Relative Performance of Different Shaped Surface Aeration Tanks

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Surface aeration experiments were conducted in tanks of length to width ratio (L/W) 1.0, 1.5 and 2 as well as circular tanks to study their relative performance on oxygen transfer and energy consumption while re-aerating deoxygenated water. An identical geometric similarity of various linear dimensions of aerators, rotor blades and rotor diameter was maintained for all sizes and shapes of aeration tanks tested. The power consumed per unit volume concept to simulate the oxygen transfer coefficient $k$ was found to be valid for all three shapes of aerators. Simulation equations to predict the oxygen transfer coefficient $k$ for any given dynamic parameter governing the theoretical power per unit volume, $X$, were developed for rectangular tanks of L/W ratios equal to 1.5 and 2. Results were compared with simulation equations (correlating $k$ and $X$) for square and circular tanks. A simulation criterion correlating the oxygen transfer coefficient $k$ with actual and effective power consumption per unit volume for three shapes of tanks was developed. Energy consumption per unit volume of water was also analyzed. Examples illustrating the application of results are presented. It has been found that the circular tanks are the most energy efficient, i.e., they produce maximum $k$ for a given effective/actual input energy, followed by square tanks, rectangular tanks of L/W = 1.5 and rectangular tanks of L/W = 2. This suggests that the circular tank performs better as far as power requirements are concerned and hence provide better economy.

Key words: aeration, aerators, oxygen transfer, power, wastewater treatment

Introduction

Aeration is one of the important processes employed in water and wastewater treatment to reduce biochemical oxygen demand. The basic phenomenon behind the process of aeration is gas transfer, in which gas molecules are exchanged between the liquid and the gas at the gas-liquid interface (Fair et al. 1971). Aeration processes are also used to remove volatile substances present in water and wastewater and improve the dissolved oxygen content.

Many types of aerators are used in practice, such as cascade aerators, spray nozzles, diffused or bubble aerators, and surface aerators. Among them, surface aerators are popular because of their comparable efficiency and ease of operation.

A typical surface aerator with six flat blades, used in this study, is shown in Fig. 1. The main component of these surface aerators is an impeller or rotor, to which the six flat blades are fitted. The rotor is rotated to create turbulence in the water body so that aeration takes place through the interface of atmospheric oxygen and the water surface. The rate of oxygen transfer depends on a number of factors—such as intensity of turbulence—which, in turn, depends on the speed of rotation, size, shape and number of blades, diameter and immersion depth of the rotor and size and shape of aeration tank, as well as on the physical, chemical and biological characteristics of water (Rao 1999).

The performance of surface aeration systems is measured in terms of their oxygen transfer rate; hence, the choice of a particular surface aeration system depends on its performance and the efficiency of its oxygen transfer rates. Therefore, there appears to be a need to study and analyze the relative performance of these aerators. As the concept of power per unit volume is an established criterion to simulate the oxygen transfer coefficient (Rao 1999 and Rao et al. 2004), while studying such a relative performance, one may consider the relationship of oxygen transfer rates in terms of the theoretical power per unit volume:

$$X = \frac{N^3 D^2}{g \rho \sqrt{\nu}}$$

where $N$ is the rotational speed of the rotor, $D$ is the diameter of the rotor, $\rho$ is the kinematic viscosity of the water and $(P/V)$ is the actual and effective power per unit volume. Rao (1999) and Rao et al. (2004) developed the simulation criteria for un baffled square and circular surface aeration tanks, correlating the

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Fig. 1. Schematic diagram of a surface aeration tank.
Relative Performance of Surface Aeration Tanks

Oxygen Transfer Rates with the Theoretical Power Per Unit Volume

Kozinski and King (1966) summarized twelve studies on cylindrical baffled tanks and reported that the overall oxygen transfer coefficient at 20°C ($K_{O2}$) is the function of the power input per unit volume as $K_{O2} = (P/V)^{1.0}$, where $P$ is the power (ppm), such that $K_{O2}$ is the saturation DO concentration which can tend towards very large values, is a constant parameter ranging from 0.2 to 0.4. Hwang (1983) developed a scale-up equation which indicates that volumetric mass transfer coefficient in cylindrical baffled tanks can be related to the power input per unit volume as $K_{O2} = 0.332(P/V)^{0.6}$. Hsieh (1991) has developed a relationship between mass transfer rates and effective power per unit volume as $K_{O2} = 0.167(P/V)^{0.6}$ for cylindrical baffled tanks. Forrester et al. (1998) has predicted the relationship between the mass transfer rates and effective power per unit volume as $K_{O2} = (76 ± 21)^{(P/V)^{0.6}}$.

Generally, unbaffled tanks are being employed for surface aeration, because these give a higher fluid-particle mass transfer rate for a given power consumption (Johnson and Huang 1956; Grisafi et al. 1994; Rao and Jothishk 1997), which is of paramount importance in designing an aeration system.

The objective of this research is to compare oxygen transfer rates and the theoretical power per unit volume ($X$) for the three shapes of aeration tanks. An additional aim is to compare their relative performance using a simulations equation (Rao 1999 and Rao et al. 2004). Lastly, this study aims to develop a relationship between oxygen transfer rates and the actual power per unit volume ($P/V$) for three shapes of unbaffled surface aeration tanks.

