Seepage losses from trapezoidal earth canals with an impervious layer under the bed

Doaa A. El-Molla\textsuperscript{a,*} and Mohamed A. El-Molla\textsuperscript{b}

\textsuperscript{a}Irrigation and Hydraulics Department, Faculty of Engineering, Ain-Shams University, Cairo, Egypt
\textsuperscript{b}Civil Engineering Department, Higher Technological Institute, 10th of Ramadan City, Egypt

\*Corresponding author. E-mail: doaa_anas@eng.asu.edu.eg

Abstract

Seepage of irrigation water from earth canals is one of the most significant causes of water loss. It should be carefully studied and controlled. This research aims to study the effect of the presence of an impervious layer under the bed of trapezoidal earth canals on the amount of seepage discharge. The finite element numerical model SEEP2D is used in the study. Different scenarios for the canal’s dimensions and side slopes are considered. Various values for the vertical distance between the canal’s bed and the impervious layer are also studied. The numerical model’s results are verified using Vedernikov’s equations. The results show that the presence of an impervious layer can reduce the amount of seepage discharge by up to 82.7% compared to that calculated without it, depending on the depth of the pervious layer, the ratio between canal’s bed width to water depth, and the side slopes. It is recommended to have boreholes at canal construction sites in order to investigate if an impervious layer exists under the canal’s bed. This helps to estimate the correct amount of seepage discharge based on the vertical distance from the canal’s bed to the impervious layer under it. Design charts are also provided.

Key words: conveyance losses, impervious layer, numerical model, SEEP2D, seepage, trapezoidal earth canals

Highlights

- Evaluates the effect of impervious layer under canals’ bed on seepage discharge.
- Simulates seepage from earth canals using SEEP2D numerical model.

ABBREVIATIONS

A = A coefficient in Vedernikov’s equation that depends on (B/H) and (m) (dimensionless).

b = Canal’s bed width (L).

B = Top width of the water in the canal (L).

g = Gravitational acceleration (LT\(^{-2}\)).

h = The total head (elevation head plus pressure head) in Laplace equation (L).

H = Depth of water in the canal (L).

K = Hydraulic conductivity of the pervious soil (LT\(^{-1}\)).

L = Width of flow at infinity (L).

m = Side slopes represented by cot of the angle \(\alpha\) (dimensionless).

q = Total seepage discharge per unit length of the canal (L\(^2\)T\(^{-1}\)).

This is an Open Access article distributed under the terms of the Creative Commons Attribution Licence (CC BY-NC-ND 4.0), which permits copying and redistribution for non-commercial purposes with no derivatives, provided the original work is properly cited (http://creativecommons.org/licenses/by-nc-nd/4.0/).
T = Vertical distance between the canal’s bed and impervious layer (permeable layer’s depth) (L).
\( \alpha \) = The angle between the canal’s sides and the horizontal plane (dimensionless).
\( \rho \) = Density of water (ML\(^{-3}\)).

**INTRODUCTION**

Water scarcity is getting more severe due to climate changes. It is essential to study and evaluate the amount of water losses and improve water management, especially in water stressed regions. Irrigation is considered one of the most important uses of water. Irrigation canals lose water through seepage and evaporation. Such losses reduce their conveyance efficiency, which is defined as the amount of water delivered by a canal divided by the amount of water diverted into the canal (Waller & Yitayew 2016).

Seepage of irrigation water from earth canals is considered one of the most significant causes of losses, while evaporation losses are very small compared to it (Garg 2005). Seepage occurs when water percolates through the canal’s wetted perimeter into the soil (Jamel 2016). Seepage can be heavy if the type of soil that the canal passes through is of high permeability and the canal is unlined. The seeping water can also cause waterlogging and raise the groundwater table. This negatively affects the soil-air-water balance in the plants’ root zone and reduces the crops’ yield. In designing a canal, the aim is usually to limit the seepage losses, so it is necessary to study seepage from canals carefully in order to be able to quantify the amount and control it (Waller & Yitayew 2016). The amount of canal seepage can be measured directly in the field, calculated analytically, computed from empirical equations, or determined using numerical models (Prakash & Saxena 2014; Christian & Trivedi 2018).

