A novel financial market for mitigating hurricane risk is described and illustrated. The structure of the market is one sided and parimutuel, so that participants buy contracts pertaining to hurricane landfall locations from an exchange rather than from other market participants, and settlements for contracts associated with the landfall location are funded by purchases in all other outcomes. Contract prices are updated automatically and objectively using a recently developed adaptive control algorithm that responds to inferred aggregate probability assessments of the market participants. The market is intended to supplement insurance by providing a mechanism to shift risk for costs not covered under existing windstorm insurance. Operation of the market mechanism is illustrated in an idealized setting and in a spatially explicit historical simulation for Hurricane Charley (2004). A companion paper in this issue describes empirical validation of this market mechanism in an experimental market setting.

1. Introduction

The risk of property damage and loss from hurricanes is a fact of life in coastal areas, and this problem is being progressively magnified by the ongoing growth of coastal populations and property values (Cutter et al. 2007; Pielke et al. 2008; Blake et al. 2011). One very significant economic consequence of these escalating losses has been the gradual degradation of the availability and quality, and increase in the cost, of windstorm insurance available to coastal residents (Derrig et al. 2008; MacDonald et al. 2010).

The main cause for this deterioration of the private windstorm insurance market is the combination of the reluctance of insurers to underwrite insurance policies for properties when the probability of a catastrophic loss is uncertain and constraints on the prices that can be charged to insure against these risks due both to regulatory controls and limits to affordability (Kunreuther and Michel-Kerjan 2009). One consequence is that in some states—particularly Florida—many major insurers are unwilling to write new policies, are greatly limiting coverage, or are withdrawing from the windstorm insurance business altogether (e.g., MacDonald et al. 2010).

Conventional weather derivative contracts (e.g., Zeng 2000; Jewson and Caballero 2003; Cao and Wei 2004; Kelly et al. 2012) require market participants to find a willing counterparty, that is, someone to take the opposite side of a contract. For example, the two participants in a conventional weather derivative contract might be a ski resort operator wishing to protect against the adverse financial consequences of a low snowfall winter and a highway authority wishing to protect against the adverse financial consequences of increased operating costs in a high snowfall winter. The former might contract to pay the latter if winter snowfall is above an agreed threshold, and the latter would pay the former if the winter snowfall were sufficiently low. In effect, both parties “bet” that adverse weather (from their individual perspectives) will occur, so that the negative impacts on their operations will be offset at least in part by the financial contract.

In order for this conventional bilateral market structure for hedging weather risk to work well, there must be
comparable numbers of individuals (or, dollars at risk) who will be hurt by the occurrence of an event (e.g., a heavy snowfall winter) and its absence (a light snowfall winter). Hurricane risk does not fit this model well, because there are many individuals and businesses that are hurt financially by hurricane landfalls, but few if any for whom the lack of landfalling hurricanes causes significant financial losses. Therefore, a conventional bilateral hurricane market will only function well to the extent that speculators with very large financial resources (e.g., “hedge” funds) take financial positions that landfalling hurricanes will not occur.

This paper describes an approach to managing hurricane risk using a novel financial market structure, which allows participants to hedge against the risk that a selected coastal county or group of counties on the U.S. Atlantic or Gulf coasts will be first hit by the next hurricane to make landfall in a calendar year. This market is not intended to replace windstorm insurance, but rather to augment it by providing home and business owners with a mechanism to shift risk for costs not covered by insurance, such as large deductibles (which are often 2%–5% or more of the insured value), outdoor damages, flood losses not covered under federal policies, and business interruption. The structure of this market differs most significantly from that of conventional bilateral markets in that it is one sided and parimutuel. That is, participants buy contracts from an exchange, in effect “betting” that a hurricane will strike the coastline in their area. The payments they receive in the event of such a hurricane strike are derived from the payments of market participants who have purchased contracts for other areas. The market structure is simple and transparent, avoids many of the pitfalls of other “derivative” markets, and may offer an attractive alternative means to address the needs of individuals, businesses, and potentially also the insurance and reinsurance industries, with respect to hedging potential financial losses from hurricanes.

