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Prediction of the discharge coefficient for the rectangular notch with different hydraulic and geometric properties

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ABSTRACT

This study aims to investigate experimentally the variation of the coefficient of discharge C_d with the rectangular notch hydraulic and geometric parameters such as water head h, notch height p, notch width B, and notch thickness t. The results show that the coefficient of discharge C_d increases with an increase of h and B while it decreases with t. There are no changes in the variation of actual discharge Q_{act} and consequently the discharge coefficient C_d with h for notch height p more than 6 cm. An empirical formula was developed based on the dimensional analysis principle that can be used to predict the coefficient of discharge C_d value for the rectangular notch with known hydraulic and geometric data (h, B, p, and t).

Key words: discharge coefficient, rectangular notch, relative notch thickness, weir: measurement

HIGHLIGHTS

- There are no Q_{act} changes and thus the discharge modulus C_{d} with h for crack height.
- The relationship between the theoretical discharge Q_{th} and vertex h to be constant for the thickness of the rectangle C_{d} increases with increasing h/p because the actual discharge Q_{act} increases with water head h.
- The discharge modulus Cd decreases with increasing slit thickness ratio t/p.

1. INTRODUCTION

A notch is defined as an obstruction in the open channel over which water flows and is considered the simple, accurate, and classical device used both in the field and in the laboratory for flow measurement in open channels based on its geometry and head on its crest (Kumar *et al.* 2011). It consists of a plate set perpendicular to the flow in a rectangular channel. The horizontal crest of the notch crosses the full channel width. This feature means that the flow is essentially two-dimensional, without lateral contraction effects (Henderson 1966). Many relationships between the head and discharge for notches were developed. Generally, the discharge Q_{act} over a sharp-crested notch under free-flow conditions in an open channel is expressed in terms of the following well-known equation (Henderson 1966).

$$Q_{\mathrm{act}} = \frac{2}{3} C_d \, B \sqrt{2g} \, h^{1.5} \tag{1}$$

where B is the notch sill width, C_{d} is the discharge coefficient, h is the head over the notch sill, and g is the acceleration due to gravity. The coefficient of discharge C_{d} depends on flow characteristics and notch geometry. The earliest experimental studies on C_{d} were carried out by Rehbock (1929). Experiments were carried out on full-width weirs and proposed the following equation for $h/p \le 5$, which does not reflect the viscous and surface tension effects:

$$C_{\mathrm{d}} = 0.611 + 0.08\frac{h}{p} \tag{2}$$

where p is the notch height.

Kandaswamy & Rouse (1957) derived a discharge coefficient similar to the Rehbock equation through a combination of the experimental measurements, as follows:

$$C_{\mathrm{d}} = \begin{cases} 0.61 + 0.08\dfrac{h}{p} & \dfrac{h}{p} < 6 \\ 1.06\left(1 + \dfrac{p}{h}\right)^{1.5} & \dfrac{p}{h} < 0.06 \end{cases} \tag{3}$$

An intermediate zone is a continuous transition between two equations. Swamee (1988) developed a generalized weir equation for sharp-crested, narrow-crested, broad-crested, and long-crested weirs by combining the equations obtained from earlier works as follows:

$$C_{\mathrm{d}} = 1.06\left\{\left(\frac{14.14\,p}{8.15\,p + h}\right)^{10} + \left(\frac{h}{h\ + p}\right)^{15} + 1,834\left\{1 + 0.2\left[\frac{(h/L)^5\ + 1,500\,(h/L)^{13}}{1 + 1,000\,(h/L)^3}\right]^{0.1}\right\}^{-10}\right\}^{-0.1} \tag{4}$$

where L is the notch length in the direction of flow. Oshima *et al.* (2013) adopted the Rehbock equation for flow calculation of full-width weirs with the limitation on weir plate height set to 1 m, and the coefficient of discharge changed slightly as follows:

$$C_{\mathrm{d}} = 0.602 + 0.083\frac{h}{p} \quad \frac{h}{p} < 4 \tag{5}$$

Kumar *et al.* (2011) conducted an experimental study on a sharp-crested weir under free-flow conditions and developed a discharge coefficient equation that is similar to the Kindsvater & Carter (1957) equation. Reviewing most of the proposed equations show that C_{d} primarily depends on the ratio h/p. Other flow characteristics may influence the discharge coefficient.

Zachoval & Rousar (2015) studied the flow characteristics over a broad-crested weir using numerical models. They found that Reynolds-averaged Navier–Stokes (RANS) equations and the two-layer shear stress transport (SST) turbulence model were suitable models. Results of their study show that numerical simulation using the Reynolds stress turbulence model gives better predictions for horizontal velocities than simulations with other turbulence models. Ghorban & Hadi (2018) experimentally examined the effect of h/y and R_{e} on the C_{d} value of a rectangular sharp-crested weir and developed a discharge coefficient equation using the optimization method. They also conducted a numerical simulation to evaluate the ability of the numerical model and analyze the flow characteristics of the notch.

