The optimization of the paddy field irrigation scheduling using mathematical programming

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ABSTRACT

In planting rice, a significant amount of irrigation water is required to prepare the farmlands and do transplanting and this is directly related to the number of machines and workers available; that is, the more the length of plowing and transplanting process due to the lack of required machinery and labor, the more the water volume consumed. Therefore, in such conditions, it is important to have an appropriate planning for the optimized allocation of machinery and labor for the agricultural lands. Determining the optimal times of opening and closing valves based on the factors directly influencing the volume of the water in paddy fields is also of great importance. To meet the above conditions and by using linear and non-linear programming, two different models are developed in this article, with the purpose of reducing the time period of plowing and transplanting operations and also lowering the water consumption, respectively. Comparing the outputs of these two models with the real irrigation situation demonstrates the efficiency of the proposed models.

Key words | irrigation scheduling, mathematical programming, optimal allocation of machinery and labor, rice production

INTRODUCTION

One of the challenges of the 21st century is to overcome the water shortage crisis. In addition to destroying water-based ecosystems, the crisis influences the food safety in the world (Kijne et al. 2003) because, along with the growth of population and industries, the request for fresh water has risen rapidly (Pimentel et al. 2004).

Agriculture, among all other industries, has allocated to itself a large percentage of the world’s sweet water (75–80%), vying with other industries in need of water (Costa et al. 2007). In fact, water is the main input in agriculture, having a noticeable role in the ongoing development of agricultural activities (Keramatzadeh et al. 2011). Among the various products available in this section, rice is regarded as one of the main and high consumption products which accounts for a noticeable amount of irrigation water (Barker et al. 2001). A total of 15% of agricultural lands under cultivation belongs to this product in the world (Ribbes 1999). It also has the maximum number of regions operated by water among the agricultural crops (Rao & Rees 1992). Apparently, more than half of 80% of sweet water sources in Asia used for agricultural purposes is applied for irrigation of 50% of water farms to grow rice (Moya et al. 2004). Rice has an important role in supplying food for people in the world (Katic et al. 2015) so that after wheat (Kijne et al. 2003), this product owns the highest area of lands under cultivation (more than one-third). In the majority of Asia, rice supplies more than 80% of total calories and 75% protein consumed by people.

In Iran, rice is also considered as one of the main and strategic products. In recent years, the consumption has increased among people and now rice is the most favorite food among the civic and rural families’ diet in the country. The area of land under rice cultivation in Iran is estimated to be 700,000 hectares, which is about 0.4% of all lands under rice cultivation in the world. Rice production plays an important role in the economy of Iranian Northern provinces. Mazandaran and Guilan provinces, with more than 75% of the farms under rice cultivation and production of more than 80% of Iran’s...
rice, along with Golestan, Khouzestan and Fars provinces are the most important rice supply bases in Iran (Najafi et al. 2009).

In comparison with global average of water resources, Iran had special conditions. The average rainfall in Iran is about one-third of the global average and its evaporation amount is three times more than the global average (Hasanli et al. 2009). There exist several reasons which decrease the amount of water needed for rice farms in Iran. These reasons are as follows: Iran’s dry and semi-dry climate which itself leads to water being the most important resource that mainly limits agricultural activities (Moriana et al. 2010), unprecedented growth in water usage in industrial and domestic sectors, successive droughts, and a decrease in the volume of water reservoirs supplying dams for the agricultural sector due to sedimentation. In Iran, 95% of water consumption with an irrigation efficiency of 30% is allocated to the agricultural sector, in other words out of 90 B m$^3$ of allocated water to agriculture, only 27 B m$^3$ is used. Bearing this in mind, agriculture, and especially sustainable cultivation of rice, in Iran would not be realized without optimizing the usage of water resource; a 10% decrease in irrigation water of rice would save about 1,500 B m$^3$ (approximately 25%) of fresh water used for non-agricultural usage (Kijne et al. 2005).