Section 2 describes the technical details of the market structure, including the necessary climatological analyses, and the adaptive control algorithm that allows fair, risk-based pricing outside of the conventional bilateral market paradigm. Section 3 illustrates the operation of this pricing algorithm and its probability convergence in a simplified and idealized setting. Section 4 describes the simulation of a hypothetical market for the 2004 Atlantic hurricane season, through the landfall of Hurricane Charley. Section 5 summarizes and provides some concluding perspectives. A companion paper (Meyer et al. 2014) demonstrates and validates this new market structure through an empirical experimental market study.

2. Market structure

a. Overview

The hurricane contracts described here are legally described as commodity options and have been named Hurricane Risk Landfall Options (HuRLOs). HuRLOs are formally commodity options under the Commodity Exchange Act and specifically are call options on whether and if so where a hurricane will make landfall. The options exercise and settle automatically and options for incorrect outcomes expire worthless. The contracts are options because every buyer’s risk of loss is limited and defined (the amount of the premium) and the right to exercise in the future is based on the contingency that the hurricane will make landfall first in a selected county or region.

Purchases of these contracts allow market participants to hedge against the risk that 1 of 78 coastal counties or adjacent county groups on the U.S. Atlantic and Gulf coasts will be the first hit by the next hurricane to make U.S. landfall in a calendar year. Figure 1 shows a map of the landfall areas (primarily, individual counties except for the mid- and northern Atlantic coast) for which HuRLOs could be purchased from an online exchange. The contracts are offered in multiple series, with Series 1 contracts pertaining to the first U.S. hurricane landfall in
a given year, Series 2 pertaining to the second, and so on. The multiple series structure has been established so that participants can receive settlements promptly following a landfall event. The definition of landfall outcomes, according to coastal segments, and the correspondence of each market to a single storm are similar to the prediction market described by Kelly et al. (2012), whose experimental study involved a conventional bilateral market structure.

In addition to the 78 explicit landfall areas, “No Landfall” HuRLOs are available in each series, which contracts pertain to the possibility that no (further) U.S. landfalling hurricanes will occur in the year to which the market pertains. For example, buyers of No Landfall contracts in all series would have been paid in 2010, because there were no U.S. landfalling hurricanes in that year. In years such as 2011, with a single U.S. landfalling hurricane, buyers of Series 2 (and higher) No Landfall contracts would be paid, but buyers of Series 1 No Landfall contracts would not.

b. Price determination

In a conventional bilateral market, price discovery is achieved through negotiations between buyers and sellers. This mechanism is not available in a one-sided market, so that an alternative, fair, risk-based pricing procedure is required. Define the pricing probabilities $\pi_i(t)$ expressing the consensus market sentiment for each of the $i = 0, \ldots, I$ possible outcomes, where $I = 78$ explicit landfall areas and $i = 0$ denotes the No Landfall event, for the $t$th transaction. The price $P_i(t)$ for a single contract in outcome $i$ is proportional to its pricing probability:

$$P_i(t) = \pi_i(t) c \exp[rj/365],$$

where $c$ is a constant defining the overall magnitudes for prices and settlements (“par” value, taken here to be $1,000 (U.S. dollars), $r$ is an annualized interest rate, and $j$ is the number of days since the market opened for the current year.

The market is initialized with the climatological (i.e., historical) probabilities $\pi_i(0)$ that, with the exception of the No Landfall probability [$\pi_0(0) \approx 0.19$], are indicated by the colors in Fig. 1 for Series 1. Estimation of these initial probabilities is described in section 2c. Once the market begins to operate, these initial climatological probabilities are dynamically updated to reflect market activity, so that probabilities (and thus also prices) for landfall outcomes being bought heavily will increase, whereas those for outcomes with little buying interest will decline. These adjustments are made using a recently introduced (Bequillard 2013, manuscript submitted to Int. J. Theor. Appl. Finance; Horowitz et al. 2013) adaptive control algorithm, which is a novel variant of the Robbins–Monro stochastic approximation algorithm (Kushner and Yin 2003).

Specifically, after each transaction the prices for all 79 options are updated to reflect the evolving market consensus regarding probabilities for all outcomes. The pricing probability for the most recently purchased outcome $i$ is updated via the recursion formula:

$$\pi_i(t) = \pi_i(t-1) + \alpha(t) \pi_i(t-1)[1 - \pi_i(t-1)], \quad (2)$$

where $\alpha(t)$ is the price adjustment parameter ($0 < \alpha(t) < 1$) that controls the rate at which prices respond to buying activity. Smaller values of $\alpha$ suppress price volatility at the expense of slowing the responsiveness of prices to changing market sentiment, whereas a larger $\alpha$ yields more prompt price responses but also higher price volatility. The rate of change of prices is greatest when $\pi_i$ in Eq. (2) is near $\frac{1}{2}$, at which point the outcome uncertainty is highest.

