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Estimating the energy dissipation of flow passing over triangular and trapezoidal plan weirs using the GMDH model

Abbas Parsaie[a,*], Sadegh Dehdar-Behbahani[b], Aaron Anil Chadee[c] and Amir Hamzeh Haghiabi[d]

[a] Faculty of Water and Environmental Engineering, Shahid Chamran University of Ahvaz, Ahvaz 6135783151, Iran
[b] Hydraulic Structures, Shahid Chamran University of Ahvaz, Ahvaz 6135783151, Iran
[c] Construction Management, University of the West Indies, St. Augustine Campus, St. Augustine, Trinidad, Trinidad and Tobago
[d] Water Engineering Department, Lorestan University, Khorramabad, Iran
*Corresponding author. E-mail: Parsaie@scu.ac.ir

ABSTRACT

Weirs are one of the most common hydraulic structures used in water engineering projects. In this research, a group method of data handling (GMDH) was developed to estimate the energy dissipation of the flow passing over the labyrinth weirs with triangular and trapezoidal plans. To compare the performance of this model with other types of soft computing models, a multilayer perceptron neural network (MLPNN) was developed. The dimensionless parameters derived from dimensional analysis, including the relative upstream head (h_o/P), the number of cycles (N_{cy}), the Froude number (F_r), and the magnification ratio (M_r) were used as input variables. The error statistical indicators of the GMDH model in the training phase were $R^2 = 0.913$, RMSE = 0.010, and in the testing phase were $R^2 = 0.829$, RMSE = 0.015. The error statistical indicators of the MLPNN in the training phase were $R^2 = 0.957$, RMSE = 0.007, and in the testing phase were $R^2 = 0.945$, RMSE = 0.009. Examining the structure of the GMDH network shows that h_o/P, N_{cy}, and M_r play more meaningful roles in the development network.

Key words: GMDH, labyrinth weir, magnification ratio, MLPNN, soft computing models

HIGHLIGHTS

- Development of GMDH to predict the energy dissipation of the flow passing over the labyrinth weirs.
- Developing the MLPNN model to predict the energy dissipation of the flow passing over the labyrinth weirs.
- Definition of most effective parameters on the mechanism of energy dissipation of flow.
- Comparison of the performance of MLPNN and GMDH models based on the Taylor diagram.

NOTATIONS

h_o/P	Relative upstream head
h_o	Head of flow over the crest
Fr	Froude number
L_{cr}	Total crest length
L_{cy}	Length of one key-cycle
M_r	Magnification ratio
N_{cy}	Number of cycles
R^2	Coefficient of determination
Cd	Discharge coefficient
E	Total flow energy
EDR	Energy dissipation ratio
g	Acceleration due to gravity
GMDH	Group method of data handling
MLPNN	Multilayer perceptron neural network
P	Weir height
RMSE	Root mean square error
V	Flow velocity
q	Discharge per channel unit width
ρ	Density

1. INTRODUCTION

Weirs are one of the most commonly used hydraulic structures for flow measurement, water level regulation, flow control in water conveyance channels and irrigation and drainage networks, and water desalination and brine treatment applications (Waller & Yitayew 2015; Panagopoulos & Giannika 2022). The weirs should be chosen based on their discharge capacity and their ability to dissipate energy. The discharge capacity of weirs is proportional to the crest length, upstream head, and discharge coefficient (Parsaie & Haghiabi 2019). One way to increase the discharge capacity of weirs based on the crest length is to use nonlinear weirs. Usually, due to the limitation of the width of the waterway, the crest length is increased (increasing the crest length at a fixed width of the waterway) (Carollo Francesco *et al.* 2012). Recently, various types of labyrinth weirs such as triangular, trapezoidal, diagonal, and parabolic have been proposed. Such weirs are usually made of one or more key cycles. Tullis *et al.* (2007) conducted several experimental studies on the labyrinth weirs with a trapezoidal plan and crest angles of 6–18 degrees. They derived the stage-discharge curve and proposed a formula for discharge coefficient (Cd) as a function of the upstream head, weir height, crest length, and angle between the key cycles. Kumar *et al.* (2011) conducted an experimental study on a triangular plan labyrinth weir. They demonstrated that, by decreasing the angle of the weir crest, the length of the interference zone increases, and the Cd decreases significantly. They also presented a formula for calculating the Cd under different vertex angles.

