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Optimization of closed underflow mini-hydrocyclones for separating sand from well water by two indices of mass efficiency and cut diameter using the Taguchi method: a case study of Mashhad, Iran

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ABSTRACT

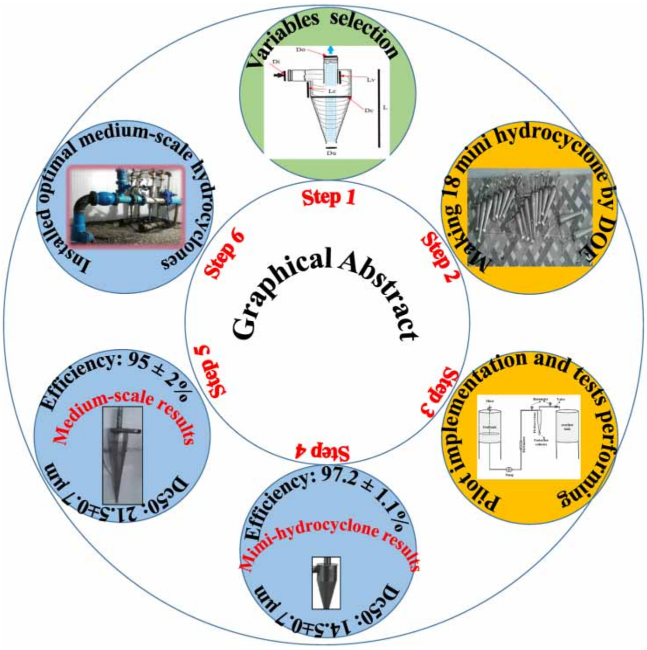
In this study, the Taguchi method was applied in order to design experiments and optimize the performance of mini-hydrocyclones by closed underflow for sand removal from well water. To optimize the hydrocyclones, two indices, mass removal efficiency and the cut diameter, were used. The first index analysis results showed that under the optimum conditions, the overall mass removal efficiency of solids will be up to 98.4%. An analysis of cut diameter data illustrated that under optimum conditions, the predicted cut diameter will be about 12.7 μm, while this diameter was estimated at about 16.3 μm under optimum mass removal efficiency conditions. The actual values of mass removal efficiency and cut diameter were determined at about 97.2 ± 1.1% and 14.5 ± 0.7 μm, respectively, for hydrocyclone manufacturing on the basis of mass removal optimum conditions. The use of the traditional estimation models such as Plitt, modified Plitt, Luz, and so on, which are applied for mining and mineral processing, illustrated that the estimated cut diameters were about 7.9, 8.4, 8.2, 0.45, and 2.33 μm, respectively. This estimation represented that these models are not suitable to predict the cut diameter of the hydrocyclone with closed underflow. Further investigation revealed that only the scale-increasing model could well predict a cut diameter of about 15.4 μm.

Key words: closed underflow hydrocyclone, hydrocyclone optimization, hydrocyclone performance, sand separation, sandy wells, Taguchi method

HIGHLIGHTS

- Mass removal efficiency and the cut diameter were used simultaneously for mini-hydrocyclone optimization.
- A total of 18 mini-hydrocyclones designed by the Taguchi method were tested to determine the optimal dimensions.
- A medium optimal hydrocyclone was designed and manufactured by the scale-increasing method experimented in a well.
- The relationship between head loss and cut diameter in the mini-hydrocyclones was investigated.

GRAPHICAL ABSTRACT



1. INTRODUCTION

Hydrocyclone, which operates on the basis of centrifugal force, is used in the removal of particles from water because of its advantages of simplicity, low cost, and high flexibility. Although the principles and fundamental design of common hydrocyclones have a more than one-century history, hydrocyclones were widely used in industry after the Second World War (Rahmani 2005; Hsu *et al.* 2011). These devices were first used in mineral processing and mining, but later they were used in other processes and industries such as water and wastewater treatment, environmental engineering, and chemical engineering, and their application is expanding (Rahmani 2005; Hsu *et al.* 2011; Hwang & Chou 2017). In the case of water treatment and especially in groundwater, the hydrocyclone can be used to remove solids such as sand grains that are heavier than water. In recent times, it has been found that water pumped from a large number of wells in some plains of Iran has a lot of sand content. Sandy wells might be formed because of several factors such as perforation of the wall pipe, corrosion, degradation of screens, excessive withdrawal, etc. So, using a simple and inexpensive method (e.g. hydrocyclone) to remove sand from water is justifiable. Various designing and operational factors alter the performance of hydrocyclones, of which the most important of them include inlet diameter, the diameter of the overflow, underflow (apex) diameter, the diameter and the height of the cylindrical section, height of the cone, overflow length, and suspended solid concentration (Svarovsky & Thew 1992; Rahmani 2005; Silva *et al.* 2013). The impact of these factors on hydrocyclone performance has been investigated by several researchers, and some of these factors and the names of researchers are given in Table 1. Most of them have examined hydrocyclone efficiency to separate most of the mud from muddy water and solid grading in mineral processing and mining projects.