Advanced numerical and experimental studies were used to investigate hydraulic phenomena. Liu *et al.* (2002) studied numerically the water surface profile on semi-circular notches using the k-ε turbulence model. Aydin *et al.* (2011) after their experimental studies proposed that the discharge in rectangular weirs can better be formulated in terms of average weir velocity, which has a universal distribution easy to fit empirically, rather than the discharge coefficient which exaggerates the experimental error by changing the curvatures. The study also proposed that for precise measurement of h, the maximum velocity in the channel should be limited to 0.55 m/s. Ferro (2012) examined the geometrical shapes of sharp-crested weirs. A stage-discharge relationship was developed for triangular sharp-crested weirs using dimensional analysis and the self-similarity theory. He concluded that a power equation can be used for establishing the stage-discharge equation with a coefficient and an exponent depending upon the weir geometry. Aydin *et al.* (2014) introduced a physical quantity known as weir velocity, i.e. the average velocity over the weir section, which is directly formulated as a function of weir geometry and head over the weir. The weir velocity plotted against the weir head has a universal behavior for constant weir width to channel width ratio which is independent of weir size. This unique behavior is described in terms of weir parameters to calculate the discharge without involving the discharge coefficient. Akoz *et al.* (2014) carried out experiments to measure the flow characteristics over a semi-cylindrical notch and compared them with those obtained numerically. Bin Shaharin (2013) showed in a numerical study that the important variable governing discharge over sharp-crested weir was the water head over weir per weir divided by weir height, *h/P*. He also highlighted the advantages of an ANSYS CFX-14 as a tool for examining velocity vectors and pressure patterns over rectangular sharp-crested weirs. In an experimental study, Zbyněk *et al.* (2014) determined a relationship for the calculation of the discharge coefficient at free overflow over a rectangular sharp-edged broad-crested weir without lateral contraction. The developed formula, expressed using the relative height of the weir, was subjected to verification made by an independent laboratory confirming its accuracy. Alwan and Al- Mohammed (2018) used a dimensional analysis technique to estimate the values of the coefficient of discharge for various rectangular notch dimensions and developed an empirical formula to estimate the discharge coefficient using a regression procedure.

Eltoukhy & Alsaydalani (2021) carried out experimental runs to study the notch thickness on the discharge coefficient for V-notch. Formulas for predicting the V-notch discharge coefficient, C_d, were developed for different vertex angles, ϴ, and then the predicted values of the discharge coefficient, C_d, using the developed formulas were plotted against the calculated values with a coefficient of determination ($R^2 = 0.9372$), showing a good agreement between the predicted and measured values.

There are no studies that deal with the rectangular notch thickness effect on the discharge coefficient. In this study, experimental runs examined the effect of the rectangular notch hydraulic and geometric data as water head h, notch width B, notch height p, and notch thickness, t on its discharge coefficient C_d value. Based on the analysis of the experimental results with the use of the dimensional analysis principle a new empirical equation is developed for predicting the coefficient of discharge C_d for given rectangular notch data (h, B, p, and t).

2. EXPERIMENTAL WORK

The experimental runs were carried out using a rectangular flume 4.0 m long, 0.30 m wide, and 0.50 m height. The Nontilting type was used in the experimental runs. The experimental setup is a self-contained one having a closed cycle for the water. A three-horse power pump makes the water circulating mechanism work efficiently in the flume. The flume is installed with glassy sheets as side walls which make the viewing of the experimental run easy. At the entrance into the channel, baffle vertical plates were installed to prevent vortex motion and regulate the flow to control the damp fluctuations at the entry of the flume. The water after the notch was then collected into a hydraulic bench (Figure 1). Actual discharge was calculated using the hydraulic bench by dividing the collected water volume in the hydraulic bench by the corresponding time. For each experimental run, the actual discharge was calculated as the average of three recording discharge values. A vernier-type gauge with accuracy ± 1 mm was used for measuring the bed elevation and water surface elevation. Calibration was done before every experimental run to avoid instrumental errors. The depth rod was adjusted accurately to the surface

Figure 1 | The flume and the hydraulic bench.

of the water to get the value of 'h'. While measuring h it was ensured that the flow in the channel was stable and constant. Discharge is maintained for an individual experimental run. Rectangular notch plates made up of fiberglass with different thicknesses were used for the experimental study. Different notch sections were selected having different sill widths B, notch thickness t, and the notch crest height P. The used notches have sill widths B of 3, 4, 6, and 8 cm, notch thicknesses t of 1, 3, 4, and 6 mm and notch crest heights P was varied as 4, 6, 8, and 10 cm only for $t = 1$ mm and $B = 3$ cm for estimating P that has a negligible effect on the discharge coefficient C_d. According to Bos (1989), the minimum notch height P was suggested as 10 cm.

Once the calibration process was completed the accuracy of discharge measurement depends on the measurement of water level h on the upstream side of the notch. The point gauge with a vernier scale having accuracy ± 1 mm was used for measuring the water level. The point gauge was fixed at an upstream distance of four times the maximum head over the notch (Bos 1989). Because the bottom boundary affects the nature of flow crossing through the weir section should be a free-flow condition, therefore notch section is used for discharge measurement and discharge can be determined by measuring the head over the notch.

2.1. Rectangular notch models

Sixteen rectangular notch models were fabricated from acrylic glass sheets with different sill widths B and thicknesses t, Table 1. Figure 2 shows the form for each model, where h is the water head, H is the height of the notch plate above the sill level, and P is the notch height. Four rectangular notches with sill widths B of 3, 4, 6, and 8 cm each with thicknesses of 1, 3, 4, and 6 mm were used in this study on different models. The experimental programs are indicated in Table 1.

Table 1 | The experimental programs

Sill width, *B* cm	Notch plate thickness, *t* mm	Notch height, *p* cm	Water heads *h* cm
3	1 3, 4, and 6	4, 6, 8, and 10 8	Six head values
4	1, 3, 4, and 6		
6	1, 3, 4, and 6		
8	1, 3, 4, and 6		

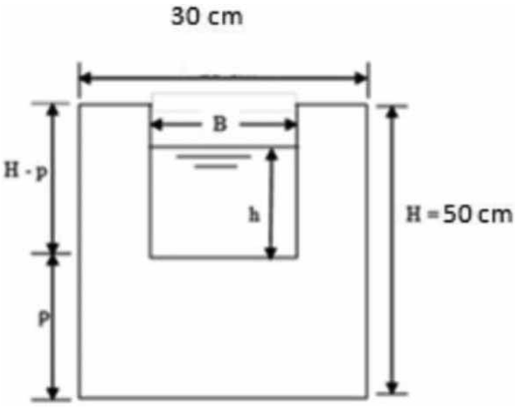


Figure 2 | General view of the rectangular notch.