However, weirs create a local disturbance in the flow structure, which causes flow energy dissipation. Estimating this feature helps to calculate the amount of flow energy downstream of the weir, which is necessary for the design of the downstream concrete slab (Haghiabi *et al.* 2022). The flow energy dissipation mechanism has been investigated in many hydraulic structures such as stepped spillways (Salmasi & Abraham 2023) and ski jump buckets (Daneshfaraz *et al.* 2021; Mollazadeh Sadeghion *et al.* 2022). Mohammadzadeh-Habili *et al.* (2018) investigated the energy dissipation mechanism of the flow regime on labyrinth weirs. They declared that the flow energy dissipation decreases linearly with increasing critical depth. Ghaderi *et al.* (2020) investigated the discharge coefficient and the flow energy dissipation of the labyrinth weir using the computational fluid dynamics technique using flow-3D software. They stated that the discharge coefficient varies between 0.4 and 0.8 considering the range of relative upstream head between 0.15 and 0.7. The labyrinth weir can dissipate between 0.6 and 0.3 of the upstream energy.

Literature review shows that a few studies have been conducted on the performance of soft computing models to estimate the energy dissipation of flow passing over labyrinth weirs. Therefore, in this research, the soft computing models including the group method of data handling (GMDH) and the artificial neural network (ANN) model are developed to estimate the performance of labyrinth weirs with triangular and trapezoidal plans. Noted that the mentioned soft computing models have already been successfully used in other aspects of hydraulic engineering (Singh *et al.* 2021), especially the energy dissipation of flow over the other hydraulic structures such as stepped spillways(Parsaie *et al.* 2018; Parsaie & Haghiabi 2021).

2. MATERIALS AND METHODS

The plan and side view of the labyrinth weirs with triangular and trapezoidal plans are shown in Figure 1. In this figure, the important hydraulic and geometrical parameters involved in the flow energy dissipation mechanism are presented, as well.

To calculate the performance of this structure, Bernoulli's equation is used for the upstream (Equation (1)) and downstream (Equation (2)) sections:

$$E_0 = y_0 + \frac{q^2}{2g(y_0^2)} \tag{1}$$

$$E_1 = y_1 + \frac{q^2}{2gy_1^2} \tag{2}$$

where q is the unit discharge per channel width. E_0 and E_1 are the total flow energy upstream and downstream of the weir. The performance of labyrinth weirs in terms of energy dissipation ratio (EDR) is estimated using Equation (3):

$$\mathrm{EDR} = \frac{\Delta E}{E_0} = \frac{E_0 - E_1}{E_0} = 1 - \frac{E_1}{E_0} \tag{3}$$