Many studies have been done concerning the optimization of irrigation scheduling. For the first time Suryavanshi & Reddy (1986) introduced mathematical (‘zero and one model’) programming for water distribution in canals. Wang et al. (1995) expanded ‘zero and one model’ and used it for Feng-Jia-Shan canal with 26 outlets in China. Using non-linear programming, Wardlaw & Barnes (1999) represented a model for optimized water allocation in complex irrigation networks in order to maximize the produced crops. The outcome of their research confirmed the practicality of the model presented. Reddy et al. (1999) represented an irrigation scheduling model using zero and one programming for lateral irrigation canals. Wardlaw & Bhaktikul (2004) used genetic algorithms with the aim of the optimum usage of water resources in a rotational irrigation system. Monem & Namdarian (2005) represented a program for optimized water distribution in Varamin irrigation canal in Iran using simulated annealing algorithm. Mathur et al. (2009) presented a program to optimize the water distribution of Feng-Jia-Shan canal in China using genetic algorithm. Brown et al. (2010) used simulated annealing algorithm to offer a new method for irrigation scheduling which, due to its adaptability with real cultivation models, considerably improved the existing methods in this field.

With respect to the research conducted in this field, it can be said that water distribution and delivery planning is a very complicated issue related to optimization; it is an issue with its limitations and variables which needs to apply powerful methods of optimization to be solved. In the studies which were done in the field of water distribution and delivery in the irrigation network, several goals have been considered: minimization of water shortage, water losses and the capacity of water distribution canals. The variables in these studies are flow rate, water delivery frequency, and the optimal number of intakes. An additional point to the above-mentioned issues which helps saving water is that a noticeable amount of irrigation water is used for farmland preparation (Gleick 1995). This fact becomes more important when there is a shortage of labor and machinery because the aforementioned shortage leads to an increase in the time of farm preparation. This increase means an increase in the amount of water. Therefore, by using a proper planning on allocating optimal machinery and work force in order to decrease the required preparation time can decrease the volume of water. So choosing a different approach from previous studies, in this research first a model is presented which considers the issue of allocating labor and machinery as a job-shop issue (meaning the allocation of M labors and machines to N lands), this model aims to decrease the time of cultivating; after that considering other affecting parameters in irrigation the second model is offered which is aimed at minimizing the opening time of valves in the time period resulting from the first model.

**PROBLEM DESCRIPTION**

**Problem statement**

The rice cultivation process is divided into five phases. The first phase is related to plowing and usually happens in winter and without any saved water utilization; the second phase is to prepare the repository and to breed the plants inside it by the farmer. The third phase, happening at the
same time as the plants’ breeding, is related to the second plowing by tractors and then flat building for transplanting. The fourth phase is related to transplanting using the workforce and transplanting machines, which occur at least 1 day after the third phase. The maximum interval between plowing and transplanting is determined based on the seed and soil condition. From the beginning of plowing until the end of transplanting, there should be a minimum level of water in each plot. Finally the fifth phase is related to irrigating and preserving the plants until the harvesting time.

The irrigation plan which is used nowadays in farms in Guilan province is the result of experts’ experiments which have been obtained through trial and error. In this approach, at first considering the soil and seed conditions, water is allocated to two main processes, i.e. plowing and transplanting successively 24 hours a day for about 35–40 days; this water is sent simultaneously to the whole area in a main canal. A decrease in the time interval of the above-mentioned processes means a decrease in the volume of water. Because of the shortage of machinery (as machinery is very expensive for farmers) and work force (as work force tend to other jobs or migrate to the cities), to do these two processes farmers usually use limited machinery and available labor force from the neighborhood in different shifts. Each section consists of couples of 2,000–3,000 m² plots which are separated by intake (Figure 1). All the sections which use the same machinery and labor force are categorized in specific groups (Figure 2). To decrease the cultivation costs and have better management, the available work force in each group are categorized into groups of five to eight people according to each plot area. Each group of people and machinery works in two shifts of 4 hours each.