Because the probabilities for all outcomes must sum to 1, pricing (or “market”) probabilities for the 78 outcomes that were not purchased in the most recent transaction are decreased proportionally:

$$\pi_k(t) = \pi_k(t-1)[1 - \alpha(t) \pi_i(t-1)], \quad k \neq i. \quad (3)$$

The structure of Eqs. (2) and (3) ensures that the updated probabilities are coherent, that is, $0 < \pi_i(t) < 1$ for all outcomes $i$, and $\sum \pi_i(t) = 1$. As shown in the appendix, the adjustment parameter $\alpha(t)$ should decrease in inverse proportion to the size of the market (i.e., to the average number of contracts per outcome), so that price responses to new purchases are progressively damped as the market size increases.

It is important to note that in Eqs. (2) and (3), the index $t$ refers not to chronological time, but rather to the sequence numbers for individual purchases. So, for example, if a block of 100 options were being purchased for a county $i$, Eqs. (2) and (3) would be iterated 100 times during this process. The result would be that each of the 100 options in this block would cost slightly more than the previous option in the block. That is, the ongoing demand for options in county $i$ would progressivly increase the price of subsequent options in that county [Eq. (2)], analogously to an ask queue in a conventional bilateral market, while depressing prices for the options in the remaining 78 outcomes [Eq. (3)].

Crucially, the adaptive control algorithm possesses the property that its adjustments to the pricing probabilities $\pi_i(t)$ converge to the consensus of market participants’ judgments about the event probabilities as...
revealed through their buying activity, which is proved in Bequillard (2013, manuscript submitted to Int. J. Theor. Appl. Finance) and Horowitz et al. (2013). Thus, apart from a modest but inevitable lag in the response of the probability adjustments (the speed of which depends on the magnitude of α), prices computed using Eq. (1) are fair and risk based, in the sense that they reflect the market consensus for the outcome probabilities at any given time, except possibly at the time of market initialization. In effect, the adaptive control algorithm for updating the pricing probabilities automatically learns investors’ probabilities for the outcomes in response to their collective actions in the market. Examples of the probability updating process are given in section 3 and 4.

c. Climatological probabilities

As noted in section 2b, the pricing probabilities \( \pi_i(0) \) must be initialized in order for prices to be defined at the time the market is opened. Ideally the hurricane market is opened early in the year, so that climatological probabilities for first U.S. hurricane landfall of the year are appropriate starting points. However, the rather fine spatial resolution of the coastal county segments indicated in Fig. 1 implies that raw climatological hurricane landfall relative frequencies exhibit too much sampling variability for this purpose.

The smoothed climatological first-strike probabilities in Fig. 1 have been obtained through Monte Carlo simulations based on the Atlantic hurricane database (HURDAT) data (Jarvinen et al. 1984) available from the National Hurricane Center (NHC) website. The procedure (proposed by Charles Neumann 2006, personal communication) is as follows. The track of each storm in the HURDAT database that reached hurricane strength is perturbed 1000 times, using displacements defined by independent random draws from the circular uniform distribution with a radius of 92.6 km (50 nautical miles). The courses of each of these perturbed tracks are then examined to find relative frequencies of crossings of the U.S. coastline segments indicated in Fig. 1 at hurricane strength. These 78 relative frequencies are then scaled so that their sum is 1 − \( \pi_{46}(0) \) ≈ 0.81. It is recognized that storms earlier in the database are less accurately portrayed, but the errors are least important near the U.S. coastline, which is the focus of the analysis. This method is similar to the Hurricane Analog (HURRAN) forecast method (Hope and Neumann 1970), which traces paths of analog historical storms displaced randomly from the current position of an existing storm, and to the forecast method described in Wilks et al. (2009), which uses the same basic methodology initialized from a forecast future storm position.