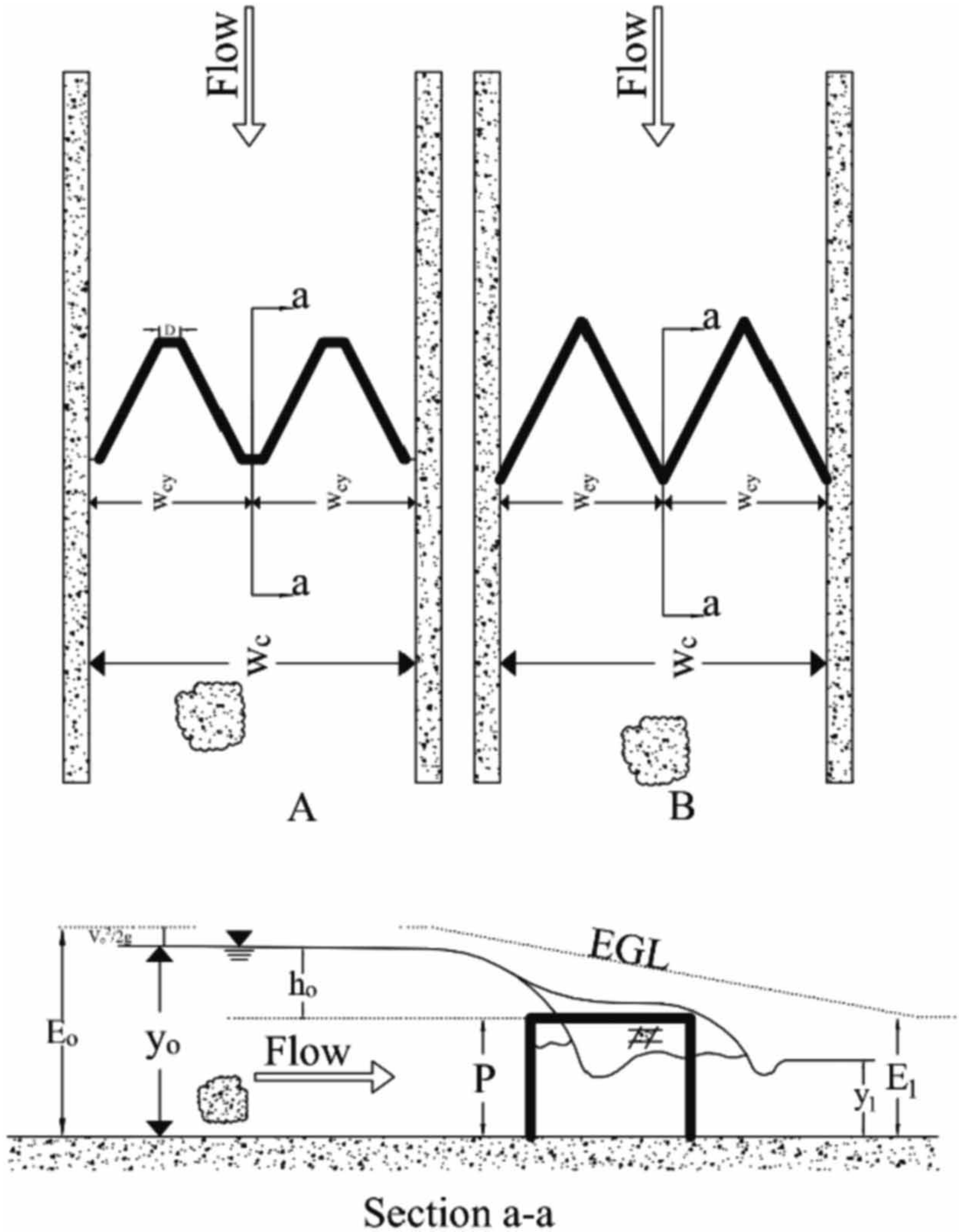

Figure 1 | Plan and side views of labyrinth weirs with triangular and trapezoidal plans, W_{cy}: width of a key-cycle, W_c: channel width, P: weir height, h_o: depth of flow on the crest, and y_o and y_1 are respectively the depth of flow upstream and downstream of the weir (Haghiabi *et al.* 2022).

The geometrical and hydraulic parameters involved in energy dissipation are summarized in Equation (4):

$$\mathrm{EDR} = f(W_c, W_{cy}, L_{cy}, L_{cr}, P, V_o, h_o, g, \rho) \tag{4}$$

where v_o is the upstream flow velocity L_{cy} is the length of one key-cycle, and L_{cr} is the total crest length. Using Buckingham's theorem as a dimensional analysis technique and taking into account V_o and P as repeated parameters, the dimensionless parameters are obtained as Equations (5) and (6). Therefore, Equation (6) can be rewritten as Equation (7):

$$\begin{aligned}
&\prod_1 (h_o) = \rho V_o P(h_o) = \frac{h_o}{P}\\
&\prod_2 (L_{cy}) = \frac{L_{cy}}{P}\\
&\prod_3 (W_c) = \frac{W_c}{P}\\
&\prod_4 (W_{cy}) = \frac{W_{cy}}{P}\\
&\prod_5 (L_{cr}) = \frac{L_{cr}}{P}\\
&\prod_6 (g) = \frac{V_o^2}{gP} \Rightarrow \sqrt{\prod_4 (g)} = \frac{V_o}{\sqrt{gP}} = \mathrm{Fr}\\
&\prod (L_{cr}) \times \frac{1}{\prod (W_c)} = \frac{L_{cr}}{P} \times \frac{P}{W_c} = \frac{L_{cr}}{W_c} = M_r\\
&\prod (L_{cr}) \times \frac{1}{\prod (L_{cy})} = \frac{L_c}{L_{cy}} = N_{cy}\\
&\prod (W_c) \times \frac{1}{\prod (W_{cy})} = \frac{W_c}{W_{cy}} = N_{cy}
\end{aligned} \tag{5}$$

$$\xrightarrow{\text{Subcritical flow}} \frac{\Delta E}{E_0} = f\left(\frac{h_o}{P}, N_{cy}, \mathrm{Fr}, \frac{L_{cy}}{W_{cy}} : M_r\right) \tag{6}$$

In Equation (6), h_o/P, N_{cy}, and M_r, respectively, indicate the ratio of flow head to weir height (relative upstream head), number of cycles, and magnification ratio.