2.2. Tests procedure

The Experimental runs procedures for each rectangular notch model were carried out as follows:

1. Installing the notch model in its allocated position at the channel of a hydraulic bench.
2. Adjust the control valve to establish the flow rate pumped to the bench.
3. After developing a stable flow, head over notch, h was measured about $4h_{max}$ away from the upstream of the weir where max is the maximum head over the weir (Franzini & Finnemore 1997), and theoretical discharge

was calculated,

$$Q_{\mathrm{th}} = \frac{2}{3} B \sqrt{2g}\, h^{1.5} \tag{6}$$

4. Recording the volume of water, V accumulated in the bench tank over time, T for each run and actual discharge was calculated, $Q_{\mathrm{act}} = V/T$.
5. Determining the coefficient of discharge, $C_{\mathrm{d}} = Q_{\mathrm{act}}/Q_{\mathrm{th}}$
6. Repeat the procedures from points 2 to 5 for further runs.
7. Repeating steps 2–6 for other notch models as indicated in Table 1.

3. DIMENSIONAL ANALYSIS

The discharge coefficient, C_{d} of the rectangular notch is a function of several parameters which is mathematically expressed by Equation (2):

$$C_{\mathrm{d}} = f(\rho, s, \mu, g, h, p, B, t) \tag{7}$$

where C_{d} is the discharge coefficient, ρ is the water density, σ is the surface tension, μ is water viscosity, h is the head over the notch sill, B is the notch width, p is the notch height, t is the notch plate thickness, and g is the gravitational acceleration. A dimensional analysis is performed to find a relation between the discharge coefficient and other parameters stated above:

$$C_{\mathrm{d}} = f\left\{R_{\mathrm{e}},\ W_{\mathrm{e}},\ \frac{h}{B},\ \frac{h}{p},\ \frac{h}{t}\right\} \tag{8}$$

where R_{e} is the Reynolds number and W_{e} is the Weber number. In most practical cases, however, the Reynolds and Weber numbers effects are negligible for water at normal temperatures and notch geometry is the main element.

Table 2 | Calibration of the bench volume tank, $t = 1$ mm, $p = 8$ cm, and $B = 3$ cm

h (cm)	*V* (l)	*T* (s)	Jar vol. (l)	Jar time (s)	Q_{act} (l/s)	Jar Q_{act} (l/s)	Error
2.74	3	15.85	3	16.17	0.189274	0.1855	2%
3.75	3	9.63	3	9.84	0.311526	0.3047	2.2%
4.88	3	6.28	3	6.39	0.477707	0.4691	1.8%

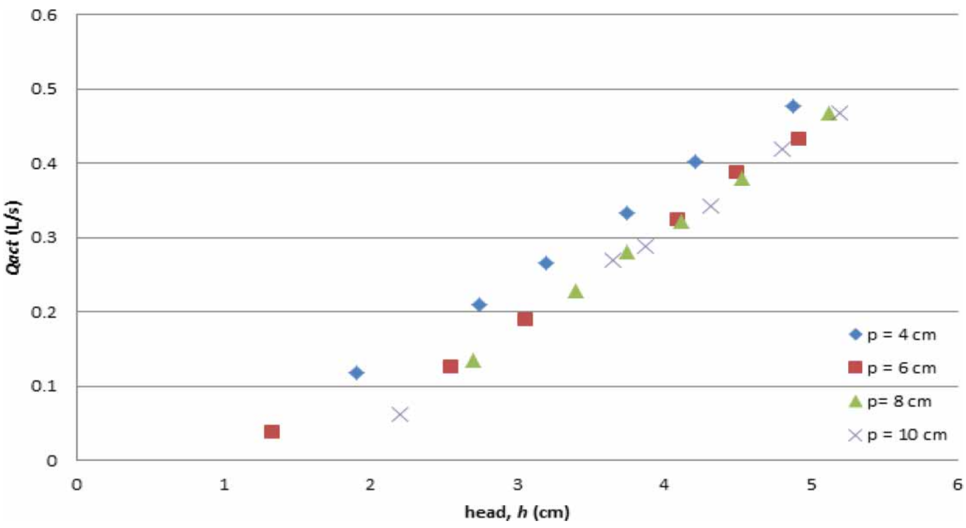
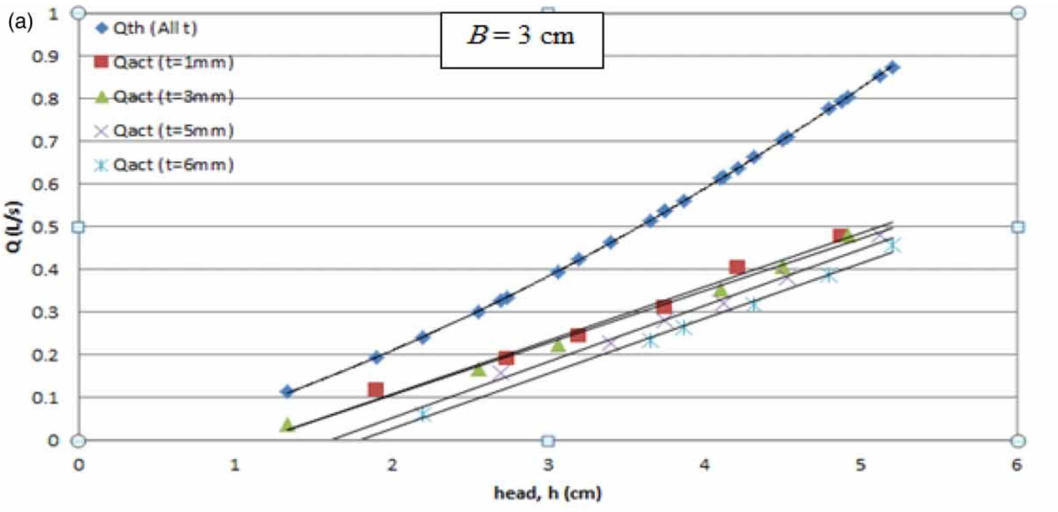
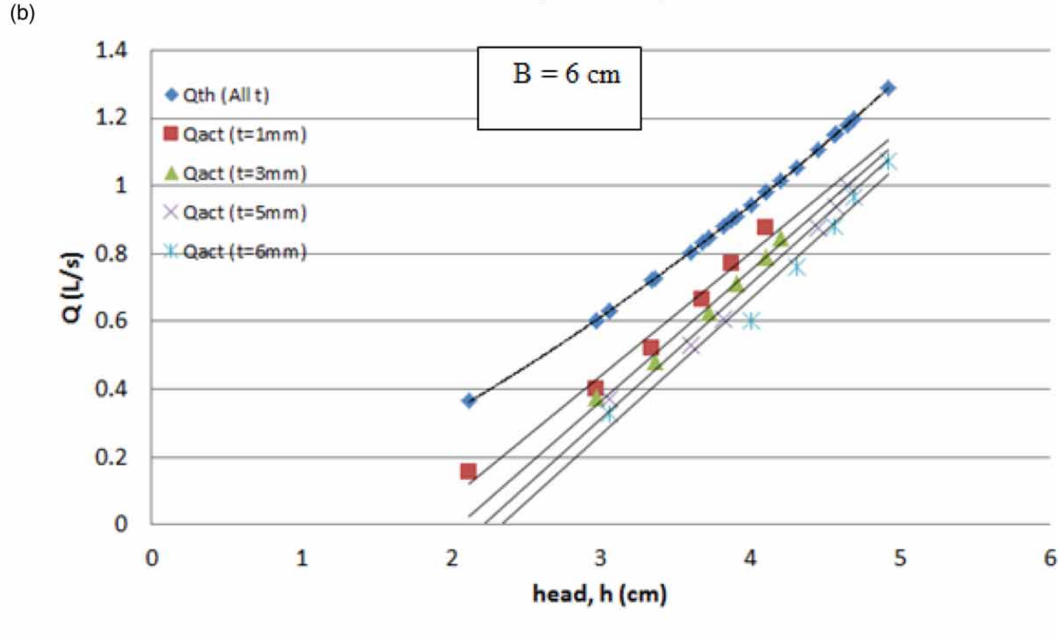