The resulting climatological probability estimates \( \pi_i(0) \) reflect the greater frequencies of tropical cyclones at the lower latitudes, the relative sizes of the counties, and the relationship of the local coastal geography to the climatological average storm path directions (shown as the offshore open arrows in Fig. 1). For example, the most likely \( \pi_{46}(0) = 0.059 \) climatological landfall location is Monroe County, Florida, which includes the Keys. These islands both present a large target and are oriented nearly perpendicularly to the climatological average storm direction in this portion of the domain. In contrast, there is a distinct probability minimum on the northeast Florida and Georgia coastlines, reflecting climatological storm directions that are nearly parallel to that portion of the coast.

d. Market seeding and initialization

Proceeds from all purchases in a given series are collected into a mutualized risk pool (MRP) for that series. In practice the market for each HuRLO series would be “seeded” with an initial stake in its MRP. The seeding institution receives an equal number of HuRLOs in each of the 79 outcomes in each HuRLO series. Because initially there are not yet market-based probabilities for the 79 outcomes, prices for the initial stakes are allocated according to the initial probabilities \( \pi_i(0) \), reflecting the historical risks. The result is that the initial number of HuRLOs in each outcome is equal, and given by

\[
n_i(0) = \frac{\text{MRP}(0)}{c},
\]

For example, if the MRP for a HuRLO series is seeded initially with \( \text{MRP}(0) = \$1,000,000 \), and \( c = \$1,000 \), then that HuRLO market begins with \( n_i(0) = 1000 \) HuRLOs in each outcome \( i \). This allocation follows from Eq. (1), with \( j = 0 \).

e. Market termination and contract settlement

Purchases of HuRLOs can continue until a possible hurricane landfall is imminent, or until 15 December, after which time the risk of U.S. hurricane landfall is vanishingly small. During the hurricane season, market activity is suspended if and when a hurricane watch for one or more of the coastline segments has been issued by the NHC, meaning that the onset of hurricane force winds are possible and that tropical storm force or stronger wind conditions are anticipated within (approximately) 48 h. At that point it is far from clear which coastline segment, if any, will receive a hurricane strike, so these suspensions prevent the parimutuel payout for the eventual landfall location becoming
diluted excessively by concentrated buying just ahead of a landfall event. This payout dilution would have the potential to occur without market suspension because of the inevitable lag in price response to the availability of new information, as the eventual landfall location becomes increasingly clear. Figure 2 shows the number of counties subtended by the NHC "cone of uncertainty" (considering cones falling fully on the coastline only) as a function of time ahead of hurricane landfall for U.S. landfalling hurricanes 2002–06. The variability in Fig. 2 derives from the different sizes of the county coastlines, and the different angles of hurricane approach to the coast. The NHC cone itself provides approximately 90% probability coverage near the time of landfall (Wilks et al. 2009).

If the storm for which the hurricane watch was issued fails to make landfall on the United States as a hurricane, trading in the suspended series is resumed. If a hurricane landfall occurs, its position for purposes of market settlement is determined by the first intersection of line segments connecting real-time NHC advisory positions with the high-resolution representation of the coastline defined by the U.S. Census Bureau database (these data, rendered in map form, are available at ftp://ftp2.census.gov/geo/maps/general_ref/stco_outline/ecn2k_pgss/).

The MRP is shared among holders of contracts for the outcome that ultimately occurs. Settlement amounts per contract are determined simply as the total dollar amount in the MRP, divided by the total number of contracts that have been sold for the coastal segment receiving the landfall. Thus, purchasers of contracts for counties that were not hit fund the payouts for the county receiving the first strike. Because the contract prices are $1,000 multiplied by the market probabilities at the time of purchase, and these probabilities have been updated continuously over the course of the market, these settlement amounts should be in the neighborhood of $1,000 per contract, provided the market is well developed (i.e., is not thin). Because real-time, operational hurricane position estimates are used to define the landfall location, holders of “in-the-money” contracts can be paid within a day or two of the qualifying landfall.

3. Pricing and probability convergence in an idealized setting

The capacity of the adaptive control algorithm to converge to market participants’ beliefs about the outcome probabilities, as revealed through their buying activity, is illustrated in this section for a simplified, hypothetical market having five outcomes. Figure 3 illustrates the probability [and, through Eq. (1), price] convergence of the adaptive control algorithm in this setting. The five “climatological” probabilities $p_i(0)$ assigned to initialize the market are equal, at which time the MRP = $1,000,000$. The price adjustment parameter has been held fixed at $a(t) = 0.001$ for all $t$.