Theory and Background Information

Oxygen Transfer Coefficient

While more advanced oxygen transfer models have been developed in the recent past (McWhirter et al. 1995, Mahendraker et al. 2005), the two film-theory developed by Lewis and Whitman (1924) seems to be satisfactory for clean water, and such a model is used in the present study.

According to this theory, the oxygen transfer coefficient at 20°C, $K_{O2}$, may be expressed as follows:

$$K_{O2} = \frac{\ln(C_s - C_0)}{t}$$

(2)

where $\ln$ represents a natural logarithm and $C_s$, $C_0$, and $C_t$ are dissolved oxygen (DO) concentrations in parts per million (ppm), such that $C_s$ is the saturation DO concentration which can tend towards very large values, $C_0$ is at $t = 0$ and $C_t$ is at time $t = 1$. The value of $K_{O2}$ can be obtained as slope of the linear plot between $\ln(C_s - C_0)$ and time $t$. The value of $K_{O2}$ can be corrected for a temperature other than the standard temperature of 20°C using the van’t Hoff-Arrhenius equation (WEF and ASCE 1988):

$$K_{O2} = K_{O2}0 (t^{-0.20})$$

(3)

where $\theta$ is the temperature coefficient 1.024 for tap water.

Dimensional Analysis

Many investigators including Schmidtke et al. (1977), Udaya et al. (1991) and Rao (1999) have successfully used the theory of dimensional analysis to describe the surface aeration process, which depends on geometric, physical and dynamic variables as explained below:

Geometric variables. The geometric variables include cross-sectional area of the tank ($A$), depth of water in the tank ($H$), diameter of the rotor ($D$), length of the blades ($L$), width of the blades ($b$), distance between the top of the blades and the horizontal floor of the tank ($h$) and the number of blades ($n$) as shown in Fig. 1.

Physical variables. The physical variables include density of air ($\rho_a$), density of water ($\rho_w$), and the kinematic viscosity of water ($\nu$).

Dynamic variables. The rotational speed of the rotor ($N$) is the dynamic variable associated with the surface aeration process.

Variables influencing the oxygen transfer coefficient at 20°C (i.e., $K_{O2}$) for a given shape of an aeration tank are therefore given by:

$$K_{O2} = f(A, H, D, L, b, h, n, g, \rho_a, \rho_w, \nu)$$

(4)

Equation 4 may be expressed in terms of non-dimensional parameters as follows:

$$k = f(\sqrt{A/D}, H/D, l/D, b/D, h/D, n, \rho_a/\rho_w, R, F)$$

(5)

where:

$$k = K_{O2} (\nu/g^2)^{1/3}$$

is the non-dimensional oxygen transfer parameter;

$$R = ND^2/\nu$$

is the Reynolds number; and

$$F = N^2D/g$$

is the Froude number.

Alternately it can be expressed as (Rao 1999 and Rao 2004):

$$k = f(\sqrt{A/D}, H/D, l/D, b/D, h/D, n, \rho_a/\rho_w, X)$$

(6)

where $X = F^{1/3}R^{1/3}$ is the parameter governing the power per unit volume. The first six non-dimensional parameters represent the “geometric-similarity” of the system and the last parameter represents the “dynamic-similarity.”

The experiments in the present study have been conducted by using six flat blades ($n = 6$) fitted to a rotor in a symmetrical manner as several investigators have followed the same number of flat blades (Schmidtke et al. 1977; Udaya et al. 1991; Rao 1999).

Therefore, the number of blades, $n$ is constant in the present experiments, also the parameter $\rho_a/\rho_w$ is considered as invariant. Thus, these two parameters are omitted in the
analysis. Therefore, the functional relationship of equation 6 can now be expressed as:

\[ k = f(\sqrt{A/D}, H/D, l/D, b/D, h/D, X) \]  

(7)

**Geometric Similarity**

Equation 7 suggests that if geometric-similarity of the first five variables on the right-hand side is maintained for any given shape of aeration tank, then \( k \) depends only on \( X \).

\[ k = f(X) \]  

(8)

An optimal solution to geometrical similarity has been investigated by Udaya et al. (1991) for a square tank by conducting a series of experiments in different sized tanks and by varying \( \sqrt{A/D}, H/D, l/D, b/D \) and \( h/D \) to a great extent. They suggested the following values to obtain a maximum \( K_{a,t,s} \) for any rotational speed \( N \) of the rotor fitted with six flat blades:

\[ \sqrt{A/D} = 2.88; \quad H/D = 1.0; \quad l/D = 0.3; \quad b/D = 0.24; \quad \text{and} \quad b/H = 0.94 \]  

(9)

The quantities listed above represent optimal geometric-similarity conditions for square tanks. To compare the performance of aerators with different shapes with square tanks, the same geometric dimensions have been maintained for all tanks in the present experiments.