2.1. Group method of data handling

The GMDH is an inductive approach based on the perceptron theory, which has been developed to identify and model complex systems (Ivakhnenko 1968). Unlike the perceptron-type structure, this model uses useful and non-useful information classification and requires less observational data; consequently, the training time is reduced. Figure 2 shows a schematic of the GMDH network. The governing function of each neuron in this model is a polynomial of degreed (Equation (7)):

$$Y = w_0 + w_1x_1 + w_2x_2 + w_3x_1^2 + w_4x_2^2 + w_5x_1x_2 \tag{7}$$

In this equation, x_1 and x_2 are inputs and y is the output. The external criterion for determining the network structure is defined using root mean square error (RMSE).

The six coefficients of the governing function of each neuron in the network are derived using the least squares approach. These steps (Assigning pairs of input variables to each neuron and deriving their coefficients) are repeated for all the neurons of the first layer and also for all the neurons of the next layers. After obtaining the coefficients from the training data, the accuracy of the neurons is calculated using the RMSE index (RMSE is calculated using the GMDH outputs and the observed data). Only the neurons with higher accuracy than the threshold of error-index value are selected to contribute to the network making (Parsaie *et al.* 2021; Yonesi *et al.* 2022).

2.2. Multilayer perceptron neural network

A neural network can be proposed as an alternative in many simulations that lead to solving a system of complex equations to find the relationship between factors affecting the system. The multilayer perceptron neural network (MLPNN) is the most widely used soft computing model in hydraulic engineering. The structure of the MLPNN consists of an input layer, one or more hidden layer(s), and an output layer. The inputs of the MLPNN are

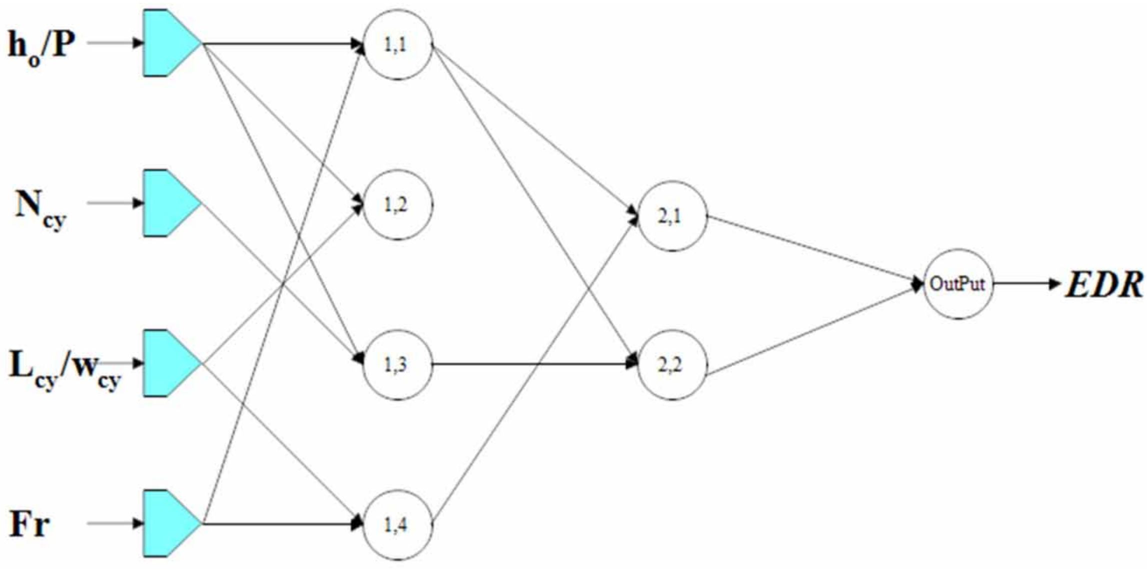

Figure 2 | The structure of the GMDH model developed to estimate EDR.

multiplied by coefficients (weight) and then added with a fixed value (bias). Then the transfer function is acted on its result. The design of the multilayer neural network model includes several steps, which include: defining the number of model layers including the input layer, hidden layer(s), an output layer, the number of neurons in each layer, defining the active transfer function, and finally choosing the training method. The purpose of MLPNN training is to determine the values of weights and constants that are multiplied and added to each input (Nou *et al.* 2022; Shen *et al.* 2022). An example of an MLPNN model is shown in Figure 3.