An initially uniform distribution for the pricing probabilities has been used here in order to illustrate the capacity of the adaptive control algorithm to respond promptly to changes in participant sentiment. The hypothetical market participants do not agree that all the outcomes are equally likely, and instead they invest money according to the following probabilities $q_i$ for outcomes $i = 1, \ldots, 5$: $q_1 = 0.30$, $q_2 = 0.25$, $q_3 = 0.20$, $q_4 = 0.15$, and $q_5 = 0.10$. This investment pattern continues until MRP = $15,000,000$, at which time new information (corresponding perhaps to updated forecast information) becomes available, indicating $q_1 = 0.6$ and $q_2 = q_3 = q_4 = q_5 = 0.1$.

Figure 3a shows the evolution of the pricing probabilities as a function of accumulated MRP. Contracts for all five outcomes are purchased within each $100,000 increment of MRP increase, with that $100,000 increment being allocated among them in proportion to the investors’ probabilities $q_i$. Because $c = 1,000$ in Eq. (1), each $100,000 increment (i.e., each additional plotting symbol in the figure) corresponds approximately to 100 new options in each of the five outcomes. After a relatively modest additional inflow of investments to the MRP, the new equilibrium corresponding to the market consensus is achieved, so that $p_i = q_i$ for each outcome $i$. When the market consensus probabilities change at MRP = $15,000,000$, adjustment of the
pricing probabilities \( \pi_i \) to the new market consensus \( q_i \) is similarly prompt. Of course the first market participants to respond to new information will have an opportunity to profit in an expected value sense during the price adjustment process, but this is a characteristic of any financial market and is not particular to the present market structure. Similarly, newly less likely outcomes will be temporarily overpriced until market adjustments bring those prices to actuarially fair levels.

Figures 3b–d illustrate that the adaptive control algorithm is robust to noise (i.e., random variations) in participants’ judgments about the outcome probabilities \( q_i \). (b) Analogous results when the participants’ investment allocations are obscured with Gaussian noise having the indicated standard deviations [Eq. (5)].

\[ m_i = 100,000q_i + \sigma z_i, \quad (5) \]

where the standard deviations \( \sigma \) are indicated in the panel legends, and the \( z_i \) are independent standard Gaussian variates updated for each new MRP increment of approximately $100,000. The result is that the market probabilities (and thus prices) fluctuate rather than reaching stable equilibria, but these fluctuations are centered on the correct values, that is, the \( q_i \). Wilks (2010) also presents results for similar simulations, in which contracts are purchased for only the most favorably priced outcome (i.e., that outcome \( i \) maximizing \( q_i - \pi_i(t) + \sigma z_i \), for each individual contract \( t \)), which yields similar but somewhat faster market adjustments to changing information and investor beliefs.

Figure 4 illustrates the trade-off between price responsiveness and volatility that must be struck when choosing \( \alpha \). Here the standard deviation \( \sigma \) in Eq. (5) has been set at $2,500 as in Fig. 3b, to which the results in Fig. 4 should be compared. Figure 4a shows the slower price response to the information changes at MRP = $1,000,000 and $15,000,000 when \( \alpha \) is reduced from 0.001 to 0.0005 and also the corresponding decrease in price volatility in response to the random investment allocations. In contrast, Fig. 4b shows quite prompt price adjustment when \( \alpha \) is increased to 0.005, but at the expense of increased price volatility relative to \( \alpha = 0.001 \) in Fig. 3b. The same random number stream has been used for all the panels in Figs. 3 and 4 in order to enhance their comparability.

This section describes a stochastic simulation for the parimutuel hurricane market in a historical and spatially explicit context, using the 2004 Atlantic hurricane season until just before the landfall of Hurricane Charley as an example. This example considers only a single series and so pertains only to the first U.S. hurricane landfall of 2004. Hurricane Charley formed in the eastern Caribbean on 9 August and tracked south of Jamaica and over western Cuba before making landfall on 13 August at Lee County (location indicated in Fig. 1) on the west coast of Florida, as a category 4 storm (Pasch et al. 2004).

The hypothetical market is initialized on 1 January using the climatological landfall probabilities $p_i(0)$ indicated in Fig. 1. The adjustment parameter decreases as the market develops, according to

$$\alpha(t) = \min[0.001, 30/\bar{n}(t)],$$

where $\bar{n}(t)$ is the average number of contracts per outcome that have been purchased at a given point $t$ in the development of the market. The initial MRP is set at $2$ million.