**Dynamic Similarity: Power Per Unit Volume Concept**

When geometric-similarity conditions are maintained, the functional relationship represented by equation 7 is reduced to a function of dynamic similarity (Rao 1999) for any shape of aeration tank (equation 8). However, the functional relationship may be different for different shapes.

The intensity of turbulence and wave action of water are major factors associated with surface aeration. Turbulence and viscous effects can be described by the Reynolds number (\( R \)) and surface wave action can be described by the Froude number (\( F \)). Hence, both \( R \) and \( F \) are important to surface aeration. It may be noted here that both \( R \) and \( F \) are implicitly expressed in \( X \) in equation 8 as \( X = F^{0.5} R^{0.5} \), where \( F = N^2 D^2/2g \), and \( R = ND^2/\nu \) such that \( X \) can be considered as a governing parameter to simulate \( k \) (Rao 1999). Hence \( k \) can be uniquely related to \( X \) as previously reported (Rao 1999, Rao et al. 2004).

Several investigators Horvath (1984), Udaya (1991) and Rao (1999) stated that if power demand (\( P \)) per unit volume (\( V \)) remains constant then there would be a similarity in oxygen transfer between geometrically similar systems in clean water systems. It is also suggested that when power demand per unit volume remains the same in geometrically similar tanks of different sizes, the oxygen transfer coefficient \( K_{a,t,s} \) remains the same in all tanks. According to Oldshue (1985) the power per unit volume can be used as a scale-up criterion for homogenous chemical reactions.

In equation 8, the parameter \( X \) governs the theoretical power per unit volume and it can be defined from hydraulics principles based on the concept that the power is related to the product of the flow discharge and head loss (Rao 1999). Therefore, one may expect a correlation between the effective (actual or measured) power per unit volume (\( P/V \)) and \( X \). Furthermore, the oxygen transfer coefficient (\( k \)) is a function of \( P/V \) because \( k = f(X) \) and \( P/V \) and \( X \) are directly related. In this functional relationship, \( P/V \) can be expressed as the non-dimensional form \( P/V \). Hence the relationship between \( k \) and \( P/V \) can be expressed as:

\[ k = f(P/V) \]  

(10)

where \( P/V \) is the non-dimensional form of:

\[ P/V = \frac{P}{V(\rho g)^{1/3}} \]  

(11)

The goal is to solve equations 8 and 10 for three shapes of tanks by conducting laboratory experiments.

**Materials and Methods**

Experiments were carried out with the objective of finding the effect of tank shape on the oxygen transfer coefficient (\( k \)) under a wide range of \( X \) values (dynamic conditions) and power requirements per unit volume for three shapes of tanks.

**Experimental Setup**

The ranges of experimental variables are listed in Table 1. Two sizes of rectangular tanks of \( L:W \) ratio 1.5 and 2 and three sizes of each square and circular tank were tested under laboratory conditions. The geometrical similarity expressed in equation 9 was maintained in all tanks.

**Determination of \( K_{a,t,s} \)**

Standard oxygen transfer tests were conducted with tap water under laboratory conditions. At first, water in the tank was deoxygenated by adding the required amount of cobaltous chloride (CoCl\(_2\)) and sodium sulfite (Na\(_2\)SO\(_3\)) (Metcalf & Eddy Inc. 2004) and thoroughly mixing the water. The deoxygenated water was re-aerated by rotating the rotor at desired speeds and maintaining the variables as per the data presented in Table 1. When the DO concentration began to rise, readings were taken at regular intervals until DO increased up to about 80% of the DO saturation value. A Lutron Dissolved Oxygen Meter was used to measure the DO concentration in water. The DO meter was calibrated with the modified Winkler’s method (APHA 1985).

Using the known values of DO measurements in terms of \( C_i \) at regular intervals of time \( t \) (including the known value of \( C_i \) at \( t = 0 \)), a line was fitted by linear regression analysis of equation 2, between the logarithm of \( (C_i - C_o) \) and \( t \), by assuming different but appropriate values of \( C_i \) such that the regression that gives the minimum “standard error of estimate” was taken. Thus the values of \( K_{a,t,s} \) and \( C_i \) were obtained simultaneously. The values of \( K_{a,t,s} \) were computed using equation 3 with \( \theta = 1.024 \) as per the standards for clean water systems.
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Relative Performance of Surface Aeration Tanks

The kinematic viscosity = 0.8 x 10^{-6} m^2/s, temperature variation of the water, T = 20 to 29°C.
water (WEF and ASCE 1988). Thus the values of $K$ were determined for different rotational rotor speeds $N$ in all of the geometrically similar tanks.

**Measurement of Power Available at the Shaft**

The current (I) and voltage (V) of the power supply given to the DC motor of the aerator were measured by a digital multimeter. The rotational speed of the rotor was measured by using a digital speed indicator. The power available at the shaft was calculated by measuring input power and deducting the losses occurring in the DC motor. Since rotational speed of the shaft was low (70 to 120 rpm) compared to the rated speed (1400 rpm) of the DC motor, only important losses (such as copper and iron losses) were considered for determining the power available at the shaft. To determine these losses, current and voltage at no load (free rotation of rotor without load) and loaded (rotation with water) conditions were measured during the experiment. The power available at the shaft was then calculated according to the method given by Cook and Carr (1947).