The performance investigation of hydrocyclones is done by using two indices, namely, (1) mass removal efficiency and (2) the cut diameter based on grade efficiency. Apart from the mass removal efficiency as an important indicator for evaluating the performance of hydrocyclones, another important indicator is the so-called cut-size diameter (d_{50c}), which determines the minimum size of the separated solids when the equipment performs at 50% separation efficiency. Rahmani (2003) used the mass removal efficiency index for the removal of sand from water by hydrocyclone. The results of his study indicate that the particles' mass removal efficiency increased with increasing the size of the particles and liquid velocity. His results reveal that the mass removal efficiency for particles above 144 μm was found between 79 and 93.5%, and for particles above 5 μm, it was 51.2–63.1% (Rahmani 2003). In some recent studies, hydocyclones have been used as a pre-treatment instrument in water and wastewater treatment (e.g. runoff and muddy seawater treatment). Yu *et al.* (2013) used hydrocyclones for treating paved-road stormwater runoff by the volumetric percentages of underflow and overflow of

Table 1 | The summary results of some research projects on hydrocyclones

Researcher	Subject	Main result	Reference
Silva *et al.*	Change in the Plitt model to simulate hydrocyclones	Input diameter is a function of the body diameter, flow, and required efficiency	Silva *et al.* (2013)
Arterburn	Appropriate hydrocyclone sizing	- Determining the overflow diameter to the body of the hydrocyclone ($D_o/D_c = 0.35$) - By increasing the solid concentration in the feed, the cut diameter will be increased.	Arterburn (1982)
Mart´ınez *et al.*	Determining the optimum height of the overflow	Determine that $L_v/L = 0.1$	Martínez *et al.* (2008)
Cilliers	The use of hydrocyclones for separation and gradation	Solid concentration increase leads to a cut diameter increase	Cilliers (2000)
Rahmani	Hydrocyclone design and construction for separating particles from water	With the increase in the input speed, the suspended solid separation efficiency increases	Rahmani (2003)

about 29 and 71%, respectively. In their study of hydocyclones by opened underflow, the corresponding percentages for total suspended solids (TSS) and chemical oxygen demand (COD) were reported at 71 and 29% and 59 and 41% (Yu *et al.* 2013). Son *et al.* (2016) applied a multihydrocyclone (consisting of three serial hydrocyclones) water pre-treatment system to reduce suspended solids and COD from muddy seawater and sewage samples. Their hydrocyclone was designed with an underflow rate of about 4% and an overflow rate of 96%. Their results indicate that COD and biochemical oxygen demand (BOD) simultaneously decreased with an increase in the number of hydrocyclone steps and the separation efficiency of the hydrocyclones asymptotically decreased in accordance with an increase in the number of hydrocyclone steps (Son *et al.* 2016). Recently, mini-hydrocyclone has been used to separate microplastics from water. In a study conducted by He *et al.* (2022), the efficiency of nine mini-hydrocyclones with different underflows and overflows was investigated and a particle recovery of 51% was obtained for microplastics with an average size of 10 μm diameter. Huang *et al.* (2021) conducted a study on the enhanced removal of fine sand by hydrocyclone in a sewage treatment plant. They used two hydrocyclones (FX100 and FX50) and determined that the removal rates of fine sand and organic matter were 61.89 and 6.89% for FX100 and 71.39 and 17.38% for FX50, respectively (Huang *et al.* 2021).

In the second method (with the cut diameter), various experimental models were developed to study the performance and efficiency of the hydrocyclones. Some of the important models for the cut diameter estimation in hydrocyclone performance are shown in Equations (1)–(5) in Table 2 (Svarovsky & Thew 1992; Rahmani 2005; Silva *et al.* 2013). Although one of the first comprehensive models that were used to predict the performance of

Table 2 | Some cut diameter estimation models for hydrocyclones (Svarovsky & Thew 1992; Rahmani 2005; Silva *et al.* 2013)

Equation number	Equation	Researcher	Year
(1)	$d_{50c} = \dfrac{50.5D_c^{0.46}D_i^{0.6}D_o^{1.21}e^{0.063\Phi}}{D_u^{0.71}h^{0.38}Q^{0.45}(\rho_s - \rho_l)^{0.5}}$	Plitt	1976
(2)	$d_{50c} = \dfrac{50.5D_c^{0.46}D_i^{0.6}D_o^{1.21}\mu^{0.5}e^{0.063\Phi}}{D_u^{0.71}h^{0.38}Q^{0.45}(\rho_s - \rho_l)^{0.5}}$	Plitt	1980
(3)	$d_{50c} = \dfrac{52.45D_c^{0.46}D_i^{0.6}D_o^{1.21}e^{0.063\Phi}}{D_u^{0.71}h^{0.38}Q^{0.45}(\rho_s - \rho_l)^{0.5}}$	Luz	2005
(4)	$d_{50c} = \dfrac{2.6892kD_c^{0.46}D_i^{0.6}D_o^{1.21}\mu^{0.5}e^{0.063\Phi}}{D_u^{0.71}h^{0.38}Q^{0.45}(\rho_s - \rho_l)^{0.5}}$	Yan	2006
(5)	$d_{50c} = \dfrac{14.8D_c^{0.46}D_i^{0.6}D_o^{1.21}e^{0.063\Phi}}{D_u^{0.71}h^{0.38}Q^{0.45}(\rho_s - \rho_l)}$	Valadão *et al.*	2007
(6)[a]	$x_{50}^2 = \dfrac{9\pi Stk_{50} \times E_u}{2 \quad k_p} \dfrac{D_c^3}{Q} \dfrac{\mu}{(\rho_s - \rho_l)} \left(\dfrac{\pi\mu D_c}{4Q\rho_l}\right)^{n_p}$	Ortega-Rivas *et al.*	1992

[a]This equation is obtained according to the references (Ortega-Rivas & Svarovsky 1992; Rahmani 2005) and by combining Stokes, Eulers, Reynolds numbers, and other equations.