After the determined time interval has passed and it is assumed that two main processes of cultivation are finished; in order to save the reserved water, the water of each section is switched on and off in a 9-day interval. This method of irrigation will be continued until the end of the period of irrigation. This method has several disadvantages such as: a decrease in farm area as the farmers cannot keep pace with this programming; high level of stress which overcomes farmers because of not being able to end the processes in the determined time; spending too much money on providing enough machinery and work force in order to avoid delays in planning; a decrease in the amount of cultivated rice; farmers’ reluctance for cultivation and, most important of all, ignoring the parameters affecting the water available in each plot (for example, evaporation, transpiration, infiltration and water loss).

Therefore, proper planning to assign the workforce and machinery in an optimized way, along with the optimum time consideration for the valves to be open, makes a big saving and causes variation in type of rice under cultivation, thus increasing the land under cultivation and triggering an economic growth of this section.

For this reason, in this paper two models are presented with the aim of decreasing time intervals of irrigation of two processes of plowing and transplanting (35–45 days).

**Assumptions**

To develop the mathematical model, regions of Foumanat irrigation network in Guilan province were selected. This area is located in west of Guilan. The area of rice farms in this region is 145,000 hectares. The main resource of water for this area is the Sefidrood dam. Water is transferred...
through Fuman canal to the farms of Foumanat irrigation network. The irrigation duration allocated to plowing and transplanting of the farms in this network is 35 days in recent conditions. Among the main canals existing in Foumanat irrigation network, BP14 canal with a length of 6,852 m is selected and some of the secondary canals have been studied (Figure 3). After studying secondary canals the following assumptions were made to formulate the models:

- All applied machinery and labor capacity demonstrate the same ability in every stage of cultivation.
- During the planning period, the number of applied machinery and labor is assumed constant.
- The area of plots in each region is 2,000 m² (Figure 1).
- The interval to move machines between different regions is assumed as 1 hour.
- Each day is divided into two 4-hour periods of time for plowing and transplanting. The beginning time of the first period is at 8 a.m. and the second at 2 p.m.
- All workers work nonstop in each 4-hour time interval.
- The start of transplanting operation will be after the plowing of all the land in each area is done.
- The type of seed for cultivation is considered the same for each region.
- The time of opening and closing the valves is at 6 a.m., which is determined based on the water level in each plot to the minimum height specified (in mm).
- Owing to the time required to irrigate each plot, which is a maximum of 2 hours, no stop is specified during plowing and transplanting at each irrigation operation.

- The number of people in each group is five.
- The minimum and maximum amount of water required in the two processes of plowing and transplanting is considered to be equal.

Symbols of the first model

- $R$ = the total number of regions considered;
- $i$ = index for each of the regions ($i = 1, \ldots, R$);
- $Tr$ = the total number of available tractors in the regions;
- $p$ = index for the number of tractors available in all regions ($p = 1, \ldots, Tr$);
- $L$ = the total amount of labor in the regions;
- $j$ = index for the amount of labor available in all regions ($j = 1, \ldots, L$);
- $Tm$ = the total number of transplanting machines available in the regions;
- $s$ = index for the number of transplantation machines available in all regions ($s = 1, \ldots, Tm$);
- $D$ = the project completion time upper bound;
- $d$ = index for the working days ($d = 1, \ldots, D$);
- $t$ = index for the working hours each day ($t = 1, 2, 3, 4, 5, 6, 7$ and $8$);
- $Vtr$ = the plowing speed of each tractor in an hour (per m²);
- $Vtm$ = the transplanting speed of each machine in an hour (per m²);
- $Vl$ = the transplanting speed of each person in an hour (per m²);
- $A_i$ = the area of region $i$ (per m²);

**Figure 3 | General scheme of BP14 canal and its branches.**
\( M_{ti} \) = the maximum interval between the end of plowing and start of transplanting in the regions;

\( M \) = large positive constant;

\( y_{ipdt} \) = a binary variable, so if the tractor \( p \) works on day \( d \) in time \( t \) in region \( i \), \( y_{ipdt} \) equals 1, and otherwise 0;

\( x_{ijdt} \) = a binary variable, so if the labor \( j \) works on day \( d \) in time \( t \) in region \( i \), \( x_{ijdt} \) equals 1, and otherwise 0;