From the measurement of no load current ($I_n$) and voltage ($V_n$), the iron loss was calculated by:

$$I_n = V_n - I_n^2 R_s$$

Total loss occurring in DC motor:

$$T_n = I_n + C_t$$

Incoming power to DC Motor:

$$P_i = I_n V_n$$

Power available at the shaft:

$$P = P_i - T_n = I_n V_n - I_n V_n - R_s (I_n)^2 + I_n^2$$

**Results and Discussion**

Experimental data were analyzed with an objective of computing three major variables, $k$, $X$, and their inter-relationships, leading to an understanding of relative performance of three shapes of tanks.

**Simulation of Oxygen Transfer Coefficient, $k$, with Theoretical Power Per Unit Volume, $X$**

Oxygen transfer coefficient in surface aeration tanks was investigated as a function of dynamic variable ($X$) to verify equation 8. The experimental data expressed in terms of $X = N^3 D^2 / (g^2 V_w)$ (equation 1) and $k = K_{sc} a r (v / g)^{0.9}$ are plotted in Fig. 2.

It is interesting to note that each set of data points pertaining to the given shape of the tank fall very closely on a unique curve, suggesting the validity of equation 8; however, the functional relationships are different for different shapes as the data fall uniquely on different curves. The simulation equation for square and circular tanks between $k$ and $X$ has been established by Rao (1999) and Rao et al. (2004) and the same is plotted in Fig. 2 along with the data from the present experiments to verify the validity of such a simulation equation.

In Fig. 2 the following equations fit well for all data points belonging to rectangular tanks with L:W ratio of 1.5:1 and 2:1 respectively.

For rectangular tanks $L:W = 1.5:1$:

$$k = \frac{K}{\sqrt{X}} = \{0.25 \exp{0.19X^{0.25}} + 8.035 \}
- 0.755 \exp[-1.85(X - 0.2)^{0.5}] \times 10^4 \sqrt{X}$$

(17)

For rectangular tanks $L:W = 2:1$:

$$k = \frac{K}{\sqrt{X}} = \{0.6275 \exp{0.5X^{0.91}} + 21.085 \}
- 20.955 \exp[-1.85(X - 0.2)^{0.5}] \times 10^4 \sqrt{X}$$

(18)

where $k$ is the oxygen transfer coefficient $k$ for the rectangular tanks. Therefore, it is confirmed that $k$ can be related with the theoretical power per unit volume parameter $X$, not only for square (Rao 1999) and circular (Rao et al. 2004) tanks but also for rectangular tanks. However, the simulation equations 17 and 18 are different depending on L/W. The equations developed earlier by Rao (1999) and Rao et al. (2004) for square and circular tanks are presented below for comparison:

For square tanks:

$$k = \frac{K}{\sqrt{X}} = \{17.32 \exp{(-0.3X^{0.05})} + 3.68 \}
- 0.925 \exp[-750(X - 0.057)^{0.5}] \times 10^6 \sqrt{X}$$

(19)

For circular tanks:

$$k = \frac{K}{\sqrt{X}} = \{10.45 \exp{(-4.5X^{0.35})} + 2.45 \}
- 0.7 \exp[-5(X - 0.35)^{0.5}] \times 10^7 \sqrt{X}$$

(20)

where $k_s$ and $k_c$ are the oxygen transfer coefficients ($k$) for square and circular tanks respectively.
Fig. 2. Simulation of oxygen transfer rate with theoretical power per unit volume.
Relative Performance of Surface Aeration Tanks with \( k \) and \( X \)

It is clear from Fig. 2 that for values of \( X \approx 0.5 \), square tanks offer a higher oxygen transfer coefficient \( k \), followed by rectangular tanks and circular tanks. But at higher values of \( X \) (say \( X = 4 \)) rectangular tanks with \( L/W = 2 \) give marginally higher values of \( k \) than the square tanks do, followed by the rectangular tank of \( L/W = 1.5 \), and with the lowest values in the circular tanks. Thus, while there seems to be no fixed trend of \( k \) for all \( X \) values, there seems to be an order for a given pair of shapes; square tanks always produce a higher \( k \) for a given \( X \) when compared to circular tanks (Rao et al. 2004) whereas rectangular tanks of \( L/W = 2 \) produce a higher \( k \) than the other rectangular tanks of \( L/W = 1.5 \).