On each simulated day through 30 July, invested funds (the daily increases in MRP) were allocated to the various outcomes according to the climatological probabilities indicated in Fig. 1 [i.e., $q_i = \pi_i(0)$, $i = 0, \ldots, 78$], but perturbed on each day with noise. Denoting $\Delta_{\text{MRP}}(j)$ as the MRP increment for day $j$, and the random relative allocation of investments to outcome $i$ on day $j$ as $g_i(j)$, the monetary allocation to outcome $i$ on day $j$ is

$$m_i(j) = \frac{g_i(j)}{\sum_{i=0}^{78} g_i(j)} \Delta_{\text{MRP}}(j).$$

Here the $g_i(j)$ are independent gamma-distributed random variables with means equal to the daily outcome probabilities $q_i$ and a coefficient of variation of 0.5 (corresponding to a gamma shape parameter of 4) for all days and outcomes. As the investments are distributed in random order among the 79 outcomes on each day, the corresponding prices fluctuate according to Eqs. (1)–(3). The result of the asymmetric gamma-distributed perturbations is that simulated investments in landfall outcomes exhibiting stronger (mean) buying interest on a given day will be more variable from day to day. Specifying gamma distributions ensures strictly positive investments in all cases.

Setting $q_i = \pi_i(0)$ through 30 July is a convenient simplification adopted in the simulation, but more realistically these climatological first-strike probabilities, conditional on no U.S. hurricane landfall occurrence to date, will change gradually once the hurricane season begins. In particular, the conditional probability of no hurricane landfalls for the rest of the season, given that none have occurred so far, gradually climbs during the hurricane season, resulting in proportional decreases from the initial climatological landfall probabilities portrayed in Fig. 1.

Beginning on 31 July, the daily outcome probabilities $q_i$ are based on county-scale disaggregations of the NHC Forecast Advisories for Hurricane Alex (31 July–2 August), Tropical Storm Bonnie (3–4 August), and Hurricane Charley (9–12 August). These county-level forecasts, computed from the NHC Forecast Advisories as described in Wilks et al. (2009), specify probabilities $f_i$, $i = 1, \ldots, 78$, that the particular storm in question will make landfall at hurricane strength at the $i$th coastal segment and the probability $f_0$ that it will either not make landfall or will do so at tropical storm strength or less. Since in this latter case the climatological first-strike probabilities would be applicable (characterizing the fate of a possible subsequent storm), the daily outcome probabilities will reflect mixtures of the climatological and forecast probabilities, according to
which define the means of the gamma distributions from which the $g_i$ in Eq. (7) are drawn.

Figure 5a shows the assumed accumulation in the MRP. It increases moderately through mid-February, representing early hedging of hurricane risk by market participants taking advantage of the more favorable prices early in the season that result from $r = 0.05$ in Eq. (1). Simulated investor interest is more modest from mid-February until the beginning of the official hurricane season on 1 June, at which time investor interest increases, especially in the first weeks of June. Two very substantial spikes in the accumulated MRP begin on 31 July and 9 August, with the issuance of NHC Forecast Advisories for Hurricanes Alex and Charley, respectively (http://www.nhc.noaa.gov/archive/2004/). When the market is suspended on 12 August in response to the imminent landfall of Hurricane Charley, the hypothetical MRP contains $2$ billion.

Figure 5b shows simulated contract prices for Carteret County in North Carolina, and the Florida counties Lee and Charlotte, during the simulated 2004 hurricane season. The locations of these three counties are indicated in Fig. 1. During the simulation, probabilities and prices for all 79 outcomes were calculated, but only these three are illustrated in Fig. 5b for clarity. Early in the simulation, through perhaps mid-February, the prices shown in Fig. 5b are fairly volatile because the MRP is still relatively small, so that occasional large random dollar allocations to one or a few of the outcomes can move prices substantially as a consequence of the price adjustment parameter $\alpha(t)$ being relatively large. This volatility is largest for the higher-probability Carteret County ($q_{33} = 0.030$, as indicated in Fig. 1), intermediate for Lee County ($q_{44} = 0.010$), and least for the relatively low-probability Charlotte County ($q_{43} = 0.002$) because of the nature of the random investment allocation perturbations. After this initial period of relatively high volatility, and through the appearance of Alex on 31 July, the prices shown in Fig. 5b exhibit smaller fluctuations around levels consistent with the probabilities indicated in Fig. 1, again with larger volatility for the more likely outcomes.