\( w_{isdt} \) = a binary variable, so if the transplanting machine \( s \) works on day \( d \) in time \( t \) in region \( i \), region \( i \), \( w_{isdt} \) equals 1, and otherwise 0;

\( STA_i \) = the start time of plowing in region \( i \);

\( FNa_i \) = the finishing time of plowing in region \( i \);

\( STb_i \) = the start time of transplanting in region \( i \);

\( FNb_i \) = the finishing time of transplanting in region \( i \);

\( c_{\text{max}} \) = the project completion time;

\( ga_{id} \) = a binary variable, so if the start time of plowing in region \( i \) is on day \( d \), \( ga_{id} \) equals 1, and otherwise 0;

\( gb_{id} \) = a binary variable, so if the finishing time of plowing in region \( i \) is on day \( d \), \( gb_{id} \) equals 1, and otherwise 0;

\( ha_{id} \) = a binary variable, so if the start time of transplanting in region \( i \) is on day \( d \), \( ha_{id} \) equals 1, and otherwise 0;

\( hb_{id} \) = a binary variable, so if the finishing time of transplanting in region \( i \) is on day \( d \), \( hb_{id} \) equals 1, and otherwise 0;

\( ka_{id} \) = a binary variable, so if the start time of plowing in region \( i \) is on days 1 to \( d \), \( ka_{id} \) in this period equals 1, and otherwise 0;

\( kb_{id} \) = a binary variable, so if the finishing time of plowing in region \( i \) is on days 1 to \( d \), \( kb_{id} \) in this period equals 1, and otherwise 0;

\( la_{id} \) = a binary variable, so if the start time of transplanting in region \( i \) is on days 1 to \( d \), \( la_{id} \) in this period equals 1, and otherwise 0;

\( lb_{id} \) = a binary variable, so if the finishing time of transplanting in region \( i \) is on days 1 to \( d \), \( lb_{id} \) in this period equals 1, and otherwise 0.

**First mathematical model**

This model is aimed at minimizing duration of the rice cultivation period through optimized allocation of labor and machinery. The formulation of the model is as follows:

\[
\text{Min } c_{\text{max}}
\]

\[
\begin{align*}
St: \\
y_{ipdt} + y_{ipdt(t+1)} & \leq 1 \quad \forall \ i, i', p, d, t(t=1, \ldots, 7), \quad t \neq 4 \text{ and } i \neq i' \\
w_{isdt} + w_{isdt(t+1)} & \leq 1 \quad \forall \ i, i', s, d, t(t=1, \ldots, 7), \quad t \neq 4 \text{ and } i \neq i' \\
\end{align*}
\]

\[
STa_i \leq FNa_i \quad \forall \ i
\]

\[
STb_i \geq 1 + FNa_i \quad \forall \ i
\]

\[
STb_i \leq FNb_i \quad \forall \ i
\]

\[
FNb_i \leq c_{\text{max}} \quad \forall \ i
\]

\[
\begin{align*}
Vtr & \left( \sum_p \sum_d \sum_t y_{ipdt} \right) \geq a_i \quad \forall \ i \\
Vtm & \left( \sum_s \sum_d \sum_t w_{isdt} \right) + VI \left( \sum_s \sum_d \sum_t x_{ijdt} \right) \geq a_i \quad \forall \ i \\
\sum_i y_{ipdt} & \leq 1 \quad \forall \ p, \ d \text{ and } t \\
\sum_i x_{ijdt} & \leq 1 \quad \forall \ j, \ d \text{ and } t \\
\sum_i w_{isdt} & \leq 1 \quad \forall \ s, \ d \text{ and } t \\
\sum_{i=2}^{i=4} x_{ijdt} & \leq M(1 - x_{ijdt}) \quad \forall \ t(t=2,3,4), i', d \text{ and } j \\
\sum_{i=5}^{i=8} x_{ijdt} & \leq M(1 - x_{ijdt}) \quad \forall \ t(t=6,7,8), i', d \text{ and } j
\end{align*}
\]
\[ x_{ijdt} \geq x_{ijd(t+1)} \quad \forall \quad t(t = 1, 2, 3, 5, 6, 7), \ i, \ j \text{ and } d \]