The relative performance of circular and rectangular tanks of \( L/W = 1.5 \) and 2 with respect to square tanks in terms of the dimensionless oxygen transfer parameter, \( k/k_s \), for a given \( X \) is shown in Fig. 3. It is clear from Fig. 3 that for any given \( X \), the ratio of \( k/k_s \) for values of a rectangular tank of \( L/W = 2 \) and rectangular tank of \( L/W = 1.5 \) (i.e., 2) is always greater than one. This means that the performance of a rectangular tank of \( L/W = 2 \) is better than a rectangular tank of \( L/W = 1.5 \) as far as \( k/k_s \) is concerned. But at higher values of \( X \) (\( X > 0.5 \)), the performance of the rectangular tank of \( L/W = 2 \) is marginally better than the square tank. It is also observed from Fig. 3 that, for any given \( X \), circular tanks produce lower values of the oxygen transfer coefficient \( k \) than square tanks. At lower values of \( X \), the performance of circular tanks is less than that of rectangular tanks; however, circular tanks produce better oxygen transfer rates than rectangular tanks of \( L/W = 1.5 \) at high \( X \) values (high speed). Non-uniform variation of the oxygen transfer coefficient, as shown in Fig. 2, may be attributed to various parameters such as turbulence intensity, input power per unit volume, non-uniform spacing around the impeller, etc., which needs further investigation.

Simulation of Oxygen Transfer on the Basis of Effective (Actual) Power per Unit Volume

As reported in the literature (Kozinski and King 1966; Hwang 1983; Hsieh 1991), the rates of oxygen transfer can be correlated with effective power consumption per unit volume. Figure 4 shows the behaviour of the oxygen transfer coefficient with input power per unit volume for square, circular and rectangular tanks. It is interesting to observe that data for an individual shape of an aerator fall on a unique curve, suggesting that the oxygen transfer rates can be simulated with the actual and effective power per unit volume. After conducting statistical analysis of the data for an individual shape, the associated relationships have been presented in equations 21 to 24.

\[
10^5 k = \left(4.64P_Ve^{-0.399/P_V}\right) + \left[0.185(P_V)^{0.5}\right] \quad (21)
\]

(For square tanks)

\[
10^5 k = \left(7.384P_Ve^{-0.189/P_V}\right) + \left[0.33(P_V)^{0.5}\right] \quad (22)
\]

(For circular tanks)

\[
10^5 k = \left(4.64P_Ve^{-0.575/P_V}\right) + \left[0.13(P_V)^{0.5}\right] \quad (23)
\]

(For rectangular tanks where \( L/W = 1.5 \))

\[
10^5 k = \left(4.77P_Ve^{-0.385/P_V}\right) + \left[0.1045(P_V)^{0.5}\right] \quad (24)
\]

(For rectangular tanks where \( L/W = 2 \))

Relative Performance of Surface Aeration Tanks with \( k \) and \( P_V \)

From Fig. 4 it is evident that for a given effective power per unit volume, circular tanks produce higher oxygen transfer coefficient \( k \) than square and rectangular tanks. As far as rectangular tanks are concerned, \( k \) values decrease as \( L/W \) values increase. The reason for such behaviour in rectangular tanks may be due to the liquid tending to move along circular trajectories, resulting in decreased relative velocities between impeller and fluid, and weak radial flows directed towards the tank walls. The presence of sharp corners (in square and rectangular tanks) interrupts circular liquid patterns, and suppresses the free surface vortex, which increases input power and small-scale mixing (because of stronger axial flow). This implies that flow energy decreases with an increasing aspect ratio. In addition, for the same value of the oxygen transfer coefficient, \( k \), square tanks require less power per unit volume of water than rectangular tanks of \( L/W \) ratio 1.5, and even less than rectangular tanks of \( L/W \) ratio 2 as shown in Fig. 4. Overall, circular tanks gave better re-aeration rates than all other tanks in terms of the input power per unit volume of water. It is clear from Fig. 4 that to achieve

Fig. 3. Relative performance of (b) rectangular tanks (2:1), (c) rectangular tanks (1.5:1) and (d) circular tanks on oxygen transfer rate, for any given \( X \), compared to (a) square tanks.
Fig. 4. Simulation of oxygen transfer rate with actual power per unit volume, $P_v$. 
the same oxygen transfer coefficient \( (k) \), while aerating the same volume of water, circular tanks utilize less power than all other tanks. The relative performance of all tanks with respect to square tanks is shown in Fig. 5.

**Relative Performance of Surface Aeration Tanks with **\( k \) **and Energy Input**

Energy requirements of surface aerators are of paramount importance while choosing and designing particular types of aerators to meet the application needs. Energy can be computed as the product of power and time required to achieve a desired level of DO concentration. As \( K_a \), has the units of inverse of time, one may characteristically express the energy with the parameter \( P/K_a \). As \( k \) is a non-dimensional form of \( K_a \), and \( P \) is the non-dimensional form of \( P/V \), the energy (parameter) per unit volume (\( \varepsilon \)) may be defined as follows: \( \varepsilon = P/k \). To get a comparative energy requirement for achieving the same oxygen transfer rates, the energy parameter \( \varepsilon \) versus \( k \) has been plotted for all the surface aerators in Fig. 6. The values of \( P \) at \( k \) have been generated by using fixed-point iteration and Newton’s method (Curtis and Patrick 2003). It is observed from Fig. 6 that circular tanks consume less energy than any other tank. For the same oxygen transfer rates, the rectangular tanks of \( \text{L/W} = 2 \) consume more energy than any other tanks.

