Prediction and optimization of volumetric power draw in an aerated and stirred vessel used in wastewater biological plant: a mathematical model developed by using central composite rotatable design analysis (CCRD)

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Abstract

In this study, the volumetric power draw P/V was determined as a factor in designing and identifying the optimal condition a successful aeration for stirred wastewater biological treatment vessels. The study was performed to characterize the volumetric power draw in the aerated stirred vessel by optimizing the operation variables. The concerning factors were improved by conjugating stirring and aeration to an efficient and economic volumetric power draw condition. The drawn volumetric power was tested and analyzed for three independent parameters; impellers rotation speed (100–200 rpm), turbine blades submergence ratio (S/W) (0.33–1.67) and wastewater height level ratio (H/D) (1.37–1.58). A mathematical model was developed in the form of a nonlinear polynomial mathematical model to predict the P/V. The optimal values of the P/V and of relevant parameters were computed through the application of the Box–Wilson technique by application of the central composite rotatable design (CCRD) model. The volumetric power draw P/V and the relevant independent parameters are presented in optimal conditions surface plots that obtained from the nonlinear mathematical model. Optimum analysis result for the independent parameters showed low levels of impellers rotation speed and turbine submergence ratio draw lower P/V while wastewater height did not have a clear effect on P/V.

Key words: aerated and stirred vessel, biological wastewater treatment, central composite rotatable design, modeling, volumetric power draw

LIST OF SYMBOLS

C propeller clearance, (m)
D rotated cone turbine or impeller diameter, (m)
d pitch blade propeller diameter (PBP, U), (m)
g acceleration of gravity (9.80665), (m2 s−1)
H water level in the vessel, (m)
N rotation speed, (s−1)
Np power number, (P/ρN3D5)
S turbine submergence, (m)
t time, (s)
T vessel diameter, (m)
To torque measured with filled vessel, (N m)
Twe torque measured with empty vessel, (N m)
V water volume, (m3)
W turbine blade width, (m)

IMPELLERS TYPES ABBREVIATIONS

IBRC inverted pitched rotated cone (developed particularly for aeration)
PBPU pitch blade propeller, up pumping
Aeration with stirring (mixing) are considered the most significant processes in gas–liquid dispersion and homogenization in wastewater treatment and in different biotechnological fields (Gimbun et al. 2009; Montante et al. 2010). Traditionally, stirred vessels are used in unit operations in wastewater biological treatment to increase flocculation rates and to prevent particle settling (Stenstrom & Rosso 2008). In wastewater treatment processes, aeration is needed for large gas handling capacity and an effective gas dispersion for generating as large an interfacial area as possible (Vasconcelos et al. 2000). The basic form of the process is a batch operation involves a single vessel which is filled, aerated, and then completely emptied (Kumar & Rao 2010).

Recently, multiple impellers are used in wastewater treatment (Shewale & Pandit 2006). Multiple impellers are preferred over a single impeller as they provide better gas utilization due to the higher gas phase residence time, narrower spread in the residence time distribution in the flow systems and higher surface area per unit liquid volume (Shewale & Pandit 2006). Different multiple impellers types and combinations are used in biological wastewater treatment. The choice depends usually on the way to gas dispersion, top to bottom liquid mixing conditions and mass-transfer potential created by the system (Stenstrom & Rosso 2008). The flow patterns in multiple impellers stirred tank is remarkably altered than that one impeller is used due to the interaction in circulation loops generated by impellers (Montante et al. 2010; Bao et al. 2012). In the biological treatment of wastewater, the aeration which accompanied with stirring can be accomplished either via air injection or by some impellers like turbines, which consume energy supplied in the form of mechanical work. Many types of the turbine can be used to achieve the aeration like rotating cones (Adachi 2015). With multiple impellers, an upper rotating cone is usually located close to the wastewater surface to achieve the aeration. While the lower impellers are placed inside the wastewater to assist the air bubbles dispersion, particle suspension, and mixing processes (Li et al. 2009).