industrial hydrocyclones was developed by Lynch and coworkers at the University of Queensland (Narasimha *et al.* 2014), the most used model for hydrocyclone-corrected cut size calculation was proposed by Plitt, and over the years, many revisions and corrections to Plitt's model have been proposed (Silva *et al.* 2013). His first model was presented in 1976, which is shown in Equation (1). Then, he modified his model in 1980, which is shown in Equation (2). In these models, d_{50c} is the cut diameter (μm), D_c is the cylindrical diameter (cm), D_i is the inlet pipe diameter (cm), D_o is the overflow diameter (cm), D_u is the underflow diameter (cm), h is the distance between the floor of the overflow to above the underflow (cm), Q is the feed flow (1/min), ρl is the fluid density (g/cm^3), ρs is the solid particle density (g/cm^3), ϕ is the percent volume of solid particles in the feed (%), and finally, μ is the fluid dynamic viscosity (*cp*). In the Yanr model (Equation (4)), k is the dimensionless calibration factor, and when there are no data to determine it, the digit 1 is used for it (Silva *et al.* 2013). Along with the use of these models, some researchers, including Son *et al.*, have applied the cut diameter equation that is used in air treatment by the cyclone, which is not mentioned here (Son *et al.* 2016). In addition, if the increasing scale factors and performance constants, including *np*, *kp*, $Stk_{50}.E_u$, are available for a hydrocyclone, the cut diameter (x_{50}) can be obtained using Equation (6) (Ortega-Rivas & Svarovsky 1992; Rahmani 2005). In Equation (6), Stk_{50} and E_u are Stoke's and Euler's numbers, respectively, that their multiplying for each hydrocyclone is constant and Q is the crossing flow of the hydrocyclone and other variables have been previously defined. It should be noted that all of these models are used in muddy water concentration, classification, and separation of particles from mineral and mining projects. Therefore, they are not necessarily used for hydrocyclones that are applied in the removal of sand from water, especially by closed underflow, necessitating the study of these models for this specific context in the current research. Investigations that have been performed by different researchers on the use of hydrocyclones indicate that they focused on mineral processing and mining as well; therefore, they have not included the simultaneous effect of all important variables. On the other hand, the hydrocyclones used for the purpose of sand removal from water are also under license and there is no coherent and complete information about them. Therefore, the main objectives of this research are as follows: (1) conducting an experimental study for investigating and optimizing mini-hydrocyclone performance by closed underflow specifically for use in sand removal from groundwater, (2) considering all important factors that simultaneously influence the performance of a hydrocyclone, (3) using two indicators, mass removal efficiency and the cut diameter, for optimizing and comparing results, (4) assessing the accuracy of the models provided for cut diameter prediction (Table 2), and (5) designing and manufacturing a medium optimal hydrocyclone by the scale-increasing method and experimenting under real and operational conditions in a well.

In order to cope with the limitations of various methods, including cost and time, various experimental design methods based on mathematical and statistical techniques have been developed, including the Taguchi method and response surface methodology (RSM). The Taguchi method includes an experimental design method to determine the effect of factors on the response and to obtain the optimal process conditions. One of the main advantages of this method is providing optimal conditions with the minimum number of experiments using orthogonal arrays, leading to cost reduction, and determining important factors in a short time (Shojaei *et al.* 2021). Therefore, for considering all of the study objectives and saving cost and time, the experimental design by the Taguchi method was applied. This method is an experimental design and statistical analysis method used in different fields, including environmental engineering and water and wastewater treatment (Gönder *et al.* 2010; Darake *et al.* 2014; Silva *et al.* 2014; Reyhani *et al.* 2015; Salgado & Silva 2017; Dehnavi *et al.* 2018). In the present study, experimental work has been conducted to find the optimal combination of considered parameters using the Taguchi technique (by Qualitek-4). To examine the contribution of these parameters, the ANOVA method was used.

2. MATERIALS AND METHODS

2.1. Factor definitions and their levels

Seven factors were considered in this study and these are shown in Figure 1. They are the input diameter (D_i), overflow diameter (D_o), underflow diameter (D_u), cylindrical section diameter (D_c), height of the cylindrical section (L_c), total height of the hydrocyclone (L), and vortex finder length (L_v). For all of these factors, three levels were considered on the basis of literature review and the geometrical dimensions of commercial hydrocyclones, which are given in Table 3. The geometric ratios used to define these levels were chosen on the basis of the

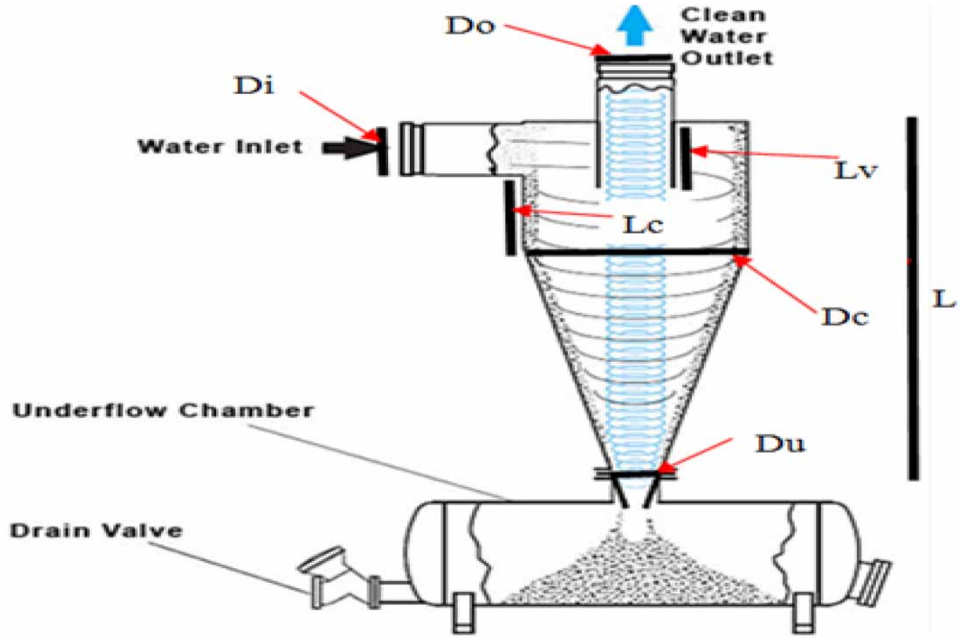


Figure 1 | Seven factors considered in this study.