\[ \sum_d g_{aid} = 1 \quad \forall \quad i \]

\[ \sum_d g_{bid} = 1 \quad \forall \quad i \]

\[ \sum_d h_{aid} = 1 \quad \forall \quad i \]

\[ \sum_d h_{bid} = 1 \quad \forall \quad i \]

\[ \sum_p \sum_{d' < d} \sum_t v_{ipdt} \leq M(1 - g_{aid}) \quad \forall \quad i \text{ and } d', \]

\[ = 2, \ldots, D \]

\[ \sum_p \sum_{d' < d} \sum_t v_{ipdt} \leq M(1 - g_{aid}) \quad \forall \quad i \text{ and } d', \]

\[ = 1, \ldots, (D - 1) \]

\[ \sum_s \sum_{d' < d'} \sum_{d < d'} w_{isdt} + \sum_{d' < d'} \sum_{d < d'} \sum_t x_{ijdt} \]

\[ \leq M(1 - h_{aid}) \quad \forall \quad i \text{ and } d' = 2, \ldots, D \]

\[ \sum_s \sum_{d' < d'} \sum_{d < d'} w_{isdt} + \sum_{d' < d'} \sum_{d < d'} \sum_t x_{ijdt} \]

\[ \leq M(1 - h_{bid}) \quad \forall \quad i \text{ and } d' = 1, \ldots, (D - 1) \]

\[ \sum_p \sum_t v_{ipdt} \geq g_{aid} \quad \forall \quad i \text{ and } d \]

\[ \sum_p \sum_t v_{ipdt} \geq g_{bid} \quad \forall \quad i \text{ and } d \]

\[ \sum_s \sum_{d' < d'} w_{isdt} + \sum_{d' < d'} \sum_t x_{ijdt} \geq h_{aid} \quad \forall \quad i \text{ and } d \]

\[ \sum_s \sum_{d' < d'} w_{isdt} + \sum_{d' < d'} \sum_t x_{ijdt} \geq h_{bid} \quad \forall \quad i \text{ and } d \]
\[ ST_i = \sum_d l_{aid} \quad \forall \quad i \]  
\[ FNb_i = \sum_d l_{bid} \quad \forall \quad i \]  

As mentioned, the main function to consider in the first model is how to minimize the total time of plowing and transplanting of the available paddy fields. As there is a 1-hour interval to move the machines (tractor and transplanting machine) from one region to another one, constraints 2 and 3 are noticed. Since the machinery can move from one region to another during the time between the first and second period with no restriction, the last hour of the first interval \((t = 4)\) has been removed from constraints. The constraints 4, 5, 6, 7 and 8 refer to the interval between the end of plowing and start of transplanting of each region. The constraints 9 and 10 indicate that all of the under-cultivation lands in the intended period should be plowed and transplanted. The constraints 11, 12 and 13 mean that each machine and labor can only be assigned to one area in an hour. The constraints 14, 15 and 16 refer to the fact that labor can only be transferred in the time interval between the first and the second 4-hour period (that is, from 12 p.m. to 2 p.m.) from one region to the other one. The variables used in the constraints 17, 18, 19 and 20 (by taking the amount of one) show that the beginning and the end of plowing and transplanting operations happen on a certain day. The constraints 21, 22, 25 and 24 refer to the permitted length of plowing and transplanting. According to these constraints, before the start and after the end of plowing and transplanting, none of machines and labors is allowed to be used. Based on the constraints 25, 26, 27 and 28, at the beginning and end of plowing and transplanting, at least one of the available machines and labors is expected to be in the considered region. Recording the start and end of each plowing and transplanting operation is very important in order to plan the irrigation process in the second model. Therefore, constraints 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39 and 40 prepare the grounds for the insertion of these numbers in equations 41, 42, 43 and 44, which are the numbers related to the start and end of plowing and transplanting operations, respectively.