The power draw of aeration or stirring in wastewater treatment had already been investigated individually by many studies to reduce the consumed energy (Davoody et al. 2015; Sun et al. 2016). Few of those studies have focused on the power draw practical application of aeration and stirring. Various trends were followed to characterize the drawn power by stirring and aeration in condition applicable for wastewater biological treatment (Zlokarnik 1979) had investigated the power consumption for four different types of the air transferator in wastewater treatment. He related the drawn power with the geometrical configuration of the implemented aerators and the wastewater surface condition. Rao et al. (2010) reached the same results for the power draw of subsurface aerators, they found that the geometrical configuration such as liquid height, tank diameter, and impeller clearance have most influences. Many studies (Taghavi et al. 2011; Scargiali et al. 2013) have obtained practical results on drawn power in aerated and stirred vessels that agree with Zlokarnik (Zlokarnik 1979). They found that with multiple impellers, the power draw is usually related with impeller geometry and wastewater surface condition. The performed studies were focused on characterizing and analysis volumetric power draw in aeration vessels for different geometries, and modes of operations. The power draw has a great dependency on the flow pattern and geometry in aerated vessels of the wastewater (Karimi et al. 2013; Wan et al. 2016). To breakthrough this relation, many studies were performed to correlate power draw with flow characteristics, the geometrical and operational parameters in aerated stirred vessels (Hiraoka et al. 2001; Issa 2016).

Different works have been made for determining optimal process parameters in aeration processes or stirring separately. Generally, the optimum parameter determination may vary as a minimum or a maximum of a parameter due to design consideration. Process parameters optimization is performed by applying multivariate statistical analysis for efficient operation and design purposes. Among the applied techniques for this purpose, the response surface methodology (RSM) or it called factorial
design is now widely used. This method is used instead of typical and conventional methods for multi-factor experimental design. Various statistical analysis methods failed to detect the true optimum conditions because some of them overlap between parameters effects and other methods need so long time to have results (Tanyildizi et al. 2005). To optimize the main response surface in engineering cases, the RSM which mixes mathematical and statistical methods can accomplish the analysis for many affecting parameters of the work (Aslan 2007). Many methods are used to design the experimental run in order modeling and analysis. These methods commonly are the full or partial factorial methods or the central composite rotatable design (CCRD) method. At minimum three levels for each variable are required to estimate the coefficients of quadratic terms in the response model (Box & Wilson 1951). Among the different RSM experimental designs, the CCRD is the more efficient method in order to find the needed information with the minimum required a number of experiments of the dependent variable (Fakheri et al. 2012). Considerable works have been conducted the CCRD method in diverse process design (Alalayah et al. 2010; Parilti 2010; Kamal et al. 2014). By applying CCRD method on aerated and stirred vessel power requirement for different impeller types (Afshar Ghotli et al. 2013) figured out this method helps to identify which impeller is more economical and efficient.

In this work the volumetric power draw in a laboratory model, which is geometrically similar to an actual stirred and aerated vessel for biological wastewater treatment is determined in term of three important relevant parameters. These parameters are impeller rotation speed, turbine submergence ratio S/W, and wastewater level ratio H/D. The aim of this work is to apply the CCRD method for analyzing and optimizing the effects of the impeller rotation speed, wastewater level, and turbine blade submergence on the volumetric power drawn in an aerated and stirred with a dual impeller in wastewater biological treatment vessel. The importance of the chosen relevant parameters comes from their evident effect on the volumetric power drawn that figured out experimentally. This step is necessary to characterize the process drawn energy that dissipated in the wastewater biological treatment vessel during the aerobic process which in turn decides the cost and efficiency of stirring and aeration processes by determining the most affecting parameters.

METHODS

RSM

RSM assumes that all variables in the process are to be measurable; then the response surface will be as illustrated in Equation (1), \( y \) represents the system response variable that needs to be optimized, and the \( x_1, x_2, \ldots, x_k \) are action variables, which also they are called factors.

\[
y = f(x_1, x_2, \ldots, x_k)
\] (1)