Figure 3 | Actual discharge, Q_{act} vs notch head over, *h*.



notch width ($B = 3$ cm)



notch width ($B = 4$ cm)

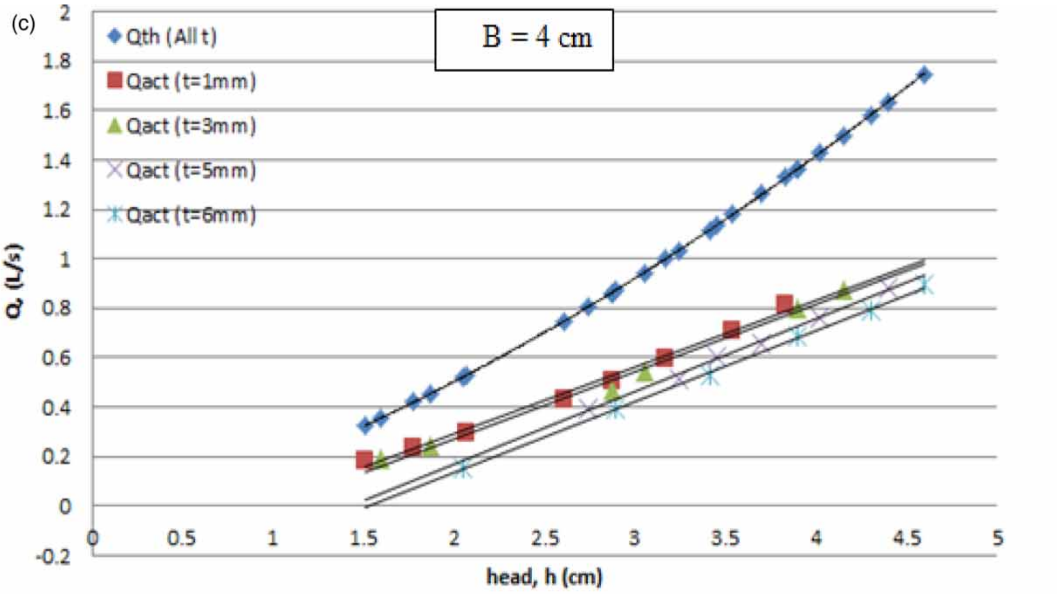
Figure 4 | Head, theoretical and actual discharge relationships for different notch width B and thickness t. (a) Notch width ($B = 3$ cm). (b) Notch width ($B = 4$ cm). (c) Notch width ($B = 6$ cm). (d) Notch width ($B = 8$ cm). (*continued.*).

4. RESULTS AND DISCUSSION

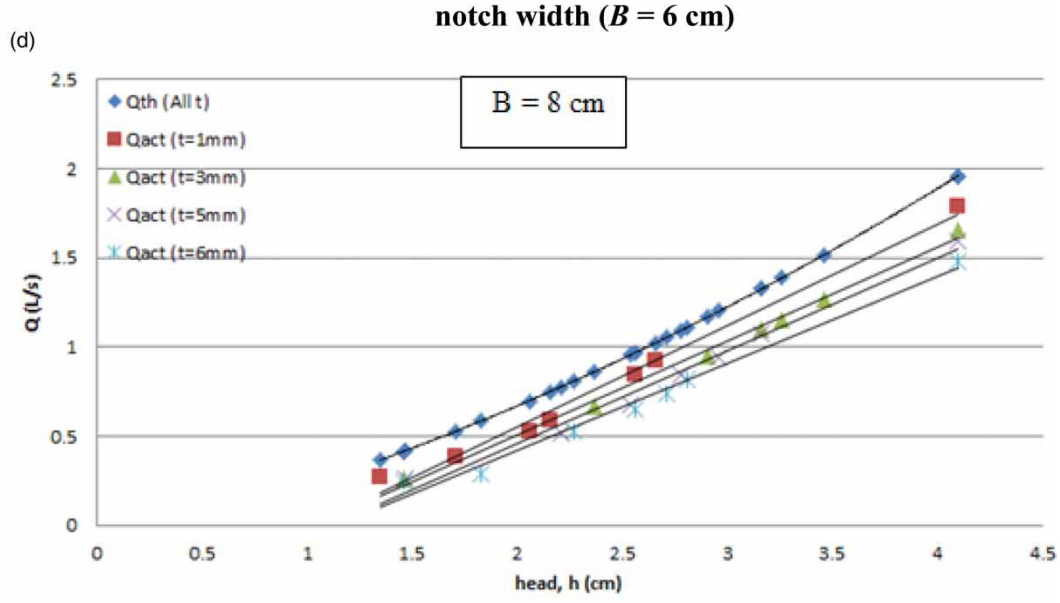
Before the commencement of the experimental runs, the volume of water, V accumulated in the bench tank over time, T which was used to calculate the actual discharge as $Q_{act} = V/T$, was calibrated with the use of a graduated jar. The calibration process was carried out three times for the same head h and the average water volume and corresponding time values were compared with V and T. The results were consistent as shown in Table 2.