Table 3 | Geometric ratios used in the literature and the present study

Geometric ratios	D_i/D_c	D_o/D_c	D_u/D_c	L_c/D_c	L/D_c	L_v/D_c	References
Minimum in the literature	0.14	0.2	0.04	0.5	3.3	0.33	Svarovsky & Thew (1992); Rahmani (2005); Martínez *et al.* (2008)
Maximum in the literature	0.28	0.37	0.28	0.55	6.93	0.55	
Minimum in the present study	0.1	0.1	0.1	0.5	3.3	0.33	Present study
Maximum in the present study	0.35	0.35	0.35	1.25	7.5	0.75	

minimum and maximum ranges of the common designs listed in Table 3. The factor levels were chosen to create the desired minimum and maximum geometric ratios to the best extent possible.

2.2. Experimental design

The experimental design was used to reduce the total number of experiments. In the present study and with the presence of three levels for each of the seven variables, 2,187 experiments were needed for one experiment replication for a full fractional method. To reduce the number of tests, cost and time, experimental design with the Taguchi method was used as one of the fractional factorial design methods. The Taguchi method was used to cut down the number of trials and thereby lower the cost. Based on this procedure and according to the desired factors as well as the levels, an M-18 orthogonal array was designed and selected. This matrix was defined using the Qualitek-4 software that is based on the Taguchi method application (Roy 2001). Qualitek-4 is a Windows software for the Taguchi experimental design technique. It can automatically design experiments on the basis of user-indicated factors and levels. The program selects the array and assigns the factors to the appropriate column. In the matrix that was designed for the present study (Table 4), each row corresponded with one experiment, and thereby it was determined that only 18 experiments instead of 2,187 experiments were required for each desired concentration and for one experiment replication. It should be noted that the experiments in this study were conducted with the maximum concentration of solids observed in the wells of the case study (Mashhad wells, Iran) and with two replications. All 18 hydrocyclones based on the compositions listed in Table 4 were manufactured of steel and were tested.

Table 4 | The M-18 orthogonal array, factors, and levels

Trial	Factors and level values (mm)						
	D_i	D_o	D_u	D_c	L_c	L	L_v
1	6	6	6	40	25	150	15
2	6	9	9	50	30	200	20
3	6	14	14	60	35	250	25
4	9	6	6	50	30	250	25
5	9	9	9	60	35	150	15
6	9	14	14	40	25	200	20
7	14	6	9	40	35	200	25
8	14	9	14	50	25	250	15
9	14	14	6	60	30	150	20
10	6	6	14	60	30	200	15
11	6	9	6	40	35	250	20
12	6	14	9	50	25	150	25
13	9	6	9	60	25	250	20
14	9	9	14	40	30	150	25
15	9	14	6	50	35	200	15
16	14	6	14	50	35	150	20
17	14	9	6	60	25	200	25
18	14	14	9	40	30	250	15

2.3. Pilot setup

To evaluate the performance of 18 mini-hydrocyclones and determine the dimensions of the optimal hydrocyclone, an appropriate pilot was designed and used, as shown in Figure 2. The setup consists of initial feed and final overflow tanks (made of galvanized sheets), an underflow collector (made of steel), Marquis model pumps creating pressure and feeding at a flow range of 5–40 L/min, 18 hydrocyclones (made of steel), a barometer with 0.05 accuracy (AT model made in Taiwan) and a flow meter.

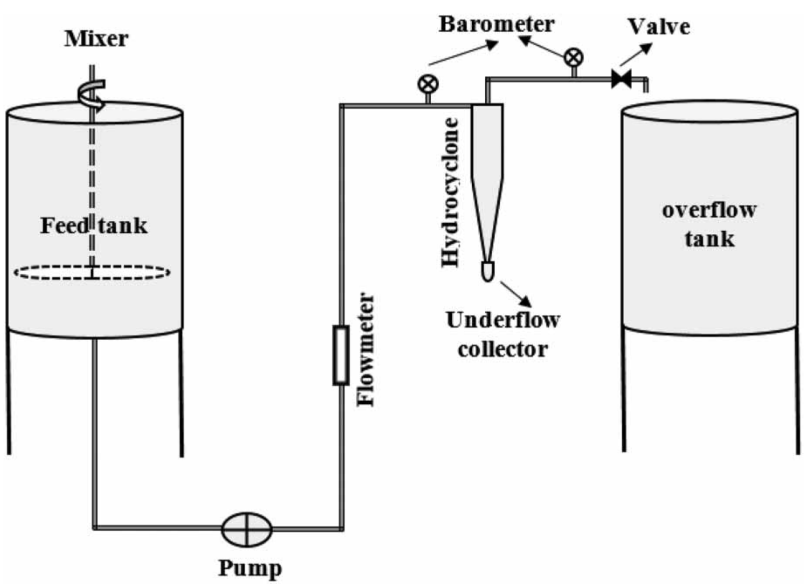


Figure 2 | Pilot setup.

2.4. Performing the experimental procedures

To perform any experiments in various replications, a certain concentration of solids was poured into the feeding tank and mixed well, so that a homogeneous mixture of sands was created. The minimum and maximum concentrations observed in the case study wells were 20 and 100 mg/L, respectively. As in another research (Dehnavi *et al.* 2018), the mentioned two concentrations were used, and it was determined that these concentrations had no effect on the results; therefore, only the maximum concentration was used in this research. Then, a homogeneous mixture with a flow rate of 15 L/min was added to consider hydrocyclone (note that the input speed of hydrocyclones was different due to several input diameters in some hydrocyclones). After performing each experiment, the weight of the accumulated sand in the underflow collector was determined on the basis of the TSS experiment and in accordance with water and wastewater standard methods (American Public Health Association 2005). After ensuring a mass balance between input, overflow, and underflow, the removal efficiency was calculated on the basis of the total weight of input suspended solids and the total weight of collecting suspended solids in the underflow collector. In addition, by using an ultrasonic sieve system analysis, the mass efficiency of each size, distribution of separated particles, and the cut diameter were determined by using a grade efficiency curve.