**Relationship between \( X \) and \( \varepsilon \)**

The relationship between theoretical power per unit volume \( X \) and the effective (actual) energy \( \varepsilon \) of all the aeration tanks is shown in Fig. 7. A common feature of all curves in Fig. 7 is that \( \varepsilon \) gradually increases as \( X \) increases and attains a maximum value, then decreases gradually. The maximum value of \( \varepsilon \) is different for each aeration tank. This value corresponds to the least efficient point, since it corresponds to the lowest oxygen transfer rate or highest effective power per unit volume. The maximum value for square, circular and rectangular tanks of \( \text{L/W} \) ratios 1.5 and 2 is 0.05, 0.15, 0.2 and 0.07 respectively. These observations show the existence of a transitional state in surface aeration systems. To overcome this transitional state, systems consume more energy. Based on data shown in Fig. 7, surface aerators should not be operated near the maximum energy point to economize their operation.

The relative energy consumption of all the other surface aeration tanks versus square-shaped tanks has been plotted in Fig. 8. It is observed from Fig. 8 that the rectangular tanks of \( \text{L/W} = 2 \) take more energy per unit volume while aerating the same volume of water at a lower range of \( X \), but at higher range of \( X \), rectangular tanks of \( \text{L/W} = 1.5 \) consume more energy than any other surface aeration tanks.

Figure 8 shows a descending trend in accordance with the aspect ratio. Rectangular tanks of \( \text{L/W} = 2 \) consume more energy per unit volume than rectangular tanks of \( \text{L/W} = 1.5 \), and the energy consumed is least in circular tanks. Based on these data, it can be concluded that circular tanks are more suitable for water and wastewater treatment than any other aeration tank.
The oxygen transfer coefficient $k$ is related to the theoretical power per unit volume ($X$) for rectangular tanks of $L/W$ ratio 1.5 and 2 as expressed in equations 17 and 18. Such equations developed earlier for square (equation 19) and circular (equation 20) tanks were also verified through further experiments.

For a given $X$, the oxygen transfer coefficient ($k$) in square tanks is always higher than that of circular tanks, whereas the rectangular tanks of $L/W = 2$ always performed better than the rectangular tanks of $L/W = 1.5$. An interesting feature of rectangular tanks of $L/W = 2$ is that they perform slightly better than the square tanks beyond a value of $X \approx 0.8$ as reflected in Fig. 2 and 3.

It has been established that $k$ can also be simulated with the effective (actual) power per unit volume ($P_v$) for a given shape of the tank. Simulation equations governing $k$ and $P_v$ were developed for all the three shapes of surface aeration tanks as given in equations 21 to 24. It is established that for a given value of $P_v$, the oxygen transfer coefficient ($k$) is the highest in circular tanks followed by square tanks ($L/W = 1$), rectangular tanks of $L/W = 1.5$ and rectangular tanks of $L/W = 2$ as shown in Fig. 4 and 5.

Among all the shapes, circular tanks were the most energy efficient as shown in Fig. 6. The next efficient shape was square followed by the rectangular tanks of $L/W = 1.5$ and the least efficient were the rectangular tanks of $L/W = 2$.

Although the square tanks were the best for quick aeration, they consumed more energy than the circular tanks, whereas circular tanks were the most energy efficient but required more time than the square tanks.

**Fig. 7.** Energy characteristics of surface aeration tanks

**Fig. 8.** Energy characteristics of (a) rectangular tanks (2:1), (b) rectangular tanks (1.5:1) and (d) circular tanks, compared to (c) square tanks.
TABLE 2. The performance of different types of aerators to re-aerate 0.347 m$^3$ of water at a constant rotational speed of the rotor, $N$$^a$.$^b$.$^c$.

<table>
<thead>
<tr>
<th>Step</th>
<th>Parameter estimation</th>
<th>Parameters</th>
<th>Square</th>
<th>Circular</th>
<th>Rectangular ($L/W = 1.5$)</th>
<th>Rectangular ($L/W = 2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equations 1 to 9</td>
<td>Geometric dimensions: Area = 1 m$^2$, Length = 1 m, Depth = —, Width = 1 m</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 m$^2$</td>
<td>1 m$^2$</td>
<td>1 m$^2$</td>
<td>1 m$^2$</td>
</tr>
<tr>
<td>2</td>
<td>Equations 17 to 20</td>
<td>$10^3k$</td>
<td>2.12</td>
<td>0.344</td>
<td>1.042</td>
<td>2.55</td>
</tr>
<tr>
<td>3</td>
<td>$K = K_0 \alpha_{20}(\sqrt{g} \Delta)^{1/3}$</td>
<td>$K = K_0 \alpha_T$</td>
<td>0.569/min</td>
<td>0.118/min</td>
<td>0.28/min</td>
<td>0.68/min</td>
</tr>
<tr>
<td>4</td>
<td>$K_0 \alpha_{20}$</td>
<td>$K_1 \alpha_T$</td>
<td>0.641/min</td>
<td>0.133/min</td>
<td>0.316/min</td>
<td>0.774/min</td>
</tr>
<tr>
<td>5</td>
<td>$K_1 \alpha_T$</td>
<td>Time $t$ required to reach 80% saturation value</td>
<td>150.5 s</td>
<td>723.32 s</td>
<td>305.52 s</td>
<td>124.84 s</td>
</tr>
<tr>
<td>6</td>
<td>Equations 21 to 24</td>
<td>Effective power per unit volume of water</td>
<td>154 W/m$^3$</td>
<td>28 W/m$^3$</td>
<td>120 W/m$^3$</td>
<td>221 W/m$^3$</td>
</tr>
<tr>
<td>7</td>
<td>$E = (t/3600) x (P/V)$</td>
<td>Effective energy consumed per unit volume of water</td>
<td>6.44 Wh/m$^3$</td>
<td>5.62 Wh/m$^3$</td>
<td>10.2 Wh/m$^3$</td>
<td>6.44 Wh/m$^3$</td>
</tr>
<tr>
<td>8</td>
<td>Energy savings using circular tanks vs. others</td>
<td>14%</td>
<td>—</td>
<td>81%</td>
<td>36%</td>
<td></td>
</tr>
</tbody>
</table>