Alex formed as a tropical depression on 31 July, offshore of the southeastern U.S. coast near 30°N, 78°W and remained nearly stationary until beginning to move northeast on 2 August. At that time hurricane warnings were posted for the outer banks of North Carolina, yielding a disaggregated forecast probability $f_{73} = 0.060$.
for Carteret County. Because the disaggregated forecast also included the probability $f_0 = 0.218$ that Alex would not make U.S. landfall as a hurricane, Eq. (8) yields the probability $q_{73} = 0.066$ that the first 2004 U.S. landfall would occur at Carteret County. As indicated in Fig. 5b, the price of Carteret contracts increases in response, but then subsequently declines as Alex passes just to the east of the coast without making landfall.

At the end of the simulation on 12 August, Hurricane Charley is located south of Cuba and is forecast to make landfall somewhere on the west coast of Florida (Fig. 6). Nearly all of the eastern coast of Florida is within the NHC cone of uncertainty at approximately 36 h ahead of landfall, because of the oblique angle of approach of this storm. Landfall probabilities $q_{44}$ for Lee County and $q_{43}$ for Charlotte County are approximately 0.11 and 0.04, respectively, for forecasts on 11 and 12 August, leading to the sharp price increases for these two counties indicated in Fig. 5b. The probabilities and therefore prices are larger for Lee than the adjacent Charlotte County because of its longer coastline. Meanwhile the landfall probability for Carteret County drops to near zero, as is also reflected in Fig. 5b.

Hurricane Charley made landfall at Lee County on 13 August. In the simulation, 2,083,147 contracts for Lee County had been purchased at the time of market suspension on 12 August, leading to a per contract parimutuel payout of $2 billion/2,083,147 contracts = $960 per contract. Even though nearly $\frac{1}{3}$ of the total MRP is invested in the final 4 days of the simulation, and primarily along the west coast of Florida, the price adjustment mechanism of Eqs. (1)–(3) was able to respond quickly enough to maintain the payout near the par level of $c = 1,000$.

5. Summary and conclusions

The novel, one-sided financial market structure described here appears to provide a promising mechanism for hedging weather risk that may not be covered by standard insurance contracts in markets for events, such as hurricanes, for which natural counterparties may be few or nonexistent. The prices for the contracts vary through time in proportion to the probability that the next U.S. hurricane landfall will occur at a particular coastline segment, as perceived by market participants, and assessed through an adaptive control algorithm that responds to different levels of buying for the different coastline segments. Proceeds of these sales are collected into a common parimutuel (“mutualized risk”) pool and holders of contracts for the coastline segment eventually experiencing the hurricane landfall are paid from this pool in proportion to the number of contracts held. This market structure efficiently spreads hurricane risks across the entire Gulf and Atlantic coasts of the United States, on the basis of the uncertainty regarding landfall location.
as quantified by participants’ probability assessments for the event. Furthermore, this result is achieved without requiring a counterparty, that is, someone other than the exchange who is willing to sell the contracts.

The landfall segments have been defined mainly as counties in order that hedgers need only pay to hedge against local risks. However, the length scale of hurricane damage is typically somewhat larger than the length of the typical county coastline, so buying contracts for adjacent counties also would generally be advisable. For example, the most extensive damage due to Hurricane Charley occurred in Charlotte and the adjacent inland Desoto counties (e.g., http://www.fema.gov/library/viewRecord.do?id=3203). A natural hedging strategy for residents in these locations would have included Lee County contracts, particularly in light of the climatological near-shore tropical cyclone directions indicated in Fig. 1.

An important issue that is faced in the setup of HuRLO markets is the choice of the price adjustment parameter \( \alpha \). Larger values yield fast price responses but also high price volatility, whereas smaller values yield more stable prices that are slower to adjust to new information. The optimization of this trade-off will depend in part on the magnitude of trading noise in a given market. This question merits further research.