2.5. Solid gradation and concentration

Solid gradation used in all of the experiments was based on the gradations observed in the studied area (three wells in Mashhad, Iran). Since the sand gradations extracted from the water of the three wells were very close to one another ($p > 0.4$), the mean of the three observed gradations was used in all experiments. In these gradations, 12.7% of sand had a size of more than 355 μm, 63.7% of sand had a size ranging between 180 and 355 μm, 13.4% had a size between 25 and 45 μm, and 10.2% had a size smaller than 25 μm, and there was no particle between 45 and 180 μm. In addition, a solid concentration of 100 mg/L was used as a maximum concentration input feed based on the gradation curve.

2.6. Analysis of results

To analyze the results based on experimental design and the M-18 orthogonal array, the Qualitek-4 software by signal-to-noise analysis and analysis of variance (ANOVA) were used (Roy 2001). Taguchi used the signal-to-noise (S/N) ratio in measurable amounts of qualitative characteristics according to the purpose of the experiments. This method uses the S/N ratio to measure the variance from the design experiment. The S/N ratio is the ratio of the mean (signal) to the standard deviation (noise). The method of calculating the S/N ratio depends on each run of the experiment. In the Taguchi method, there are 3 characteristic values converted into the S/N ratio that describe different characteristics of quality according to the nature of the problem. S/N ratio characteristic values are 'Nominal is the best', 'Larger is the better', and 'Smaller is the better' (Roy 2001). The Qualitek-4 software performs the three basic steps in analysis: main effect, ANOVA, and optimum studies. Analysis can be performed using standard or S/N ratios of results for smaller, bigger, nominal, or dynamic characteristics. Since the mass removal efficiency and the cut diameter were used as indices for the evaluation of the hydrocyclone performance, the quality index of 'larger is better' and 'smaller is better' was used in the analysis based on Equations (7) and (8), respectively. In these equations, n is the number of experiments and Y is the response of the variables (Roy 2001).

$$\frac{S}{N} = -10LOG\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{Y_i^2}\right) \tag{7}$$

$$\frac{S}{N} = -10LOG\left(\frac{1}{n}\sum_{i=1}^{n}Y_i^2\right) \tag{8}$$

2.7. Confirmatory and supplementary experiments

To perform a confirmatory experiment, an optimal mini-hydrocyclone was built and tested again by the above experimental procedures. In addition, and for supplementary tests based on the scale-increasing results (row 6 in Table 2), the optimal medium-scale hydrocyclone was built and tested for a 5 L/s flow rate and the above experimental procedure. Finally, four medium-scale hydrocyclones were made, installed and tested in a Mashhad well. Figure 3 shows the conceptual flowchart of the present study.

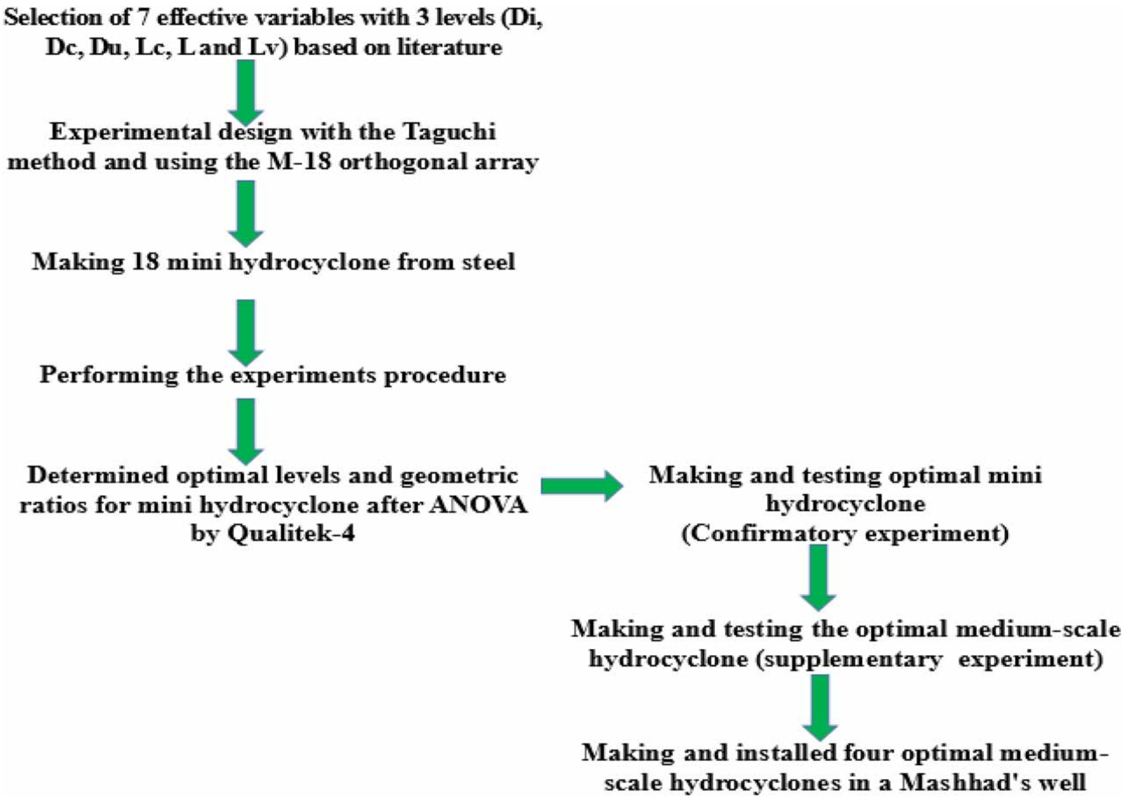


Figure 3 | Study conceptual flowchart.