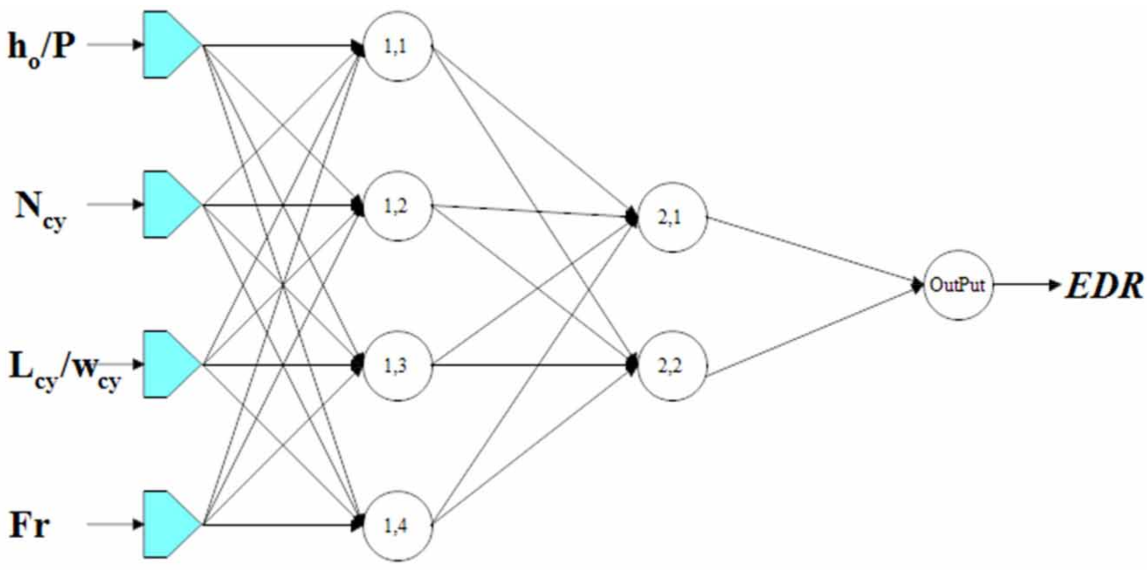

Figure 3 | Schematic view of the structure of an MLPNN.

2.3. Approaches of modeling and data

Soft computing models are data-driven models. This means that to use a soft computing model, information and data related to the under-study phenomenon be collected. In this regard, the results of Mohammadzadeh-Habili *et al.* (2018) were used to develop (calibration: training and validation: testing) both applied soft computing models. The collected data was divided into two categories: training and testing. According to the nature of the problem, data can be randomly assigned to each category. Usually, 80% of the data is allocated to training and the remainder (20%) to testing. The range of collected data in the training and testing phases of the model is presented in Table 1.

Table 1 | Statistical characteristics of data allocated for the training phase

Parameters	Min	Max	Average	St. Dev.
Training				
h_o/P	0.012	0.063	0.034	0.015
N_{cy}	1.000	2.000	1.488	0.503
L_{cy}/W_{cy}	1.000	4.570	2.899	1.249
Fr	0.490	2.800	0.783	0.526
$\Delta E/E_o$	0.670	0.862	0.748	0.032
Testing				
h_o/P	0.013	0.062	0.037	0.016
N_{cy}	1.000	2.000	1.333	0.483
L_{cy}/W_{cy}	1.000	4.570	2.849	1.102
Fr	0.490	2.620	0.806	0.545
$\Delta E/E_o$	0.701	0.820	0.754	0.031

3. RESULTS AND DISCUSSION

In this section, the results of modeling and estimating the energy dissipation of the flow passing over the labyrinth weirs with triangular and trapezoidal plans using GMDH and MLPNN are presented. As mentioned, soft computing models are data-driven, so in the first step, the statistical characteristics of the data collected from the desired phenomenon and the correlation between the input and output variables should be determined. Statistical characteristics are given in Table 1 and the correlation coefficient is given in Table 2.