An additional aspect of the HuRLO markets that has not been mentioned is that the exchange can also support a secondary market, in which bilateral trades of previously purchased HuRLOs in a conventional bid-ask setting may be made. Neither short sales nor margin sales are allowed in either the primary or the secondary markets, so that many of the problems associated with conventional financial derivatives markets, such as excessive leverage and lack of pricing transparency (e.g., Duffie 2009), are avoided. The market structure described here might also be applied to other natural hazard events whose occurrences cannot be influenced by human actions, such as major earthquakes.

The results presented here have all been model-based simulations and were limited to the primary one-sided market. Empirical validation results derived from an economic laboratory simulation setting, including the operation of the accompanying secondary market, are described in a companion paper (Meyer et al. 2014). To date the market described here has operated outside of a laboratory setting only on a limited, preliminary basis. To the extent that it may attract broad participation in future years, it will be possible to compare the market probabilities with those from various forecast counterparts (Kelly et al. 2012; Wolters and Zitzewitz 2004), as well as to study other empirical properties of the market.

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APPENDIX

Magnitude of the Adjustment Parameter \( \alpha \) at Equilibrium

In the case of a market in equilibrium, when the ratio of price to payout equals the pricing probability \( \pi_i \),

\[
\pi_i(t-1) = \frac{P_i(t-1)}{W_i(t-1)} = \frac{\pi_i(t-1)c \exp(rj/365)n_i(t-1)}{MRP(t-1)}
\]  

(A1)

holds at time \( t-1 \) for all outcomes \( i \). Here \( W_i(t-1) = MRP(t-1)/n_i(t-1) \) is the indicative payout for outcome \( i \) at time \( t-1 \) (i.e., the settlement amount if outcome \( i \) was to occur with no further market activity after this time). All other symbols are as defined in the main body of the paper. Equation (A1) expresses the condition that all pricing probabilities \( \pi_i \) correctly reflect the current risk as expressed by the ratio of price to indicative payout.

Subsequently in this appendix, explicit indication of outcome \( i \) using subscripts will be suppressed for notational simplicity. Let \( \nu(t) = P(t)/W(t) \) be the outcome probability implied by the ratio of risk (price) to potential reward (indicative payout). We wish to increase the pricing probability \( \pi(t) \), using Eq. (2), to match the increase in \( \nu(t) \) resulting from the payout dilution for this outcome produced by the purchase of one additional option. Therefore,

\[
\pi(t) = \pi(t-1) + \alpha(t)\pi(t-1)[1 - \pi(t-1)] = \nu(t) = \frac{\pi(t-1)c \exp(rj/365)}{W(t)} = \frac{\pi(t-1)c \exp(rj/365)[n(t-1) + 1]}{MRP(t-1) + \pi(t-1)c \exp(rj/365)}
\]

\[
= \frac{\pi(t-1)c \exp(rj/365)[n(t-1) + 1]}{n(t-1)c \exp(rj/365) + \pi(t-1)c \exp(rj/365)} = \frac{\pi(t-1)[n(t-1) + 1]}{n(t-1) + \pi(t-1)}.
\]  

(A2)
Here use has been made of the fact that, because of the equilibrium expressed in Eq. (A1) at step \( t - 1 \),

\[
\alpha(t) = \left\{ \frac{\pi(t-1)[n(t-1) + 1]}{n(t-1) + \pi(t-1)} \right\} \{\pi(t-1)[1 - \pi(t-1)]\}^{-1} = \frac{1}{n(t-1) + \pi(t-1)} \approx \frac{1}{n(t-1)}.
\]

This final approximation will be a very close one, because in realistic cases \( n(t) \gg \pi(t) \).

If the market is out of equilibrium, which condition might be brought on by changes in external information (e.g., newly updated forecasts for the meteorological situation), a larger adjustment parameter than the equilibrium value in Eq. (A3) will be more appropriate, in order to induce faster price responses in the direction of the new equilibrium. Simulations (not shown) have indicated that

\[
\frac{30}{\overline{n}(t-1)} \leq \alpha(t) \leq \frac{40}{\overline{n}(t-1)}.
\]

where \( \overline{n}(t-1) \) is the average number of options over all outcomes at time \( t \) produce a smoothly operating market with acceptably small price volatility.

REFERENCES


Horowitz, K. A., A. L. Bequillard, A. P. Nyren, P. E. Procter, and D. S. Wilks, 2013: Activity relating to ongoing financial events. MRP\( (t - 1) = n(t - 1) c \exp(rj/365) \). Solving for the equilibrium adjustment parameter,

\[