3. RESULTS AND DISCUSSION

The raw results for five initial compositions of the mass removal efficiency index are given in Table 5. In addition, the results of the cut diameter of these five experiments based on an ultrasonic sieve analysis on the second replication of experiments are given in the same table.

Table 5 | Raw results of five initial experiment combinations of mass removal efficiency and cut diameter indices

	Mass removal efficiency (%)		
Trial	Replication 1	Replication 2	Cut diameter of Replication 2 (μm)
1	87.2	93.2	24
2	80.3	92.6	22
3	79.6	91.6	16
4	89.9	94.4	18
5	85.9	93	20

Along with the 'Taguchi' technique, the ANOVA method is an established method that is utilized to verify the percentage contribution (PC) of each process parameter on the desired outputs. Moreover, some ANOVA results of the mass removal efficiency and the cut diameter indices are given in Table 6. Based on the PC (shown in columns 4 and 6 of Table 6), it is specified that three first variables of the input, overflow, and underflow diameter constitute 79.2 and 82% of the result, separately in terms of the two-mode analysis of mass removal efficiency and the cut diameter, respectively. Therefore, these three factors have a significant role to play in hydrocyclone performance based on both indices. It is noted that in this study, the underflow diameter represents the angle of the conical hydrocyclone due to the presence of several diameters in Dc and Du. Yang *et al.* (2010) showed that the angle change of the cone has a significant effect on the value of some indicators such as separation efficiency, energy consumption and separation sharpness. Also, with the increasing input speed by decreasing the input diameter, the separation efficiency of suspended solids increases (Rahmani 2003).

Table 6 | Some statistical indicators of experiment analysis based on two evaluation indices

		Mass removal efficiency					Cut diameter				
Factor	Freedom Degrees	Sum of Squares	Variance	F-Ratio	Pure Sum	PC	Sum of Squares	Variance	F-Ratio	Pure Sum	PC
D_i	2	1.36	0.68	66.77	1.34	27.6	34.36	17.2	58.92	33.78	27.94
D_o	2	1.62	0.81	79.76	1.60	33.01	24.11	12.06	41.65	23.53	19.46
D_u	2	0.93	0.46	45.51	0.91	18.66	42.36	21.18	72.64	41.78	34.55
D_c	2	0.46	0.23	22.72	0.44	9.10	0.78	0.39	1.33	0.19	0.16[a]
L_c	2	0.18	0.1	9.08	0.16	3.39	2.53	1.26	4.33	1.94	1.61
L	2	0.20	0.1	9.76	0.18	3.67	4.69	2.35	8.05	4.11	3.4
L_v	2	0.07	0.04	3.47	0.05	1.03[a]	11.19	5.60	19.20	10.61	8.78
Error/ Others	2	0.029	0.009	–	–	3.57	0.87	0.29	–	–	4.10
Prediction results at optimum conditions		98.4%					12.7 μm				

[a]At 95% confidence level, the level of these factors within three defined levels is not significant, and therefore, any factor-level selection is acceptable.

Interestingly, the total PC of these three factors in two modes of analysis is very close together (79.2 and 82%.) In addition, at a 95% confidence level, the vortex finder height (L_v) and cylindrical hydrocyclone diameter (D_c) at the defined three levels do not cause a significant change in the results of the mass removal efficiency and the cut diameter, respectively. Thus, the selection of any one of their defined levels in this study was acceptable, and therefore, the use of each level did not result in a significant change in hydrocyclone performance. More investigations on the mass removal efficiency index showed that the predicted removal efficiency could be about 98.4% for the optimal hydrocyclone. To investigate this efficiency, an optimal hydrocyclone was required to be constructed and tested (confirmatory experiment). Besides predicting the optimal mass removal efficiency, an optimal cut diameter was also estimated by analyzing 18 cut diameters of the M-18 array. The results revealed that the cut diameter predicted under optimal conditions would be 12.7 μm, which suggested that the optimal hydrocyclone had an appropriate and reasonable level of efficiency in the removal of fine particles from ground water. This figure indicated that the optimal hydrocyclone based on the cut diameter index was capable of separating 50% of particles with diameters less than 12.7 μm. This diameter proved very convenient to remove sand from well water, especially in the study area. It is clear that as this diameter is small, the hydrocyclone will have better efficiency in the separation of finer particles (Svarovsky & Thew 1992; Rahmani 2005).

Further investigations indicated that this diameter was 27 μm under worst conditions (exactly the opposite of the optimal condition) based on the analysis conducted. Hence, it was at least two times bigger than the cut diameter under optimal conditions. In Table 7, the geometric dimensional ratios corresponding to optimal levels are given separately for two analyses. In addition, the geometric dimensional ratios of common hydrocyclones are compared in the form of minimum and maximum ratios in the same table.

Table 7 | Geometric dimensional ratios for two evaluation indices

	In this study		In other research (Svarovsky & Thew 1992; Rahmani 2005; Martínez *et al.* 2008)	
Geometric ratios	Mass removal index	Cut diameter index	Minimum	Maximum
D_i/D_c	0.225	0.223	0.133	0.28
D_o/D_c	0.225	0.223	0.2	0.34
D_u/D_c	0.15	0.223	0.04	0.28
L_c/L	0.12	0.14	0.048	0.214
L_v/L	0.08	0.1	0.1	0.3