4.1. Notch height *p*

First of all, for the prediction of discharge coefficient, C_d for the rectangular notch with different thicknesses, 24 experimental runs were carried out to estimate the notch height, p which is used for achieving the purpose of this study. Notch models with thickness, t of 1 mm, notch height, p values of 4, 6, 8, and 10 cm, and six head, h values for each p were used for p estimating. The actual discharge, Q_{act} – head relationships were presented in Figure 3,



notch width ($B = 6$ cm)



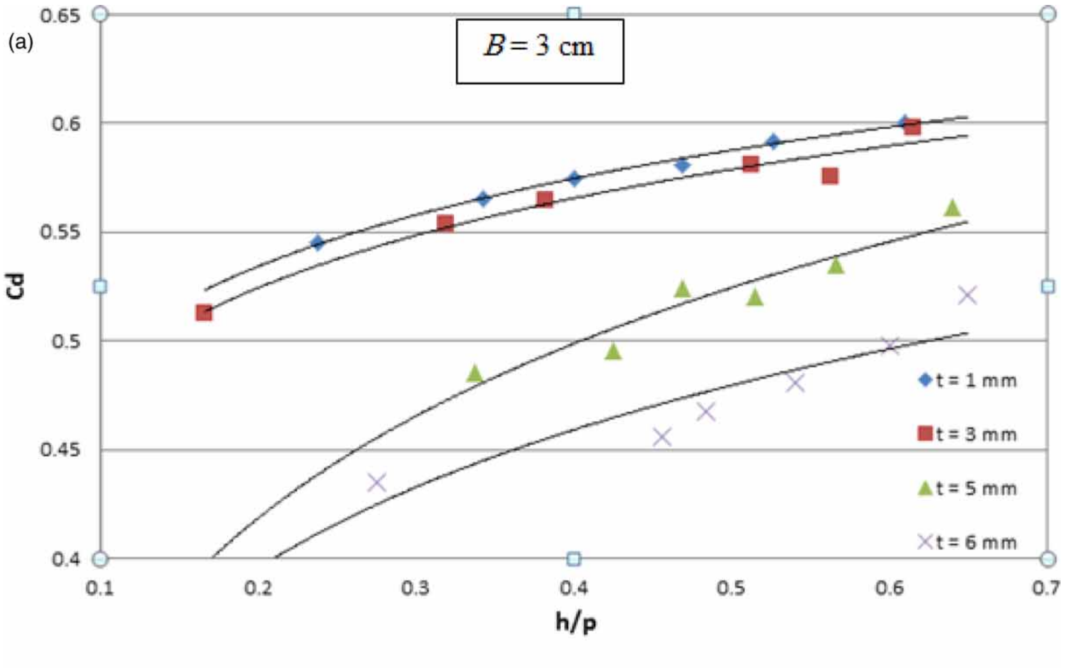
notch width ($B = 8$ cm)

Figure 4 | Continued.

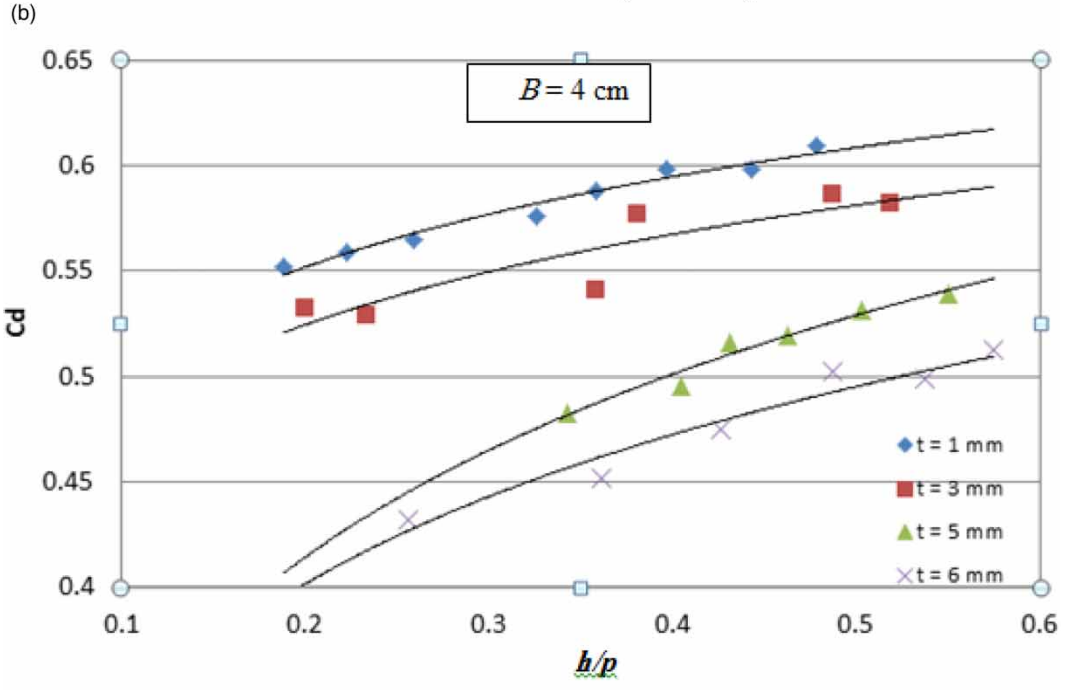
which shows that the actual discharge, Q_{act} increases with the head, h. Also, it was shown that there are no changes in the variation of Q_{act} and h for p more than 6 cm i.e. $p = 8$ and 10 cm. The notch height, $p = 8$ cm was selected to diminish the effect of the bottom boundary. So the discharge coefficient, C_d will become independent of the value of h/p. This is consistent with Sisman's (2009) and Bos's (1989) observations. Thus, any p-value greater than the recommended value will hydraulically imply that the flow over the notch is no longer relying on the height of the notch. In addition, it is realizable that the chosen p may remain valid for the experimental range of water heads only. Above this range, it can be expected that larger notch plate heights might be required to suppress boundary layer development.