Field measurements can be either performed by the ponding method, the inflow – outflow method, or using a seepage meter. The ponding method is performed by closing the ends of a canal and measuring the rate of decline of the canal’s water surface (Sarki *et al.* 2008; Waller & Yitayew 2016; Khan 2019). Field measurement of seepage using the ponding method cannot occur while the canal is working as it requires the flow to be cut (Khan 2019). This is practically impossible in large essential canals due to the continuous flow of water and the large widths of such canals (Singh *et al.* 2013). The inflow – outflow method measures the inflow and outflow discharges of a canal and uses the water balance equation to determine the amount of seepage discharge. It is not suitable for measuring seepage in short reaches of earth canals due to the small differences between the inflow and outflow discharges. A seepage meter is a restricting cylinder pressed into the side or bottom of a canal. It is used to calculate the infiltration rate on a small separate location. Measuring seepage using a seepage meter may require performing a large number of measurements and taking an average value. It cannot be used for canals having a velocity more than 0.6 m/sec (Singh *et al.* 2013; Khan 2019).

Analytical solutions have several assumptions and are used for simple problems that are rarely met in the field (Singh *et al.* 2013). Empirical equations are based on previous observations relating the water losses to the canal’s hydraulic parameters. Some empirical formulas are established for very restricted situations while others can guess the amount of seepage in more general situations. Empirical equations can require the knowledge of the canal’s discharge, the velocity of flow, the hydraulic properties of the soil, the depth to groundwater, and the canal’s cross section. The required data may not be available in some conditions (Khan 2019). Numerical models are becoming more commonly used as they are less time consuming and require less effort compared to direct field measurement. They also have the ability to solve more complicated problems than analytical methods. Numerical models can be used to study many different situations and require fewer inputs compared to empirical equations.

Previous researchers (Sarmad 1980; Sarki *et al.* 2008; Nam *et al.* 2016; Khan 2019) studied seepage from earth canals by measuring it directly from existing canals. Others (Sharma & Chawla 1979;
Chahar 2001, 2007; Ghazaw 2002; Osman & Rahman 2008; Carabineanu 2012; Uchdadiya & Patel 2014) performed analytical studies to evaluate the problem of seepage from simple canals. Empirical equations were also used and compared with each other in other previous studies (Mowafy 2001; Saha 2015). Numerical modeling has been recently implemented in many canal seepage studies. Jamel (2016) used the SEEP/W model to study the downward seepage from lined and unlined triangular open channels, considering different scenarios for the canal section and soil properties. Aghvami et al. (2013) used the SEEP/W model along with the evolutionary polynomial regression modeling to study seepage from Qazvin and Isfahan Channels. Dolatkhah et al. (2015) used the SEEP/W model to study the amount of seepage from the earth canals of Moghan irrigation and drainage network. Tavakoli et al. (2017) performed a case study to estimate seepage from the earth canal of Boldaji using empirical equations and the SEEP/W model. Hosseinzadeh Asl et al. (2020) used the SEEP/W model to study the factors affecting seepage from trapezoidal, rectangular and triangular earth canals.

The literature review shows that the previous studies were either case studies performed on certain canals, studies aiming to evaluate a specific method or compare empirical equations, or studies focusing on the effect of the canal’s section parameters on the seepage discharge. No enough attention was given to the effect of the existence of an impervious layer under the canal’s bed on the amount of seepage discharge, which is the main scope of this study.

In this research, a 2D finite element numerical model, SEEP2D, is used to study seepage from trapezoidal earth canals considering different scenarios for the ratios between the canal’s bed width and water depth, and the angle of the canal sides with the horizontal plane. The effect of the presence of an impervious layer and its distance from the canal’s bed on the seepage discharge is also evaluated. The numerical model’s results are verified by using Vedernikov’s equations.

GMS (Groundwater Modeling System) is a numerical model from Aquaveo that simulates and solves different 2D and 3D groundwater problems. The model has many different modules that can simulate seepage in porous media, ground water flow, stratigraphic modeling, and many other applications. The SEEP2D finite element numerical model is a module in GMS. It can model a variety of confined and unconfined 2D steady-state seepage problems. SEEP2D is used to simulate cross-section models by considering a vertical section through the flow system that does not change in the third dimension. The model has been previously used to study many problems of seepage under hydraulic structures and through earth dams (Aquaveo 2019). The literature review did not show any studies using SEEP2D to study seepage from earth canals. So this paper highlights a different use for the SEEP2D model and verifies its results.