An important assumption of this method is that the conducted experiment has insignificant errors when it deals with independent variables in which they are continuous and controllable. The task then is the variable of response surface should have an accurate practical relation with independent variables. The design consists of factorial design points with \( 2^k \) runs, \( 2^k \) axial or star points, and \( n \) replications at the center of the design (Kwak 2005; Fakheri et al. 2012). Usually; RMS method implements a second-order polynomial to represent the mathematical relationships for response variables. Multiple linear regression analysis is applied to estimate the developed mathematical models for each response surface variable. Various terms of a quadratic model are resulting like linear, squared, and interaction (Kwak 2005).
The CCRD was first found by Box and Wilson and then developed by Box and Hunter (Box & Wilson 1951; Box & Hunter 1957). CCRD method requires a fewer number of experiments than the full factorial method, provides much information, and has described the steady-state responses of various operations (Aslan 2008). According to these merits, in this study, it was decided to use CCRD to design and optimize the volumetric power draw P/V in the aerated and stirred wastewater vessel. The affecting parameters under a steady state condition are impeller rotational speed N (rpm), turbine blades submergence ratio S/W and water height level ratio H/D. The required number of experiments in CCRD method covers the standard $2^3$ factorials with its origin at the center, $2^3$ points fixed axially at a distance from the center to generate the quadratic terms, and repeat runs at the center; the axial points are chosen such that they allow rotatability (Aslan 2007). Repeats of the experimental runs at the center level are essential to obtain an independent estimate of the experimental error (Box & Hunter 1957). The optimization of interested parameters may require many tests in ordinary ways to measure the impact of investigated factor on the oxygen transfer rate and to study the interactions; one parameter is varied and the other parameters are kept constant.

Experimental setup

The experimental runs were carried out in a cylindrical flat bottom vessel with inside diameter of 0.8 m. The vessel is made of transparent fiberglass. The vessel was fabricated geometrically similar with to an actual stirred and aerated vessel for biological wastewater treatment. The schematic diagram of the system is shown in Figure 1. Three baffles of width b (0.1 T) are used with our experimentation that to prevent or lessen the tangential circulatory flow created by the system, (the baffles have the same height of the vessel see Figure 1).

![Figure 1](https://iwaponline.com/wpt/article-pdf/11/3/590/380959/wpt0110590.pdf)  
**Figure 1** | Schematic diagram of the experimental setup.

The types of impellers were used in the study are: a turbine of the type of an inverted pitched rotated cone (IBRC) (used particularly for surface aeration), and a pitch blade propeller, up pumping flow (PBPU). This multiple impeller combination is used typically for mixing and aeration purposes with its up-pumping in the biological wastewater treatment operation, where the turbine (IBRC) is placed at the wastewater surface. in all these cases an acceptable mixing will provide good contact interfacial area between the contents, where the function of the lower propeller (PBPU) is re-directing the liquid flow toward the upper turbine (IBRC) for ensuring continuous feeding intake and to achieve
the well-mixing and distribution of the contents of the treatment vessel. The geometrical ratio for the propeller \((d/T) = 0.15\). A number of blades of the turbine (IBRC) of is 12 (blades width 0.24 m), with diameter ratio \(D/T = 0.24\). Propeller clearances ratio were kept constant with the ratio \(C/T = 0.2\). It is necessary to mention that impellers choice were made on the basis of the requirements found at the real plant and result in a sufficient oxygen supply for the operation conditions in an aerobic wastewater vessel.

**Power consumption calculation**

The consumed power of the system was calculated by torque measurement. Actual consumed power is calculated by applying the following equation

\[
P = 2\pi N(T_o - T_{oe})
\]

where, \(T_o\) and \(T_{oe}\) are the measured torques in filled and empty vessel respectively in (N.m). An torque meter was used with torque capture transducer.

**Experimental design**

To apply CCRD design it requires to codify all independent variable levels, this is made by converting the real values to coordinates inside a scale with dimensionless values. The independent variable levels \((X_i)\) are coded as a dimensionless values \(x_i\) while \(X_0\) corresponds to the central value of the independent variables and \(\Delta X_i\) is the step change (Tanyeli et al. 2005) as follows.

\[
x_i = \left(\frac{X_j - X_0}{\Delta X_j}\right) i = 1, 2, 3, \ldots, k
\]

The coefficients of the yielded second order polynomials were calculated and analyzed by using mathematical model equations that derived by computer simulation programming with the applying least squares method using Matlab R2014a software. Depending on the preliminary experimental results, the chosen levels for the independent variables, impellers rotational speed \((x_1)\), turbine blades submergence ratio \((x_2)\), and wastewater height level ratio \((x_3)\) are listed in Table 1. CCRD method of the experiments design was used where the values of independent variables were coded as the variables, \(x_i\), in the range of +1 and -1 levels. The dependence of drawn volumetric power \(P/V\) on the parameters has been determined with the first and second-grade polynomials like linear and non-linear models.