### Symbols of the second model

- \( i \) = index for each region;
- \( D = \) index for working days \((d = 1, \ldots, D')\);
- \( D' = \) the total number of time periods considered for each region (Figure 3);
- \( R = \) the total number of regions considered;
- \( L_{ai} = \) the maximum level of water in the region \( i \) (in mm);
- \( D_{oid} = \) a binary parameter, so if there is enough rainfall on day \( d \), \( D_{oid} \) equals 1 (because the minimum daily water requirements can be provided), otherwise 0;
- \( L_{bi} = \) the minimum water level required in region \( i \) (in mm);
- \( E_{id} = \) the rate of evaporation, evaporation and transpiration, depth percolation and loss of the water in each plot, each day for each region (in mm);
- \( M = \) large positive constant;
- \( \alpha_1, \alpha_2 = \) distinct digits between zero and one;
- \( W_{dai} = \) the level of water out of rainfall (in mm) on day \( d \) in region \( i \);
- \( L_{si} = \) the level of water in region \( i \) (in mm) at the start of the plan;
- \( M_{pi} = \) the minimum time permitted to start the irrigation plan;
- \( M_{psi} = \) the maximum time permitted for water to reach the last plot in each region \((M_{pi} > M_{psi});\)
- \( z_{id} = \) a binary variable, so if the region \( i \) needs water on day \( d \), \( z_{id} = M_{psi} \) equals 1, and otherwise 0;
- \( w_{id} = \) the amount of water in each plot at the end of each day, regardless of the minimum and maximum amount permitted;
- \( w_{id} = \) the amount of water in each plot at the end of each day \( (0 \leq w_{id} \leq L_{ai}) \);
- \( y_{aq} = \) a binary variable, so that this variable, by taking the amount of 1, prevents the level of water from being negative at the end of each day;
- \( y_{bi} = \) a binary variable, so if \( w_{id} \) at the end of each day is between 0 and the maximum amount determined, this variable, by taking the amount of 1, makes \( w_{id} = w_{id} \);
- \( y_{ci} = \) a binary variable, so if the water level in region \( i \) on day \( d \) becomes more than the maximum specified;
- \( y_{ci} = 1 \) acts like the last valves of each plot by taking the amount of one and makes the level of water stand at the maximum allowed \((w_{id} = L_{ai})\);
$x_{id} = \text{a binary variable which, by taking the amount of 1 each day, determines that the outlet valve be open on that day;}$

$xx_{id} = \text{decision variable for maximizing } x_{id} \text{ in objective function.}$

**Second mathematical model**

After determining the optimum length of plowing and transplanting through the first model, the optimal timing of valve opening during this interval is determined through the second model (Figure 4). This model is formulated as follows:

$$\min \alpha_1 \times \sum_i \sum_d z_{id} + \alpha_2 \times \sum_i \sum_d xx_{id} \quad (45)$$

**St:**

$$w_{i1} = (Ls_i + Wd_{i1} - E_{i1}) \quad \forall \ i \quad (46)$$

$$W_{id} = (Wd_{id} + \omega w_{i(d-1)} - E_{id}) \quad \forall \ i \text{ and } 2 \leq d \leq M_{p_i} \quad (47)$$

$$W_{id} = \left( \left( Wd_{id} - E_{id} \right) + \left( L_{ai} \times z_{i(d-(M_{p_i}+1))} \right) + \left( \omega w_{i(d-1)} \times (1 - z_{i(d-(M_{p_i}+1))}) \right) \right) \quad \forall \ i \text{ and } d \quad (48)$$

$$w_{id} \leq L_{ai} + M(1 - y_{ai}) \quad \forall \ i \text{ and } t \quad (50)$$

$$w_{id} \geq -M(1 - y_{id}) \quad \forall \ i \text{ and } t \quad (51)$$

$$w_{id} > L_{ai} - M(1 - y_{ci}) \quad \forall \ i \text{ and } t \quad (52)$$

$$ya_{id} + yb_{id} + yc_{id} = 1 \quad \forall \ i \text{ and } d \quad (53)$$

**Table 1 | Input for first model**

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The objective function (45) in this model is composed of two parts. The first part consists of minimizing the number of opening the valves and the second part involves determining the time span of the outlet valves being open. Since the first part of the objective function is the main goal of this plan, so \( \alpha_1 > \alpha_2 (\alpha_1 + \alpha_2 = 1) \).