**DESCRIPTION OF THE MODEL**

In the present research, the numerical model SEEP2D is used to study seepage from trapezoidal earth canals. Different scenarios are considered for the ratios between the bed width and water depth in the canal as well as the canal side slopes and the presence of an impervious layer at different depths from the canal’s bed. A total number of 84 runs are performed.

SEEP2D software is a module in GMS10 that is used to model seepage using 2D finite element (steady state) method. The two dimensions are the horizontal and vertical dimensions (i.e., the vertical profile). SEEP2D can be used to model confined and unconfined flow for profile models. In order to model a typical problem using GMS10-SEEP2D, the user should draw the problem, set the material properties, assign the boundary conditions, select the mesh cell size, generate the mesh, execute SEEP2D, then display the results.

The governing equation used in the SEEP2D models is the Laplace equation (Equation (1)) which is used to model seepage in porous media following Darcy’s law. Darcy’s law is only applicable for low
velocities and laminar flow which is the case in the present study. Transient condition cannot be modelled using SEEP2D. SEEP2D output results include plotting the phreatic line for unconfined flow, drawing flow vectors, contouring of the total head equipotential lines, drawing the flow lines, as well as displaying the total flow rate (SEEP2D Primer 1998).

$$\frac{\partial}{\partial x} \left( K_{xx} \frac{\partial h}{\partial x} + K_{xy} \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial y} \left( K_{yy} \frac{\partial h}{\partial y} + K_{yx} \frac{\partial h}{\partial x} \right) = 0$$

where $h$ is the total head (elevation head plus pressure head) and $K$ is the hydraulic conductivity.

Many seepage case studies were performed using the SEEP2D numerical model and it was found to be efficient and reliable for both confined and unconfined seepage problems.

**DIMENSIONAL ANALYSIS**

The variables involved in the research are defined in the abbreviations section and presented in Figure 1. They can be expressed as shown in Equation (2):

$$\theta(b, B, H, K, L, q, \rho, g, T, m) = 0$$

![Figure 1](image)

Figure 1 | The variables involved in the study. (a) infinite previous layer under the canal, (b) impervious layer under the canal.

Applying Buckingham’s $\pi$ Theorem, and considering a constant value for $b$, $g$, $K$ and $\rho$ throughout the study, the final functional relation of dimensionless $\pi$ terms becomes:

$$\Theta \left( \frac{b}{H}, \frac{B}{H}, \frac{L}{H}, \frac{q}{KH}, \frac{m}{T}, \frac{T}{b} \right) = 0$$

**MODEL VERIFICATION**

In order to verify the numerical model, a simple trapezoidal earth canal passing through pervious soil of infinite depth is studied using the numerical model SEEP2D. The model’s results are compared to those obtained from Vedernikov’s equations. Vedernikov used the inversion of hodograph and conformal mapping techniques to obtain an exact analytical solution that calculates the steady state seepage from trapezoidal canals and ditches. Vedernikov considered that the canal passes through an unconfined, homogeneous and isotropic porous medium of infinite depth (Harr 1962; Jamel 2016).

Using SEEP2D numerical model, the seepage from the canal is considered to move downwards. Four ratios between bed width and water depth ($b/H$) are considered ($b/H = 1, 2, 3.5$ and $5$). Three values for the side slopes ($m$) are selected to cover the most widely used slopes for all soil
types \( (m = 1, 1.5 \text{ and } 2) \). The soil that the canal passes through is selected to be homogeneous isotropic sand of hydraulic conductivity \( (K) \times 10^{-4} \text{ m/s (8.64 m/day).} \) After trying different cell sizes for the finite element mesh, a 0.5 m cell size is selected in order to ensure accurate results without unnecessary increase in the analysis time. Considering the case of trapezoidal canal passing through a pervious layer of infinite depth, the model’s boundary conditions are entered as the total head at the sides and bed of the canal and the exit faces. The selected cell size for the SEEP2D finite element mesh and the boundary conditions for the case of pervious layer of infinite depth are shown in Figure 2. Figure 3 shows a sample of the model’s output. The model calculates the seepage discharge and plots the flow lines and equipotential lines. The width of flow at infinity \( (L) \) can also be measured.