4.2. Actual and theoretical discharges variation with head

Once the notch height was decided to be kept at 8 cm, experimental runs continued with different notch widths. There were four different notch widths were tested in this study ($B = 3$, 4, 6, and 8 cm) and results were recorded. Figure 4 shows the obtained results points for actual Q_{act} and theoretical Q_{th} discharges at different water heads,



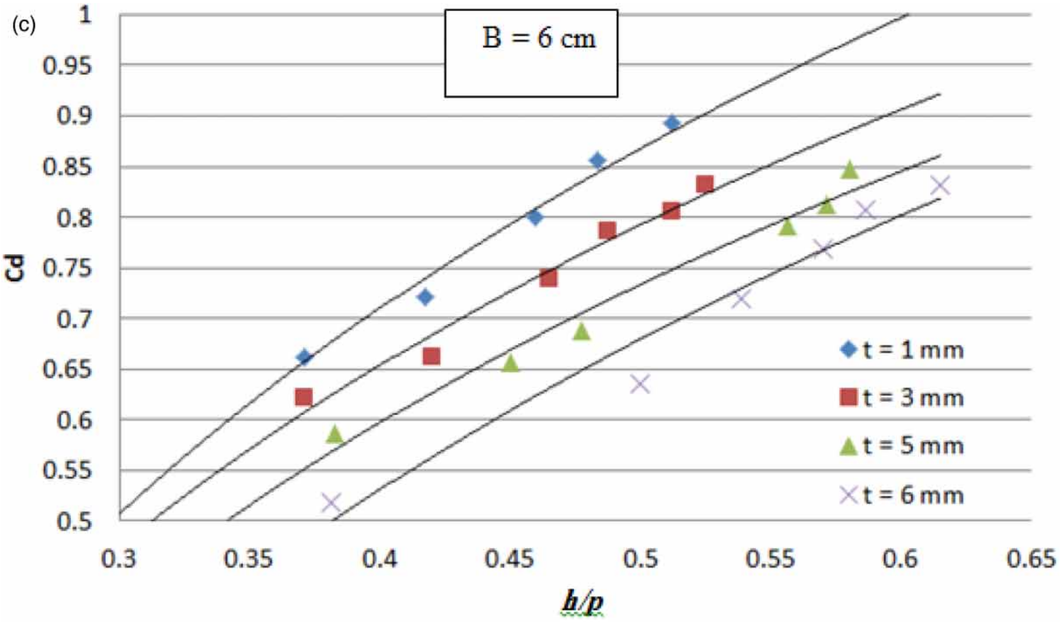
notch width ($B = 3$ cm)



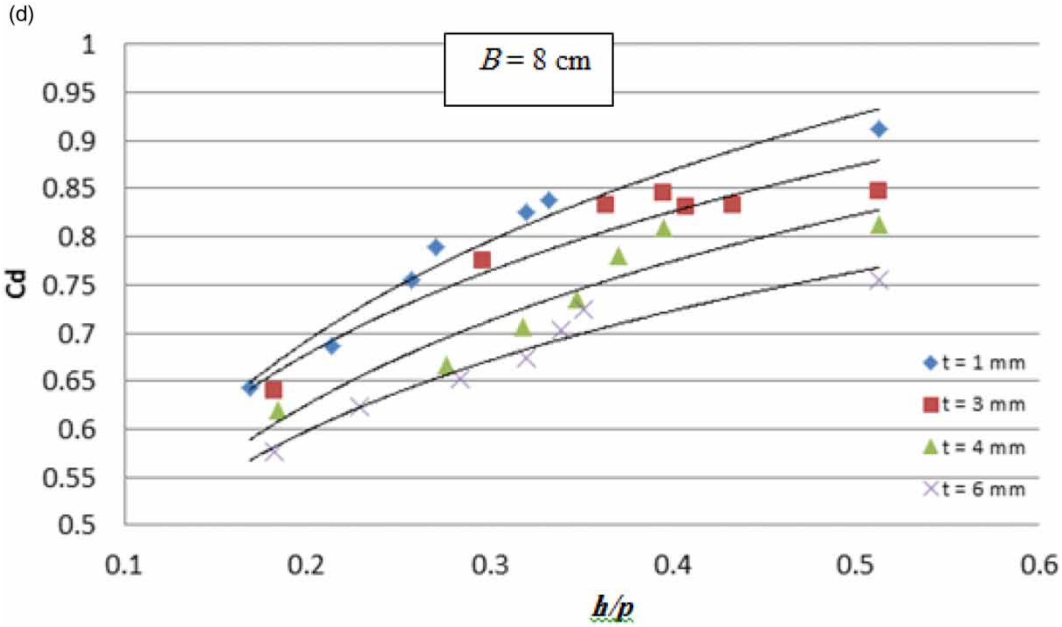
notch width ($B = 4$ cm)