To design the experiments, the operating ranges for three variables is first identified as; impellers rotation speed (100–200 rpm), turbine blades submergence ratio \(S/W\) (0.33–1.67) and wastewater height level ratio \(H/D\) (1.37- 1.58).

To develop a mathematical model, coded values for the operating levels of the variables are used. The relationships between the coded levels \((x_i)\) and the corresponding real process variables \((X_i)\) according to Equation (3) for three variables are as follows:

\[
x_1 = \frac{X_1 - 150}{28.868}, \quad x_2 = \frac{X_2 - 1.0}{0.387} \quad \text{and} \quad x_3 = \frac{X_3 - 1.48}{0.0606}
\]

A three level factorial can be used to define the nonlinear response, however, easier application of second order designs can be made using the Box–Wilson assuming that the response can be represented by the following equation (Box & Hunter 1957).

\[
Y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{23}x_2x_3
\]
For the general form of the quadratic second-degree polynomial model (Equation (5)), the surface contains linear terms in \( x_n \), square terms in \( x_{nn} \) and cross-product term in \( x_n x_j \).

For 2\(^3\) factorial designs, the total number of the treatment combinations is \((2^n + 2n + 1)\). However for three variables, the required experiments are 15 units. The rotatable most likely to be useful in practice belong to a series that is also central. The design is subdivided into three parts; (1) the eight points constitute a 2\(^3\) factorial, (2) the six points are the extra points included to form a central composite design, and (3) five points are added at the center. According to Equation (5) and upon the completion of each test a total 19 runs were planned to be carried out due to the applied statistical method (see Table 1).

**RESULTS AND DISCUSSION**

Development of a three-variable model depending upon the experimental design

The coded values for the first eight experiments were at two levels –1 and +1 only which are 2\(^3\) factorial design representing the linear components. Seven extra points were added to the first eight points. These seven points include; one point added the center of the design which was repeated 5 times to estimate the external errors and to facilitate the interpretation of the functional relationship. The other six points were added in pairs along the three coordinate axes at a distance of \( a = 1.73 \) from the center of the \( x_1, x_2, \) and \( x_3 \). This arrangement was found to develop a suitable second order equation fitted to any observed response with studied variables.

The experimental results for the various combinations of the tested variables for the 19 experimental runs are shown in Table 2. Depending on these obtained results the second order polynomial

**Table 1** Coded experimental design points for the drawn volumetric power in aerated and stirred wastewater biological treatment vessel by using central composite design (Three – variable)

<table>
<thead>
<tr>
<th>Run No.</th>
<th>Coded Variables</th>
<th>Real Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( x_1, N, ()</td>
<td>( x_2, S/W, ()</td>
</tr>
<tr>
<td>1</td>
<td>–1</td>
<td>–1</td>
</tr>
<tr>
<td>2</td>
<td>–1</td>
<td>–1</td>
</tr>
<tr>
<td>3</td>
<td>–1</td>
<td>+1</td>
</tr>
<tr>
<td>4</td>
<td>–1</td>
<td>+1</td>
</tr>
<tr>
<td>5</td>
<td>+1</td>
<td>–1</td>
</tr>
<tr>
<td>6</td>
<td>+1</td>
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<tr>
<td>7</td>
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</tr>
<tr>
<td>8</td>
<td>+1</td>
<td>+1</td>
</tr>
<tr>
<td>9</td>
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<td>0</td>
</tr>
<tr>
<td>10</td>
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<td>0</td>
</tr>
<tr>
<td>11</td>
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</tr>
<tr>
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<td>–1.73</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
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<td>0</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
<td>–1.73</td>
</tr>
<tr>
<td>17</td>
<td>0</td>
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<tr>
<td>18</td>
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<td>0</td>
</tr>
<tr>
<td>19</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

For the general form of the quadratic second-degree polynomial model (Equation (5)), the surface contains linear terms in \( x_n \), square terms in \( x_{nn} \) and cross-product term in \( x_n x_j \).
coefficients in Equation (5) were calculated using a computer simulation programming by applying least squares method using Matlab R2014a software. Statistical analysis of the model was performed to evaluate the predicted P/V by Equation (5) (see Table 2).