A comparison of the geometric ratios specified in the current study with the ratios of common hydrocyclones shows that in most cases, the ratios of this study based on two evaluation indices are in the range of minimum and maximum values given in Table 7. Furthermore, the geometric ratios determined on the basis of the two indices show that in most cases, there are no significant differences between the results. Therefore, and for further investigation, the manufacture and experiment of the optimal hydrocyclone were done only on the basis of the dimensional geometry of the mass removal efficiency index. After making the optimal hydrocyclone, it was examined in two different replications for both indices. The results indicated that the mass removal efficiency was about 97.2 ± 1.1%, which was close to the value predicted by the Taguchi method (98.4%). This showed that the use of the Taguchi experimental design method, in spite of its limitations in investigating the interaction effects of factors, proved reliable in the present study, because the results predicted by it matched with the real results. Huang *et al.* (2021) used two mini-hydrocyclones (FX100 and FX50) with D_i, D_o, D_u, and D_c equal to 25, 32, 16–22, and 100 for FX100; and 15, 20, 6–12 and 50 for FX50. Their research results revealed that the removal rates of fine sand were 61.89 and 71.39% for FX100 and FX50, respectively. However, the results of the current study show that the optimal hydrocyclone efficiency is much higher than that described in Huang *et al.*, which can be attributed to the size of the particles and optimization in this study in relation to the research by Huang *et al.* Yu & Fu (2020) investigated the separation performance and potential for the treatment of rice starch wastewater of two mini-hydrocyclones (by D_c of 8 and 10 mm). The results revealed that the total separation efficiencies of the 8-mm mini-hydrocyclone and of the 10-mm mini-hydrocyclone for rice starch were 71.4 and 58.4%, respectively, when the operating conditions are as follows: feed flow rate = 1,440 L/h, split ratio = 0.38, and feed concentration = 0.5% (w/v). In a recent study, up to 95% of microplastics within 5–50 μm were removed from water by using several mini-hydrocyclones (Liu *et al.* 2022). The removal efficiency obtained in the Liu *et al.* study is significant and interesting considering the density of microplastics compared with sand density. For more investigations, the optimized hydrocyclone that was constructed was examined to determine its cut diameter to compare with the cut diameter in different estimation models. In the first step and by using the Taguchi method, it was predicted that the cut diameter of the optimized hydrocyclone was 16.3 μm. To verify this prediction, this diameter was assessed in two replications that determined it to be 14.5 ± 0.7 μm, which showed a slight difference with the figure predicted by the Taguchi method. A comparison of this figure with the value in Table 6 showed a slight difference in the geometric ratios of the two analyses; however, the cut diameters of the two modes had no significant difference (12.7 and 14.5 μm). This slight difference also confirms that the geometric ratios determined by the two modes were not significantly different. This was confirmed by examining the actual value of the geometric ratios for the two analyses that are shown in Table 7. For further investigation and to compare the determined cut diameter with the cut diameter predicted by various common models (Table 2), the dimensions and specifications of the optimal hydrocyclone (based on the mass removal efficiency) were used on the basis of Equations (1)–(6). In Table 8, the results of these prediction models are shown. In all these models, the density of particles was considered 2.65 g/cm^3 and the dynamic viscosity of water at 15 °C was considered 1.129° centipoise. The results of rows 1–5 show that the values predicted by conventional hydrocyclone models (by opening underflow) do not match with the values predicted by the Taguchi method (row 7) and the actual value (row 8) that was determined in this study for hydrocyclone by closed underflow. Therefore, these models cannot be applied in the hydrocyclones used for the removal of

Table 8 | Cut diameter estimation result based on different conventional models

Row	Model	Equation number	Estimated cut diameter (μm)
1	Plitt	(1)	7.9
2	Plitt (modified)	(2)	8.4
3	Luz	(3)	8.2
4	Yan	(4)	0.45
5	Valadão *et al.*	(5)	2.33
6	Scale increasing	(6)	15.4
7	Taguchi estimation value	This study	16.3
8	Actual determined value	This study	14.5

sand from water by closing underflow. The lack of match between estimated values and actual values can probably be attributed to two reasons. The first and the most basic reason is the closing or opening underflow in the hydrocyclones which is used in the removal of sand from water or in mining and mineral processing, respectively. Closing of underflow reduces the mass efficiency and increases the cut diameter (Rahmani 2005). In addition, a slight concentration of solids in removing sand from water and a relatively high concentration in mineral projects can probably be another factor that influences the actual results and the results predicted by these models. It has been observed that the performance of a hydrocyclone is affected by the solid concentration in the feed (Sabbagh *et al.* 2017). Therefore, we should be careful in using the models predicting the cut diameter for water purification projects so that they are not used without calibration. In addition to using five models (row 1–5 in Table 2), the scale-increasing model was also used (row 6 in the same table).

To use this model, constants corresponding to the actual conditions must be used. For this purpose, and by conducting several experiments of optimal hydrocyclone, the constants *np*, *kp*, and $Stk_{50}.E_u$ were determined as 0.603, 8.853, and 0.147, respectively, to be used in Equation (6). With the application of these constants in Equation (6), the cut diameter was determined as 15.4 μm, which was very close to that of the Taguchi method (16.3 μm – row 7) and that the actual measured value (14.5 μm – row 8). This showed that in the case of determination of constants based on the experiment of the considered hydrocyclone, the scale-increasing model can be used to determine the cut diameter. This model, unlike others, can be used in predicting the cut diameter of hydrocyclones used in the removal of sand from water by closed underflow. This reason can be sought in determining the considered constants according to operational conditions. In other words, closeness of underflow as one of the basic differences or other differences impact the determination of these constants. Therefore, the results of this model in predicting the cut diameter are very close to the actual values. It should be noted that the constants *np* and *kp* have been determined in the range of 0–0.323 and 316–6,381 in different mineral studies, respectively (Rahmani 2005). Ritema reported two constants 0.134 and 316 by experimenting a 75-mm hydrocyclone, while Bradley reported 0.323 and 446.5 by experimenting one hydrocyclone with a 38-mm diameter (Rahmani 2005). In addition, Mozley, in two separate studies with hydrocyclones with diameters of 22 and 44 mm, reported *np* to be 0 and *kp* to be 6,381 and 4,451, respectively (Rahmani 2005). These values indicated that the range of these constants was very open, and therefore, it became necessary that each hydrocyclone was determined, presented, and used separately. The scale-increasing model and constants *kp* and *np* were used to increase the scale of the optimal mini-hydrocyclone to medium size in order to use it in actual projects. This issue and the comparison of the results of the optimal mini-and-medium-scale hydrocyclone were investigated by the author. A medium-scale hydrocyclone was designed, manufactured, and tested for a flow rate of 5 L/s in actual conditions. The results showed that the mass efficiency, the cut diameter, and the head loss were 95 $\pm$ 2%, 21.5 $\pm$ 0.7 μm, and 0.9 $\pm$ 0.05 bar, respectively. The actual results of the medium scale hydrocyclone compared with those of the mini-scale one showed that, although the cut diameter slightly increased, the mass efficiency of the two small- and medium-scale hydrocyclones was close to each other and that increasing the scale did not have a significant effect on efficiency and hydrocyclone performance. The optimal medium-scale hydrocyclone (four hydrocyclones per well) installed in a Mashhad well is shown in Figure 4.