Table 2 | The correlation coefficient between involved parameters

	h_o/P	N_{cy}	L_{cy}/W_{cy}	Fr	EDR
h_o/P	1				
N_{cy}	0.109	1			
L_{cy}/W_{cy}	0.115	0.886	1		
Fr	0.010	−0.290	−0.489	1	
EDR	−0.325	−0.155	−0.103	−0.417	1

As it is clear from Table 2, the correlation coefficients of EDR (as the output variable) and independent dimensionless parameters including h_o/P, N_{cy}, L_{cy}/N_{cy}, and Fr (as input parameters) is negative. In other words, the relationship between EDR and other parameters is inverse and as these parameters increase, the EDR decreases.

As mentioned in the review section of the GMDH model, only two variables can be entered in each neuron, and considering that the number of input variables is four, therefore, a maximum of six neurons are prepared in the first hidden layer. After evaluating the performance of these six neurons, four of them had an appropriate accuracy and were chosen for forming the next layer. In the second layer, from the four neurons of the previous layer, a maximum of six neurons can be prepared, but due to the stricter development criteria in the second layer, only two of them were qualified. In the output layer, from these two neurons of the previously hidden layer (second hidden layer), one neuron was prepared. The network of the GMDH model to estimate energy dissipation is shown in Figure 2. As shown in this figure, the GMDH network has two hidden layers, where there are four and two neurons in the inner hidden layers. The performances of the GMDH model in different phases of training and testing are shown in Figures 4 and 5. The statistical indicators of the GMDH model in the training phase are $R^2 = 0.913$, RMSE $= 0.010$, and in the test phase $R^2 = 0.829$, RMSE $= 0.015$. Examining the structure of

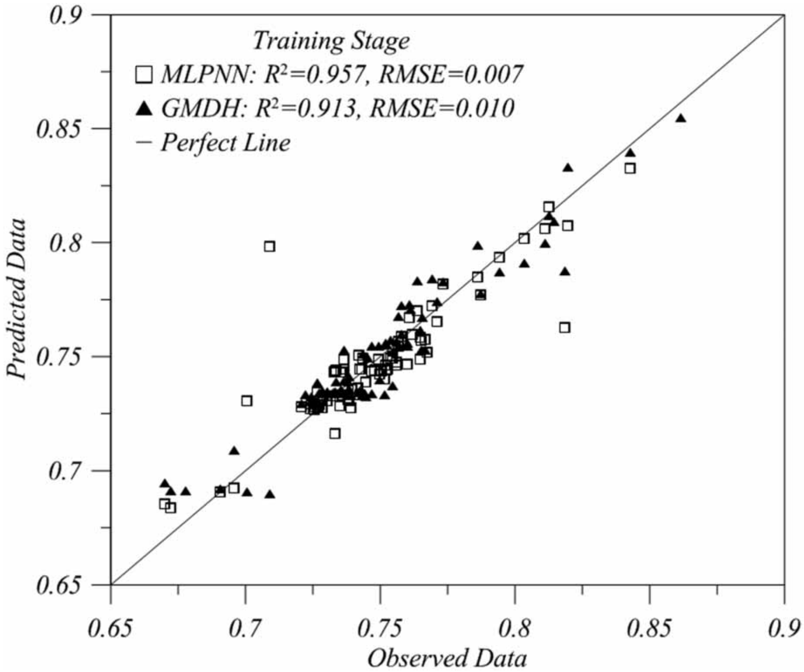

Figure 4 | The performance of the developed models in the training phase.