The coefficients, i.e. the main effect ($x_i$) and two-factor interactions ($x_{ij}$) were estimated from the experimental results using a computer simulation applying the method of least squares in Matlab R2014a software. From the experimental results in Table 2, the second-order response functions representing P/V can be expressed as a function of the three operating parameters of P/V, namely the N, S/W and H/D. The relationship between responses drawn volumetric power (P/V) and operating parameters were obtained in a coded unit as follows:

$$P_V = 75.752 + 1.422x_1 + 19.458x_2 + 3.806x_3 - 1.871x_1^2 - 1.418x_2^2 - 0.487x_3^2 + 5.405x_1x_2 + 0.621x_1x_3 - 1.2x_2x_3$$

The response factors at any regime in the interval of our experimental design can be calculated from Equation (6). The predicted values for P/V with observed values are given in Table 2. The observed values and predicted values of P/V obtained using the model of Equation (6) are presented in Figure 2. As shown in this figure, there is a good agreement between predicted values and the obtained data points ($R^2$ value of 0.9723 for P/V) and F-value equals 148.05.

### Effect of variables on volumetric power draw (P/V)

The interrelation in the developed model of (Equation (6)) is shown in the 3-D response surface plots (See Figures 5-7).
Figure 2 | The relation between experimental and predicted P/V values using Equation (6).

Figure 3 | Response surface predicting recovery from the developed model (Equation (6)) effect of impellers rotation speed and turbine blades submergence at center level of wastewater height.

Figure 4 | Response surface predicting recovery from the developed model (Equation (6)) effect of turbine blades submergence and wastewater height at center level of impellers rotation speed.
In Figure 3, the effect of impellers rotation speed and turbine blades submergence on the P/V at the center level of water height can be seen. A higher volumetric power draw can be consumed with increasing the turbine submergence and impellers rotation speed. Figure 4 shows the effect of water height and turbine blades submergence on P/V at center level of rotation speed. The general form of the three-dimensional relationship shows high P/V occurs with a high level of turbine blades submergence for all wastewater height levels.

Figure 4 shows that higher levels of turbine blade submergence have a remarkable effect on P/V. This effect comes from that the immersed blades draw more power to reach the required rotation compared with other conditions. It can be seen also that higher wastewater height cannot reduce the consumed P/V. Figure 4 shows that the center level of impellers rotation speed is in good condition to obtain a lower P/V.

Figure 5 shows the effect of impellers rotation speed and wastewater height on P/V at center level of turbine blades submergence. It can be seen that as the rotation speed is increased, the P/V is increased progressively. It can be noticed that levels of wastewater height are in acceptable condition to obtain a moderate P/V but the extreme levels of impellers rotation speed are highly power consumable.

From the 3-D response surface figures that impellers rotation speed has a considerable impact on P/V. The minimum level of impeller rotation speed is determined as an optimum to achieve minimum P/V at 6-20 watt/m$^3$. The second important operating parameter is the turbine blades submergence while wastewater height has a minor effect on P/V.

**CONCLUSION**

A derived mathematical model was derived by using the experimental data and by application of CCRD for optimizing the surface aerator performance. CCRD was used to design an experimental program for modeling the effects of rotation speed, wastewater level, and turbine blades submergence on the performance of surface aerator volumetric power draw in an agitated tank.

Predicted values of P/V from the developed mathematical model (Equation (6)) were found to be in good agreement with the observed values of P/V ($R^2$ value of 0.9545). The results show that impellers rotation speed has a significant effect on the obtained P/V, whilst the turbine blades submergence was less significant and wastewater height has a small effect on P/V. The optimum value of impellers rotation speed is at its minimum level 100 rpm, at this level lower volumetric power was drawn.
(0–20 watt/m$^3$). Generally, Low turbine submergence ratio keeps the P/V at low levels (0–20 watt/m$^3$). For the third independent variable (H/T) it is hard to distinguish an optimum level for it in term of the other variables. The described analysis here will basically help to find the best performance for the used impellers.

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