\[ q = K (B + AH) \]  
\[ L = B + AH \]  

where \( q \) is the seepage discharge from the canal \( (\text{in } \text{m}^3/\text{day/m}^2) \), \( B \) is the top width of water in the canal \( (\text{in m}) \), \( H \) is the depth of water in the canal \( (\text{in m}) \), \( K \) is the hydraulic conductivity of pervious soil \( (\text{in m/day}) \), \( L \) is the width of flow at infinity \( (\text{in m}) \) and \( A \) is a coefficient that depends on \( B/H \) and \( m \) \( (\text{dimensionless}) \), and \( m \) is the side slope represented by cot of the angle \( (\alpha) \) \( (\text{dimensionless}) \).
Table 1 shows the inputs and results of Vedernikov’s equations and the SEEP2D numerical model. Comparing the discharges and widths of flow at infinity obtained from the numerical model SEEP2D to those resulting from Vedernikov’s equations show noticeable agreement. The results of \((q/KH)\) and \((L/H)\) obtained from numerical model almost coincide with Vedernikov’s method, as shown in Figure 4.

### Table 1 | Inputs and results of Vedernikov’s equation and SEEP2D numerical model

<table>
<thead>
<tr>
<th>B/H</th>
<th>m</th>
<th>A</th>
<th>(q) (m³/d/m²) (Model)</th>
<th>(q) (m³/d/m²) (Ved.)</th>
<th>(q/KH) (Model)</th>
<th>(q/KH) (Ved.)</th>
<th>(L) (m) (Model)</th>
<th>(L) (m) (Ved.)</th>
<th>(L/H) (Model)</th>
<th>(L/H) (Ved.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.00</td>
<td>1</td>
<td>3.3</td>
<td>89.36</td>
<td>88.99</td>
<td>1.47</td>
<td>1.47</td>
<td>10</td>
<td>10.30</td>
<td>2.00</td>
<td>2.06</td>
</tr>
<tr>
<td>5.50</td>
<td>1</td>
<td>3.1</td>
<td>106.1</td>
<td>106.12</td>
<td>2.23</td>
<td>2.23</td>
<td>11</td>
<td>12.28</td>
<td>2.20</td>
<td>2.46</td>
</tr>
<tr>
<td>4.00</td>
<td>1</td>
<td>2.75</td>
<td>146.24</td>
<td>145.80</td>
<td>4.22</td>
<td>4.22</td>
<td>16</td>
<td>16.88</td>
<td>3.20</td>
<td>3.38</td>
</tr>
<tr>
<td>3.00</td>
<td>1</td>
<td>2.5</td>
<td>237.57</td>
<td>237.60</td>
<td>9.17</td>
<td>9.17</td>
<td>25</td>
<td>27.50</td>
<td>5.00</td>
<td>5.50</td>
</tr>
<tr>
<td>8.00</td>
<td>1.5</td>
<td>3</td>
<td>95.09</td>
<td>95.04</td>
<td>1.38</td>
<td>1.37</td>
<td>11</td>
<td>11.00</td>
<td>2.20</td>
<td>2.20</td>
</tr>
<tr>
<td>6.50</td>
<td>1.5</td>
<td>2.8</td>
<td>114.63</td>
<td>114.76</td>
<td>2.04</td>
<td>2.04</td>
<td>12.4</td>
<td>13.28</td>
<td>2.48</td>
<td>2.66</td>
</tr>
<tr>
<td>5.00</td>
<td>1.5</td>
<td>2.52</td>
<td>162.16</td>
<td>162.43</td>
<td>3.76</td>
<td>3.76</td>
<td>18</td>
<td>18.80</td>
<td>3.60</td>
<td>3.76</td>
</tr>
<tr>
<td>4.00</td>
<td>1.5</td>
<td>2.28</td>
<td>271.2</td>
<td>271.50</td>
<td>7.85</td>
<td>7.85</td>
<td>29.5</td>
<td>31.40</td>
<td>5.90</td>
<td>6.28</td>
</tr>
<tr>
<td>9.00</td>
<td>2</td>
<td>2.75</td>
<td>101.66</td>
<td>101.52</td>
<td>1.31</td>
<td>1.31</td>
<td>10.5</td>
<td>11.75</td>
<td>2.10</td>
<td>2.35</td>
</tr>
<tr>
<td>7.50</td>
<td>2</td>
<td>2.6</td>
<td>124.3</td>
<td>124.63</td>
<td>1.92</td>
<td>1.92</td>
<td>13.6</td>
<td>14.42</td>
<td>2.72</td>
<td>2.88</td>
</tr>
<tr>
<td>6.00</td>
<td>2</td>
<td>2.3</td>
<td>179.91</td>
<td>179.28</td>
<td>3.46</td>
<td>3.46</td>
<td>19.5</td>
<td>20.75</td>
<td>3.90</td>
<td>4.15</td>
</tr>
<tr>
<td>5.00</td>
<td>2</td>
<td>2.1</td>
<td>309.21</td>
<td>306.72</td>
<td>7.10</td>
<td>7.10</td>
<td>33</td>
<td>35.50</td>
<td>6.60</td>
<td>7.10</td>
</tr>
</tbody>
</table>