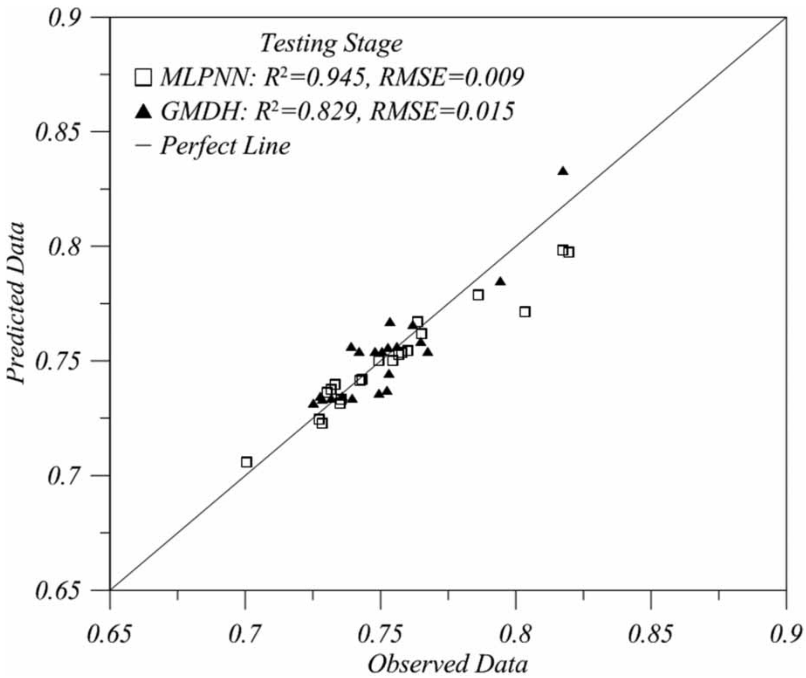

Figure 5 | The performance of the developed models in the test phase.

GMDH shows that the h_o/P, N_{cy}, and M_r play the greatest role in the development and formation of the GMDH network.

To compare the performance of the GMDH model with other soft computing models, the MLPNN model was chosen and developed in this regard. To establish the logical conditions of comparison, the MLPNN model with two hidden layers was considered; so that there are four and two neurons in the first and second hidden layers, respectively.

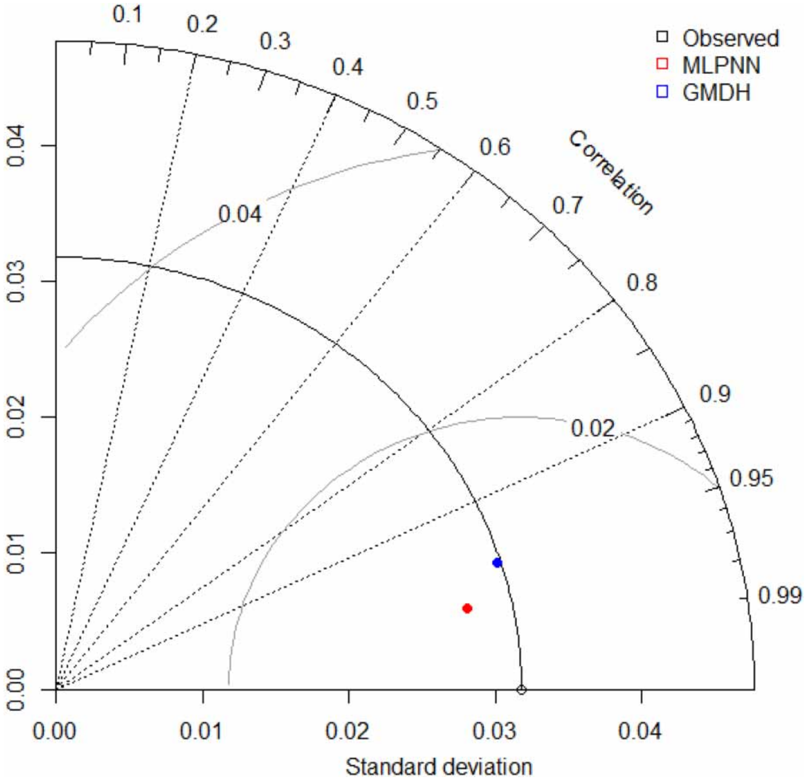

Figure 6 | Taylor diagram of MLPNN and GMDH models in the training stage.