$^a$Constant rotor speed, $N = 80$ rpm.

$^b$The rotor is rotated at a constant speed until DO concentration, $C_o$, attains 80% of the saturation value. Assume the initial DO concentration $C_o = 0$ at water temperature $T = 25^\circ C$.

$^c$Known variables: $T = 25^\circ C$, $v = 0.88 \times 10^{-4}$ m$^3$/s, $g = 9.81$ m/s$^2$; volume of water = 0.347 m$^3$; $X = N^3D^2/g^{2/3}v^{1/3} = 1.42$.
TABLE 3. The performance of different types of aerators to re-aerate 0.347 m$^3$ of water at constant effective power supply$^{a,b,c}$

<table>
<thead>
<tr>
<th>Step</th>
<th>Parameter estimation</th>
<th>Parameters</th>
<th>Square</th>
<th>Circular</th>
<th>Rectangular (L/W = 1.5)</th>
<th>Rectangular (L/W = 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$1 \text{ m}^2$</td>
<td>$1 \text{ m}^2$</td>
<td>$1 \text{ m}^2$</td>
<td>$1 \text{ m}^2$</td>
</tr>
<tr>
<td>1</td>
<td>Equations 1 to 9</td>
<td>Geometric dimensions</td>
<td>Area</td>
<td>Length</td>
<td>Depth</td>
<td>Width</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$1 \text{ m}$</td>
<td>—</td>
<td>$1.128 \text{ m}$</td>
<td>—</td>
</tr>
<tr>
<td>2</td>
<td>Equations 21 to 24</td>
<td>$10^5k$</td>
<td>$1.94$</td>
<td>$3.92$</td>
<td>$1.47$</td>
<td>$1.07$</td>
</tr>
<tr>
<td>3</td>
<td>$k_{L,T}(v/g^2)^{1/3}$</td>
<td>$k_{L,T}$</td>
<td>$0.52/\text{min}$</td>
<td>$1.056/\text{min}$</td>
<td>$0.39/\text{min}$</td>
<td>$0.29/\text{min}$</td>
</tr>
<tr>
<td>4</td>
<td>From $k_{L,T}$</td>
<td>$K_{L,T}$</td>
<td>$0.588/\text{min}$</td>
<td>$1.189/\text{min}$</td>
<td>$0.45/\text{min}$</td>
<td>$0.32/\text{min}$</td>
</tr>
<tr>
<td>5</td>
<td>From $K_{L,T}$</td>
<td>$t$ for 80% saturation</td>
<td>$164 \text{ s}$</td>
<td>$81 \text{ s}$</td>
<td>$216 \text{ s}$</td>
<td>$297 \text{ s}$</td>
</tr>
<tr>
<td>6</td>
<td>$E = (t/3600) \times (P/V)$</td>
<td>Effective energy consumed per unit volume of water</td>
<td>$6.56 \text{ Wh/m}^3$</td>
<td>$3.25 \text{ Wh/m}^3$</td>
<td>$8.68 \text{ Wh/m}^3$</td>
<td>$11.91 \text{ Wh/m}^3$</td>
</tr>
<tr>
<td>7</td>
<td>Energy savings versus circular tanks vs. others</td>
<td>$101%$</td>
<td>—</td>
<td>$167%$</td>
<td>$266%$</td>
<td></td>
</tr>
</tbody>
</table>

$^a$Effective power supply = 50 watts or $P/V = 144.1$ watts/m$^3$.

$^b$The rotor is rotated at a constant speed until DO concentration, $C_0$, attains 80% of the saturation value. Assume the initial DO concentration $C_0 = 0$ at water temperature $T = 25^\circ\text{C}$.

$^c$Known variables: $T = 25^\circ\text{C}$; $\nu = 0.88 \times 10^{-5} \text{ m}^2/\text{s}$; $g = 9.81 \text{ m/s}^2$; volume of water = 0.347 m$^3$. 