In addition, with the aim of predicting the cut diameter with the head loss of the optimal mini-hydrocyclone, the cut diameter altered with changes in the overall head loss of hydrocyclone is shown in Figure 5. As Figure 5 shows, a reduction in the cut diameter is possible by increasing the head loss. This means that by increasing the input flow to one hydrocyclone with certain dimensions associated with increased head loss, the cut diameter can be reduced. It is clear that this process is not linear, and by an excessive increase in flow, the cut diameter will not decrease significantly by depending on the Reynolds number. Figure 5 shows changes in the Reynolds number in the hydrocyclone inlet pipe.

Yang *et al.* (2010) showed that when the Reynolds number is too high or too low in the hydrocyclone inlet pipe, it would reduce the removal efficiency. An investigation of the Reynolds number in the inlet shows that increasing this number to values above 50,000 will have no significant impact on reducing the cut diameter. Complementary information on the impact of this number on the removal efficiency of optimal hydrocyclone has been studied by Dehnavi et al. (2018). According to Figure 5 and considering the costs of energy supply, it seems that a head loss of about 100–130 kPa (equivalent to 10–13 m-H_2O) leads to a reasonable cut diameter of about 10–12 μm. In addition, by increasing the head loss to more than this value, not only will energy supply costs increase, but also it would have no significant effect on cut diameter reduction. Further investigations showed that one exponential equation could be fitted to the changes in the cut diameter against the head loss shown in the same figure.

Figure 4 | The optimal medium-scale hydrocyclone installed in a Mashhad well.

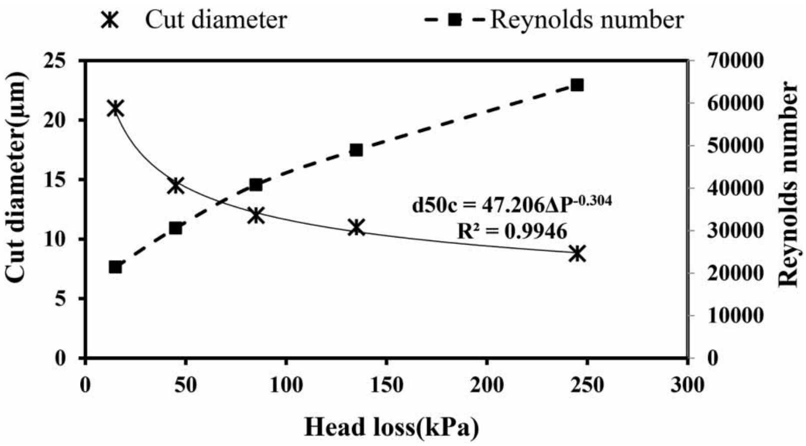


Figure 5 | Cut diameter and Reynolds number variations with total head loss.

This equation with correlation higher than 0.99 can predict the cut diameter in each head loss, and/or per each cut diameter, hydrocyclone head loss can be estimated. In this equation, the head loss is based on kPa and the cut diameter is based on micron.

4. CONCLUSION AND RECOMMENDATION

In the present study, all variables affecting the performance of hydrocyclones in the separation of sand from water were investigated. With this objective and based on the experimental design by the Taguchi method, 18 different mini-hydrocyclones were constructed and tested for 100 mg/L of suspended solid concentration for optimization. Finally, the geometric ratios under optimum conditions for two indices, namely, mass removal efficiency and cut diameter, were determined and compared. A comparison of these ratios showed that in most of the cases (except for D_u/D_c), geometric ratios based on the two indices were close to each other. Such being the case, an optimal mini-hydrocyclone based on the mass removal efficiency for the two indices was constructed and examined. The result of this investigation in two similar replications for the mass removal efficiency was 97.2 $\pm$ 1.1%, which was very close to the result predicted by the Taguchi method (98.4%). On the other hand, investigating the cut diameter in this hydrocyclone showed that this diameter was 14.5 ± 0.7 μm, which was close to the value predicted by the Taguchi method for this hydrocyclone (16.3 μm). In addition, it was determined in the present study that the common models used in the determination of the cut diameter were not applicable for a closed underflow hydrocyclone used in the removal of sand from water, but a scale-increasing model with hydrocyclone constants could be used to predict the actual cut diameter. This model with the constants 0.603, 8.853, and 0.147, respectively, for np, kp, and $Stk_{50}.E_u$, could predict a value of 15.4 μm for the cut diameter, which was very close to the actual value (14.5 μm) as well as the value predicted by the Taguchi method (16.3 μm).

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DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

CONFLICT OF INTEREST

The authors declare there is no conflict.

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