Figure 5 | Discharge coefficient variation with h/p for different notch width B and thickness t. (a) Notch width ($B = 3$ cm). (b) Notch width ($B = 4$ cm). (c) Notch width ($B = 3$ cm). (d) Notch width ($B = 3$ cm). (*continued.*).

different notch widths B, and notch plate thickness t. All discharge–head relationships have the same trend and the theoretical discharge Q_{th} is independent of t, as in Equation (6). On the other hand, the Q_{th} and Q_{act} increase as the notch width B increase. The theoretical discharge Q_{th} increases with B with the same increasing ratio. For example at the same head h and notch thickness t, increasing B from 4 to 6 cm (50%) the Q_{th} at $B = 6$ cm equals $6/4 \times Q_{th}$ at $B = 4$ cm (50%). But the actual discharge Q_{act} increases with a different ratio, for example at $h =$ 3.8 cm and $t = 1$ mm, increasing B from 4 to 6 cm (50%), the Q_{act} increases from 0.810811 to 0.898356 l/s (10.79%).



notch width ($B = 3$ cm)



notch width ($B = 3$ cm)

Figure 5 | Continued.

4.3. Variation of the discharge C_d coefficient with *h/p*

Figure 5 shows the variation of the discharge coefficient C_d with h/p for different notch widths B and thicknesses t. It can be seen that C_d which equals Q_{act}/Q_{th} changes abruptly with changes in h/p. At the same notch width B and notch thickness t, C_d increases as h/p increases this is because the actual discharge Q_{act} increases with water head h with a value more than that of the theoretical discharge Q_{th}. For example for $B = 4$ cm and $t = 1$ mm, for increasing h/p from 0.2588 to 0.4788 (0.85%), Q_{act} increases from 0.293 to 0.81081 l/s (177%), and Q_{th} increases from 0.5276 to 1.328 l/s (152%).

4.4. Effect of the notch thickness ratio *t/p* on the discharge coefficient C_d

Throughout the analysis of the obtained results of the experimental runs using the different notch models with thicknesses of 1, 3, 4, and 6 mm, the effect of the notch thickness ratio t/p on the discharge coefficient C_d is presented in Figure 6. It can be seen that the discharge coefficient and the notch thickness ratios have the same trend for different notch widths, where the discharge coefficient decreases as the notch thickness ratio increases. For example, at $h/p = 0.512$, increasing the notch thickness t by 100% i.e. from 3 to 6 mm results in decreasing in the actual discharge Q_{act} by 17.22, 13.08, 21.13, and 10.9% for notch width B of 3, 4, 6, and 8 cm, respectively, and decreasing the discharge coefficient C_d by the same ratios of the actual discharge decreasing. This is because the theoretical discharge has a constant value at a given head for different notch thicknesses.

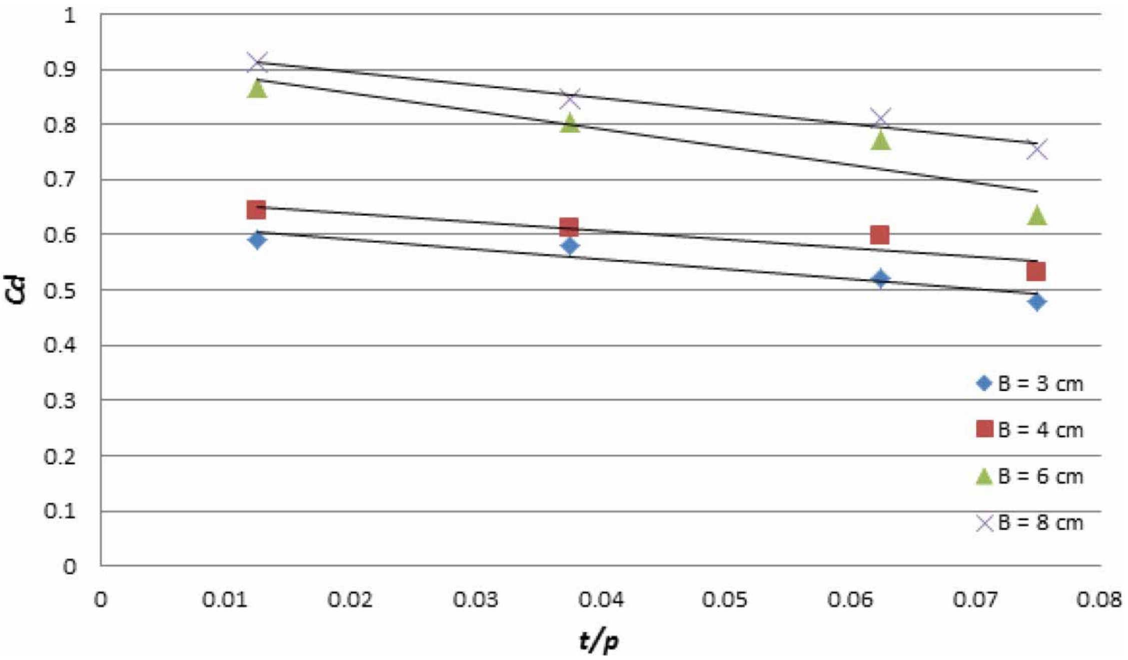


Figure 6 | Variation of discharge coefficient C_d with notch thickness ratio t/p for different notch width B.