Relative Performance of Surface Aeration Tanks
TABLE 4. The performance of different types of aerators to re-aerate 0.347 m$^3$ of water at a constant oxygen transfer rate, $K_{1a20}$

<table>
<thead>
<tr>
<th>Step</th>
<th>Parameter estimation</th>
<th>Parameters</th>
<th>Geometric dimensions</th>
<th>Effective power per unit volume of water</th>
<th>Effective energy consumed per unit volume of water</th>
<th>Energy savings using circular tanks vs. others</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equations 1 to 9</td>
<td>Area</td>
<td>1 m$^2$</td>
<td>1 m$^2$</td>
<td>1 m$^2$</td>
<td>1 m$^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Length</td>
<td>1 m</td>
<td>—</td>
<td>1.224 m</td>
<td>1.414 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Depth</td>
<td>—</td>
<td>1.128 m</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Width</td>
<td>1 m</td>
<td>—</td>
<td>0.82 m</td>
<td>0.707 m</td>
</tr>
<tr>
<td>2</td>
<td>$k = K_{1a20} (v/\rho g)^{1/3}$</td>
<td>10$^3$k</td>
<td>1.756</td>
<td>1.756</td>
<td>1.756</td>
<td>1.756</td>
</tr>
<tr>
<td>3</td>
<td>Equations 21 to 24</td>
<td>Effective power per unit volume of water</td>
<td>139.92 W/m$^3$</td>
<td>77.67 W/m$^3$</td>
<td>159.83 W/m$^3$</td>
<td>181.9 W/m$^3$</td>
</tr>
<tr>
<td>4</td>
<td>$E = (t/3600) \times (P/V)$</td>
<td>Effective energy consumed per unit volume of water</td>
<td>4.49 Wh/m$^3$</td>
<td>2.59 Wh/m$^3$</td>
<td>5.32 Wh/m$^3$</td>
<td>6.05 Wh/m$^3$</td>
</tr>
<tr>
<td>5</td>
<td>From equations 17 to 20</td>
<td>X</td>
<td>1.09</td>
<td>5.83</td>
<td>3.81</td>
<td>0.96</td>
</tr>
<tr>
<td>6</td>
<td>Energy savings using circular tanks vs. others</td>
<td>73%</td>
<td>—</td>
<td>105%</td>
<td>133%</td>
<td></td>
</tr>
</tbody>
</table>

$^a$Constant oxygen transfer rate, $K_{1a20}$, is approximately 0.5/min.

$^b$Known variables: $T = 25^\circ\text{C}$; $\nu = 0.88 \times 10^{-6}$ m$^2$/s; $g = 9.81$ m/s$^2$; volume of water = 0.347 m$^3$; $K_{1a20} = 0.5$/min.
References


Notations

The following symbols are used in this paper:

- $A$: cross-sectional area of an aeration tank (L$^2$)
- $b$: width of the blade (L)
- $C_0$: initial concentration of dissolved oxygen at time $t = 0$ (ppm)
- $C_i$: concentration of dissolved oxygen in the liquid bulk phase (ppm)
- $C_r$: copper losses
- $C_s$: saturation value of dissolved oxygen at test conditions (ppm)
- $C_t$: concentration of dissolved oxygen at any time $t$ (ppm)
- $D$: diameter of the rotor (L)
- $F$: $N^2 D/g$, Froude number
- $g$: 9.81 m/s$^2$ acceleration due to gravity (L/T$^2$)
- $H$: depth of water in an aeration tank (L)
- $b$: distance between the top of the blades and the horizontal floor of the tank (L)
- $h_i$: head loss due to rotational movement of the water in an aeration tank (L)
- $I_1$, $I_2$: input current at no load and loading conditions respectively
- $I_r$: iron losses
- $k$: $K_s a_{rot}(V g^{1/3}) = $ non-dimensional oxygen transfer coefficient
- $k_r$: non-dimensional oxygen transfer coefficient for rectangular tanks
- $k_c$: non-dimensional oxygen transfer coefficient for circular tanks
- $k_{c1}$: non-dimensional oxygen transfer coefficient for square tanks
- $K_{t1}$: overall oxygen transfer coefficient at room temperature T$^\circ$C of water
- $K_{t2}$: overall oxygen transfer coefficient at 20$^\circ$C
- $L$: size of rectangular tank (L)
- $l$: length of the blade (L)
- $N$: rotational speed of the rotor with blades (1/T)
- $n$: number of rotor blades = 6
- $p$: power available to the rotor shaft (ML/T$^2$)
- $P/V$: Power per unit volume (ML/T$^3$)
- $P_v$: $P/[(V g^{1/3})]^{1/3} = $ Non-dimensional power per unit volume
- $Q$: discharge of water being pumped by rotor rotation (L/T)
- $R$: $N D^2/\pi$, Reynolds number
- $V$: volume of water in an aeration tank (m$^3$)
width or breadth of the surface aeration tanks

armature resistance of DC motor

input voltage at no load and loading conditions respectively

\[ X = N'D'/(g^{4/3}v^{1/3}) = F^{4/3}R^{1/3} \] = theoretical power per unit volume parameter

\( \theta = 1.024 \); constant for pure water used in equation 3

kinematic viscosity of water \((M^2/T)\)

mass density of air \((M/L^3)\)

mass density of water \((M/L^3)\)

energy consumption per unit volume