffness matrix $\mathbf{K}$ is symmetric. An interesting phenomenon in this study of axially moving sandwich beams is that the gyroscopic matrix $\mathbf{G}$ is no longer skew-symmetric because the ratio of the elastic and shear modulus exists due to the soft core.

The natural frequencies can be determined by the fact that the determinant of the coefficient matrix of the linear algebraic equations (25) is zero to make sure that we have nontrivial solutions for the system.

By studying the real and imaginary parts of complex natural frequencies varying with the axially velocity, we can determine the natural frequencies and critical speed beyond which the system loses its stability. In Fig. 2, the first two complex natural frequencies are plotted for different axially traveling velocities for the ratio of the elastic and shear modulus $Y=0$ and $Y=0.01$, respectively. For a homogeneous material, i.e., $Y=0$, it is well known that the system presents a sequence of phenomena such as stable-divergence-stable-flutter with the increase of axial velocity. Starting from the velocity $c=0$, the real parts of all orders remain zero while the imaginary parts decrease. At $c=\pi$, which is called the critical speed, the imaginary part of the first order natural frequency vanishes and the real part begins positive, where divergence occurs. In a small range beyond $c=2\pi$, the real part becomes zero again and the system regains stability. We can find both the positive real part and positive imaginary parts when the natural frequencies of the first and second orders coincide. Hence, the flutter phenomena occur and the system loses stability for the second time in this case. For the axially moving sandwich beam, the natural frequencies are lowered because of the introduction of a soft core. The critical axial velocity is also reduced accordingly. It appears that it is less stable for the axially traveling sandwich material than the homogeneous material. In this investigation, the thin face layers of the sandwich beam are assumed to bear axial loading while the core layer bears only shear deformation. With the increase of the thickness of the core layer, the overall bending stiffness will increase because of the distance between the two layers. Hence, on one hand, the natural frequencies are lowered if the core layer is softer and, on the other hand, the natural frequencies will be greatly increased because of the introduction of the soft layer, which leads to the far distance of the face layers.

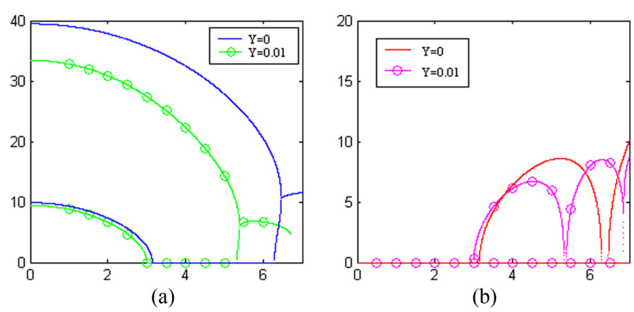

Fig. 2 Complex natural frequencies versus axially moving velocity: (*a*) imaginary part, and (*b*) real part

3.2 Complex Mode Method. Another method used to determine the natural frequencies of the axially moving material is the complex mode method, which has been used by many authors [16,27].

The solution to Eq. (21) can be assumed as

$$w = e^{i\omega t} e^{i\beta x} \tag{27}$$

in which ω is the natural frequency of the system and β is the complex wave number. Substituting Eq. (27) into Eq. (21) yields a dispersion relation

$$\beta^4 - (1 - iY\beta)\left(+c^2\beta^2 + 2c\omega\beta + \omega^2\right) = 0 \tag{28}$$

The nth modal function corresponding to Eq. (21) can be expressed as

$$\phi_n(x) = C_{n1}e^{i\beta_{n1}x} + C_{n2}e^{i\beta_{n2}x} + C_{n3}e^{i\beta_{n3}x} + C_{n4}e^{i\beta_{n4}x} \tag{29}$$

where β_n are the quadruple roots of Eq. (28). To satisfy the boundary conditions of Eq. (23), the following equation:

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ e^{i\beta_{n1}} & e^{i\beta_{n2}} & e^{i\beta_{n3}} & e^{i\beta_{n4}} \\ \beta_{n1}^2 & \beta_{n2}^2 & \beta_{n3}^2 & \beta_{n4}^2 \\ \beta_{n1}^2 e^{i\beta_{n1}} & \beta_{n2}^2 e^{i\beta_{n2}} & \beta_{n3}^2 e^{i\beta_{n3}} & \beta_{n4}^2 e^{i\beta_{n4}} \end{pmatrix} \begin{pmatrix} C_{n1} \\ C_{n2} \\ C_{n3} \\ C_{n4} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{30}$$

must yield nontrivial solutions of C_n. Hence, the determinant of the coefficient matrix should be zero, i.e.,

$$\begin{aligned} &\left[e^{i(\beta_{n1}+\beta_{n2})} + e^{i(\beta_{n3}+\beta_{n4})}\right]\left(\beta_{n1}^2 - \beta_{n2}^2\right)\left(\beta_{n3}^2 - \beta_{n4}^2\right) \\ &+ \left[e^{i(\beta_{n3}+\beta_{n1})} + e^{i(\beta_{n2}+\beta_{n4})}\right]\left(\beta_{n3}^2 - \beta_{n1}^2\right)\left(\beta_{n2}^2 - \beta_{n4}^2\right) \\ &+ \left[e^{i(\beta_{n2}+\beta_{n3})} + e^{i(\beta_{n1}+\beta_{n4})}\right]\left(\beta_{n2}^2 - \beta_{n3}^2\right)\left(\beta_{n1}^2 - \beta_{n4}^2\right) = 0 \end{aligned} \tag{31}$$