**Figure 4** | Comparison between the results of SEEP2D and Vedernikov’s equation.
MODEL APPLICATION

In this section, the numerical model SEEP2D is applied to study a trapezoidal earth canal passing through a soil layer of finite depth and resting on an impervious layer. The seepage discharge is checked and the effect of the presence of the impervious layer as well as its distance from the canal’s bed are evaluated.

Four values are studied for the ratio between the canal’s bed width and the water depth (b/H) (b/H = 1, 2, 3.5 and 5). For each (b/H) ratio, three values for the canal’s side slopes (m) are investigated (m = 1, 1.5 and 2). Six ratios for the depth of the pervious layer (T/b) are also studied (T/b = 1, 2, 3, 4, 5 and 6), where the depth of pervious layer (T) is the vertical distance measured from the canal’s bed to the impervious layer. A total number of 72 scenarios are studied by suggesting different combinations of (b/H) ratio, side slopes (m), and depth of the pervious layer (T/b). Figure 5 shows the boundary conditions when an impervious layer is considered to exist under the canal’s bed and a sample of the numerical model’s output.

RESULTS AND DISCUSSION

Effect of presence of an impervious layer under the canal’s bed and the depth of the pervious layer on seepage discharge

The numerical model's results show that the presence of an impervious layer under the canal’s bed reduces the value of seepage discharge (q/KH) by a percentage that ranges from 1.1% and up to 82.7% compared to the value obtained from the case of infinite soil layer depth. The percentage of reduction depends on the depth of pervious layer (T/b), the (b/H) ratio and the value of side slopes (m).

The pervious layer’s depth (T/b) has positive association with the seepage discharge (q/KH). The amount of seepage discharge decreases for smaller (T/b) depths. This may be because the decrease in depth of the pervious soil layer reduces the total head difference, which leads to less seepage from the canal. The effect of (b/H) ratio on seepage discharge (q/KH) also decreases as the thickness of soil layer decreases.

For canals with side slopes m = 1, it was found that the presence of an impervious layer can reduce seepage by 1.5% at (b/H) = 5 and (T/b) = 6 and up to 74.7% at (b/H) = 1 and (T/b) = 1, compared to its value in the case of infinite pervious layer.

For canals with side slopes m = 1.5, it was found that the presence of an impervious layer can reduce seepage by 1.1% at (b/H) = 5 and (T/b) = 6 and up to 76.9% at (b/H) = 5 and (T/b) = 1, compared to its value in the case of an infinite pervious layer.
For canals with side slopes $m = 2$, it was found that the presence of an impervious layer can reduce seepage by $1.4\%$ at $(b/H) = 5$ and $(T/b) = 6$ and up to $82.7\%$ at $(b/H) = 1$ and $(T/b) = 1$, compared to its value in the case of an infinite pervious layer.

**Figure 6** shows the effect of different pervious layer’s depths $(T/b)$ ranging from 1 to 6 on the seepage discharge $(q/KH)$ for $(b/H)$ ratios ranging from 1 to 5 and side slopes $(m) = 1, 1.5$ and 2. The chart shows that as $(T/b)$ decreases, the effect of the impervious layer on reducing seepage discharge $(q/KH)$ gets more noticeable for all $(b/H)$ ratios and all slopes $(m)$.