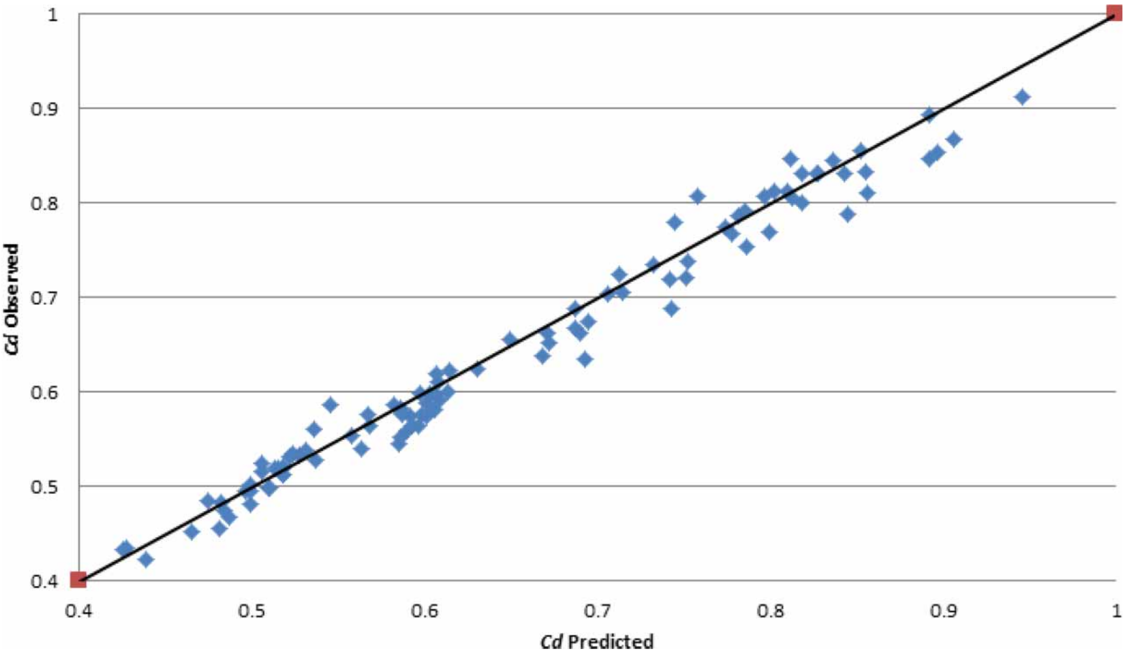


Figure 7 | Comparison of formula (9) with the experimental results for discharge coefficient.

4.5. Discharge coefficient based on dimensional analysis technique

Now the effect of the notch thickness t was checked on the discharge coefficient C_d. Then, the results from the four notch thicknesses were analyzed. Using nonlinear regression analysis and all the observed data, the relation for Equation (8) can be found and the suggested relation is shown in Equation (9) (Figures 7 and 8).

$$\text{percent error} = \frac{\text{observed } C_d - \text{predicted } C_d}{\text{observed } C_d} \times 100 \tag{9}$$

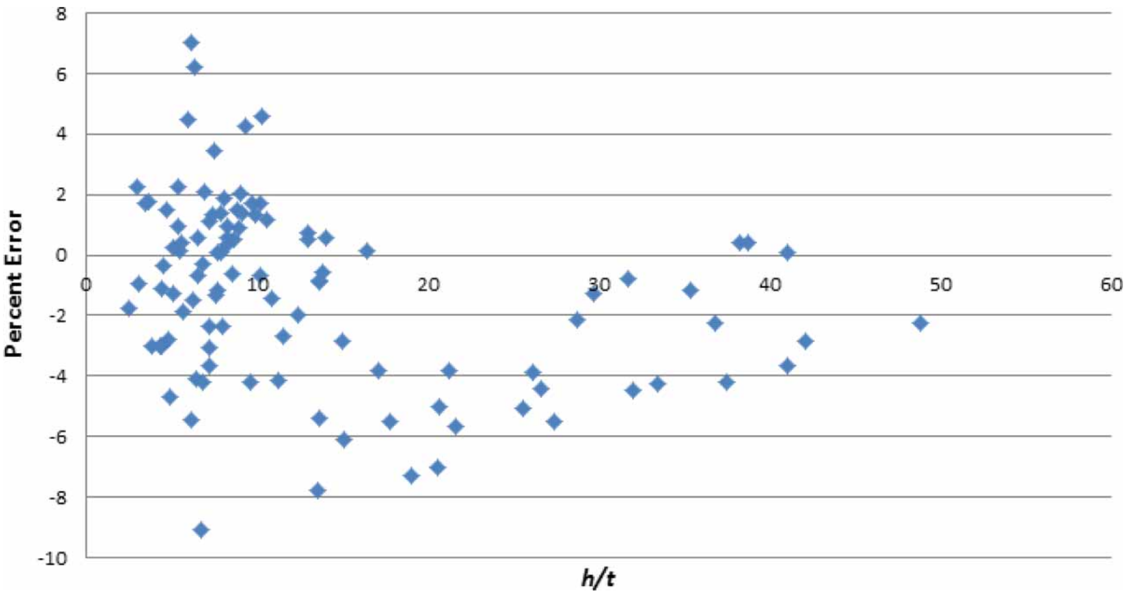


Figure 8 | Relative error of the discharge coefficient C_d in relationship to the relative notch thickness h/t.

5. CONCLUSIONS

A series of experiments were carried out to investigate the effect of water head, h, and rectangular notch geometry (width B, height p, and thickness t) on the discharge coefficient C_d values. Based on the analysis of the experimental run results of this study the following conclusions were obtained:

- There are no changes in the variation of actual discharge Q_{act} and consequently the discharge coefficient C_d with h for notch height p more than 6 cm, i.e. $p = 8$ and 10 cm.
- Theoretical discharge Q_{th} – head h relationship was found to be constant for different rectangular notch thicknesses t but the actual discharge Q_{act} – head h relationship varies with the notch thickness.
- Theoretical discharge Q_{th} increases with the same percent as the rectangular notch width B increases, but the actual discharge increases with different percent.
- At the same notch width B and notch thickness t, C_d increases as h/p increases; this is because the actual discharge Q_{act} increases with water head h with a value more than that of the theoretical discharge Q_{th}.
- The discharge coefficient C_d decreases as the notch thickness ratio t/p increases.
- The developed empirical formula Equation (9) can be used for predicting the discharge coefficient C_d value for the rectangular notch with known hydraulic and geometric data (h, B, p, and t).

DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

CONFLICT OF INTEREST

The authors declare there is no conflict.

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