In a finite internal of real numbers, we can propose a numerical approach to locate some values of ω satisfying that the four roots of Eq. (28) makes Eq. (31) hold. This numerical technique can help us to solve for the first several real natural frequencies ω_n and corresponding complex wave number β_n. By a similar method proposed by Öz and Pakdemirli [27], the nth mode function for asimply supported axially moving beam can also be calculated from Eq. (30) as

$$\begin{aligned} \phi_n(x) = c_1\Bigg\{ & e^{i\beta_{n1}x} - \frac{\left(\beta_{n4}^2 - \beta_{n1}^2\right)\left(e^{i\beta_{n3}} - e^{i\beta_{n1}}\right)}{\left(\beta_{n4}^2 - \beta_{n2}^2\right)\left(e^{i\beta_{n3}} - e^{i\beta_{n2}}\right)} e^{i\beta_{n2}x} \\ & - \frac{\left(\beta_{n4}^2 - \beta_{n1}^2\right)\left(e^{i\beta_{n2}} - e^{i\beta_{n1}}\right)}{\left(\beta_{n4}^2 - \beta_{n3}^2\right)\left(e^{i\beta_{n2}} - e^{i\beta_{n3}}\right)} e^{i\beta_{n3}x} \\ & - \Bigg[1 - \frac{\left(\beta_{n4}^2 - \beta_{n1}^2\right)\left(e^{i\beta_{n3}} - e^{i\beta_{n1}}\right)}{\left(\beta_{n4}^2 - \beta_{n2}^2\right)\left(e^{i\beta_{n3}} - e^{i\beta_{n2}}\right)} \\ & - \frac{\left(\beta_{n4}^2 - \beta_{n1}^2\right)\left(e^{i\beta_{n2}} - e^{i\beta_{n1}}\right)}{\left(\beta_{n4}^2 - \beta_{n3}^2\right)\left(e^{i\beta_{n2}} - e^{i\beta_{n3}}\right)}\Bigg] e^{i\beta_{n4}x}\Bigg\} \end{aligned} \tag{32}$$

3.3 Critical Speed. From Sec. 3.1, we know that with the increase of the axially velocity there exists a critical speed, beyond which the system loses its stability by divergence. To analytically determine the critical speed, we may study the static terms of Eq. (21). Neglecting the time-dependent terms, we obtain the following equation

$$\frac{\partial^4 w}{\partial x^4} + c^2\frac{\partial^2 w}{\partial x^2} - Yc^2\frac{\partial^3 w}{\partial x^3} = 0 \tag{33}$$

The solution to Eq. (33) can be assumed as

$$w = \sum_{j=1}^{4} C_j e^{r_j x} \tag{34}$$

where r_j are the four roots of the following characteristic equation

$$\left(1 - c^2 Y\right) r^4 + c^2 r^2 = 0 \tag{35}$$

When $Yc^2 < 1$, the four roots are

$$r_{1,2} = 0, \quad r_{3,4} = \pm\alpha i \tag{36}$$

where

$$\alpha = \sqrt{\frac{c^2}{1 - Yc^2}} \tag{37}$$

The static solution now can be expressed as

$$w = C_1 + C_2 x + C_3 \cos\alpha x + C_4 \sin\alpha x \tag{38}$$

Substituting Eq. (38) into the boundary conditions (22) yields

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \cos\alpha & \sin\alpha \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{39}$$

For the divergence case, the preceding linear algebraic equations have nontrivial solutions. Hence, the determinant of the coefficient matrix should be zero. Thus, in the case of $Yc^2 < 1$, the minimum value of the axial velocity satisfying the preceding condition is

$$c_{\rm cr} = \sqrt{\frac{\pi^2}{1 + \pi^2 Y}} \tag{40}$$

It can be verified that when $Yc^2 > 1$, the system is always unstable. Hence, the critical speed can be determined by the minimum value for divergence in Eq. (40).

Figure 3 shows the variation of critical speed with the ratio of the elastic and shear modulus Y. It can be found that with the increase in the ratio number Y, the critical speed decreases. From Eq. (20), we know that Y is the ratio of the elastic modulus of the face layer and the shear modulus of the core layer. Therefore, with the increase of the shear modulus of the core layer, the

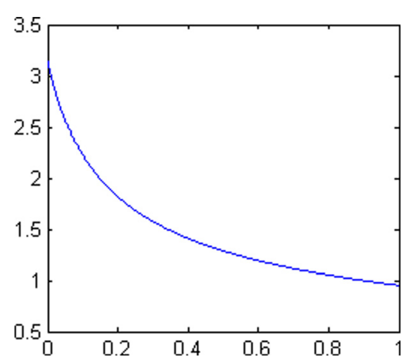

Fig. 3 The critical speed versus the ratio of the elastic and shear modulus *Y*

critical speed will increase. The critical speed recovers to π when $G = \infty$, i.e., the shear strain is neglected because it is rigid, which is the result of the homogeneous beam.

4 Summary

The natural frequencies and critical speed of an axially moving sandwich beam have been analytically studied in this paper. The face layers are assumed to be of a membrane model bearing only axial loading but no bending. Pure shear deformation is considered in the soft core layer. The governing partial equation was derived by Newton's second law and then transferred into dimensionless form. The Galerkin method and the complex mode method were used to study the natural frequencies. It was found that with the increase of the axially traveling velocity, all of the natural frequencies decrease. The critical speed is defined when the first natural frequency vanishes, which triggers the instability of divergence. The gyroscopic matrix for the axially traveling sandwich is no longer skew-symmetric because of the introduction of the soft core, while the gyroscopic matrix is always skew-symmetric for the homogeneous axially traveling material. The critical speed for the divergence of the axially traveling sandwich beam has been analytically obtained. It was found that with the increase of the shear modulus of the core layer, the critical speed increases. This investigation may provoke further studies of axially moving composite materials.

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