**Figure 7** shows the effect of pervious layer’s depth on seepage discharge for different side slopes $(m)$. It can be used to obtain the seepage discharge $(q/KH)$ for different $(b/H)$ ratios ranging from 1 to 5. The figure covers the case of a pervious layer of infinite depth as well as the case of the presence of an impervious layer at a distance from the canal’s bed $(T/b)$ ranging from 1 to 6, for side slopes $(m) = 1, 1.5$ and 2 respectively.

**Effect of side slopes on seepage discharge**

For the case of a trapezoidal earth canal passing through an infinite soil layer and considering downward seepage, flatter side slopes lead to more seepage discharge than steep side slopes. This occurs due to the greater wetted perimeter that is caused by flattening the side slopes. Every increase in the value of $(m)$ by 0.5 causes an increase in the seepage discharge by $6.41\%$ for channels with $(b/H) = 5$ and up to $14.16\%$ for $(b/H) = 1$. For the case of an infinite soil layer, it can be clearly
seen that increasing the value of (m) causes the seepage discharge term (q/KH) to increase. For the case of the presence of an impervious layer under the canal, the depth of the pervious soil layer that the canal passes through is found to influence the effect of side slopes on seepage discharge. The results show that for large depths of pervious soil layer, the flatter side slopes lead to more seepage discharge than steep side slopes. This effect keeps decreasing as the depth of pervious soil layer decreases till it is reversed for smaller depths such as (T/b = 1) where the seepage discharge in flatter slopes become less than steeper slopes by a value of 3.51% to 6.25%. Figure 8 shows the relation between the side slopes and the seepage discharge (q/KH) for different depths of pervious soil for (T/b = 1, 3, 6) compared to the case of an infinite soil layer.

Figure 7 | Effect of pervious layer’s depth on seepage discharge for different side slopes (m).
CONCLUSIONS

SEEP2D numerical model is an accurate and reliable tool that can be used to study seepage from trapezoidal earth canals. The presence of an impervious layer under the canal’s bed can significantly affect the amount of seepage discharge. It causes the quantity of seepage discharge ($q/KH$) to decrease by a percentage that ranges from 1.1% and up to 82.7% compared to the case of infinite pervious layer depth. The percentage of seepage reduction depends on the depth of pervious layer ($T/b$), the $(b/H)$ ratio and the value of side slopes ($m$). It is recommended to have boreholes at canal construction sites in order to investigate if an impervious layer exists under the canal’s bed. This helps to estimate the correct amount of seepage discharge based on the vertical distance from the canal’s bed to the impervious layer under it.

Flatter side slopes lead to more seepage discharge than steep side slopes for the case of an infinite soil layer. Every increase in the side slopes by a value of 0.5 causes an increase in the seepage discharge by 6.41% for channels with $(b/H) = 5$ and up to 14.16% for channels with $(b/H) = 1$. For the case of the presence of an impervious layer under the canal’s bed, flatter side slopes lead to more seepage discharge than steep side slopes for large $(T/b)$ ratios. This effect decreases till it is reversed for smaller $(T/b)$ ratios such as $(T/b = 1)$, where the seepage discharge in flatter slopes becomes less than steeper slopes by 3.51% to 6.25%.

Design charts to estimate the amount of seepage discharge ($q/KH$) are provided in Figures 6 and 7. The charts can be used for the case of a pervious layer of infinite depth as well as the presence of an impervious layer under the canal’s bed at a depth $(T/b)$ ranging from 1 to 6. The charts also cover a range of $(b/H)$ ratio from 1 to 5 and side slopes ($m$) = 1, 1.5 and 2, which are the most widely used slopes for all soil types.

DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

REFERENCES


Khan, A. A. 2019 *Evaluation of Water Losses in Unlined Canal: A Case Study of Malik Branch Canal*. MSc Thesis, Faculty of Engineering, Department of Civil Engineering, Capital University of Science and Technology, Islamabad, Bahawalnager, Pakistan.


Sarmad, M. 1980 *Measuring Seepage Rates From Irrigation Canals*. MSc Thesis, Faculty of the Graduate College of the Oklahoma State University.


First received 13 November 2020; accepted in revised form 2 February 2021. Available online 16 February 2021.