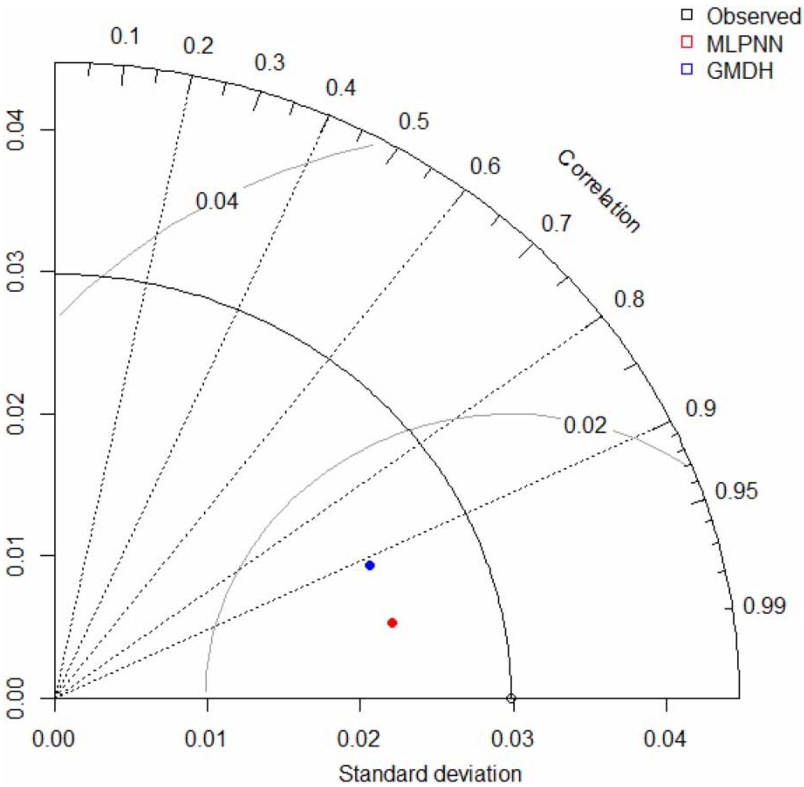

Figure 7 | Taylor diagram of MLPNN and GMDH models in the testing stage.

The structure of the MLPNN model is shown in Figure 3. In the development of the MLPNN, the sigmoid tangent function was used as the activation function for the neurons. The performances of the MLPNN model in the training and testing phases are shown in Figures 4 and 5. The statistical indices of the MLPNN in the training phase are $R^2 = 0.957$, RMSE $= 0.007$ and in the testing phase, they are $R^2 = 0.945$ and RMSE $= 0.009$. Comparing the performance of these two models shows that, in the training phase, the performance of the models is almost close together. However, in the testing phase, the accuracy of the GMDH model is slightly reduced. Of course, the GMDH structure has a lower computational cost compared to the MLPNN.

To further investigate the performance of both models and also to make a more accurate comparison between the MLPNN and GMDH models, Taylor's diagram was drawn for both training (Figure 6) and testing (Figure 7) stages. In this figure, observed data, and MLPNN and GMDH models are presented. According to Figure 6, the results of the MLPNN model are closer to the observational data, and the results of the GMDH model are farther away from the observational data even than the MLPNN model. It should be mentioned that Figure 7 also presents the same content.

3.1. Comparing the performance of soft computing models used in this study with models used in previous studies

Mahdavi-Meymand & Sulisz (2022) developed a support vector machine (SVM) to predict the energy dissipation downstream of labyrinth weirs. To train the SVM model, they applied three modern optimization techniques including multi-tracker optimization algorithm (MTOA), particle swarm optimization (PSO), and differential evolution (DE) algorithms. They declared that the average R^2 of SVM models is 0.98 which is close to the MLPNN model and is more accurate than the GMDH model.

Dutta *et al.* (2020) prepared three statistical and machine learning models including multiple linear regression (MLR), SVM, and ANN to predict the performance of triangular plan labyrinth weirs regarding energy dissipation (ΔE). They stated that the best accuracy is related to the SVM model such as away its average value of R^2 is 0.96 which like the results of Mahdavi-Meymand & Sulisz (2022) is close to the MLPNN model and is more accurate than the GMDH model.

4. CONCLUSION

The performance of a labyrinth weir with triangular and trapezoidal plans in the energy dissipating of the passing current was modeled and estimated using both a GMDH and an MLPNN. Our results showed that the GMDH model with two hidden layers, in which there are four and two neurons in the first and second hidden layers respectively, can predict the flow energy dissipation of such weirs in the validation phase with acceptable statistical indicators of $R^2 = 0.829$, RMSE $= 0.015$. The MLPNN model also like the GMDH model has two hidden layers where the first and the second layers contain four and two neurons, respectively. Examining the performance of the MLPNN model shows that this model, with the sigmoid tangent function as the activation function of the neurons, can predict the flow energy dissipation with the statistical indices $R^2 = 0.945$ and RMSE $= 0.009$. In general, it was found that both models have an acceptable accuracy in estimating the flow energy dissipation, however, the GMDH model is slightly less accurate.

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DATA AVAILABILITY STATEMENT

All relevant data are available from an online repository or repositories. Available from: https://link.springer.com/article/10.1007/s40996-017-0088-6.

CONFLICT OF INTEREST

The authors declare there is no conflict.

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