The constraints 46, 47 and 48 specify the water available in each plot at the end of each day in mm. As the water in each plot should not be less than zero and more than \( H_{ai} \), \( w_{id} \) and the constraints 49, 50, 51, 52, 53 and 54 are considered. To prevent the valves from opening at the time of severe rainfall, the constraint 55 has

\[
ww_{id} = (yb_{id} \times w_{id}) + (yc_{id} \times L_a) \quad \forall \ i \text{ and } d
\]  

(54)

\[
z_{i(t-M_{pi})} + Do_{it} \leq 1 \quad \forall \ i \text{ and } d \geq M_{pi}
\]  

(55)

\[
ww_{id} \geq L_{bi} + (M \times z_{i(t-M_{pi})}) + (M \times Do_{id}) \quad \forall \ i \text{ and } d \geq M_{pi}
\]  

(56)

\[
ww_{id} - (M \times Do_{id}) < L_{bi} + (M \times (1 - z_{i(t-M_{pi})})) \quad \forall \ i \text{ and } d \geq M_{pi}
\]  

(57)

\[
z_{id} \times \sum_{d} (x_{id} - 1) = 0 \quad \forall \ i \text{ and } d
\]  

(58)

\[
xx_{id} \geq x_{id} \quad \forall \ i \text{ and } d
\]  

(59)

Table 2 | Input for second model

<table>
<thead>
<tr>
<th>Region((i))</th>
<th>(D)</th>
<th>(W_{id})</th>
<th>(E_{id})</th>
<th>(D_{oi})</th>
<th>(\alpha_1)</th>
<th>(\alpha_2)</th>
<th>Group</th>
<th>(i)</th>
<th>(L_{ai})</th>
<th>(L_{bi})</th>
<th>(L_{ci})</th>
<th>(M_{psi})</th>
<th>(M_{pi})</th>
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Table 3 | The results obtained from conducting the first model

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<th>Group</th>
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<th>(ST_{ai})</th>
<th>(FN_{ai})</th>
<th>(ST_{bi})</th>
<th>(FN_{bi})</th>
<th>(C_{max})</th>
<th>Cultivation period</th>
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<td>13</td>
<td>14</td>
<td>19</td>
<td>26</td>
<td>18</td>
</tr>
</tbody>
</table>
been considered, because severe rainfall disrupts the operations of plowing and transplanting not to mention that the open valve means the loss of the stored water. The constraints 56 and 57 determine the right time for the valves to be open. The planning for each region is based on the last available plot in the region and the value of \( w_{w/d} \) is calculated at the end of each day. Each plot reaches the maximum level of water determined for a maximum of 4 hours. Therefore, the length of the days for the outlet valve to be open is equal to the maximum time specified for the water to reach the last available plot in each region (\( M_{psi} \)), plus a day added to irrigate the intended region supplied by the constraints 58 and 59.

### Table 4: The optimal timing of valve opening during the process of plowing and transplanting

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</tbody>
</table>
RESULTS AND DISCUSSION

To evaluate the abilities of the presented model, the information related to five groups from BP14 canal in Foumanat irrigation network has been used. In groups 1, 2 and 3 there exists two regions and in groups of 4 and 5, there are three regions. The details related to the plowing and transplanting speed of machinery, the workforce transplanting speed and the area of the regions are obtained from Agricultural Organization of Guilan province. The number of machinery and workforce is acquired through field studies (Table 1), the details of evaporation, transpiration and the volume of rainfall are obtained from Iran Meteorological Organization, the details of cultivation phases in need of water and the water infiltration from Rice Research Institute of Iran and the details of available water for farmlands from Guilan Regional Water Company (Table 2).

To solve the presented models, two software programs have been used: Cplex and Lingo. These both have high ability in solving linear and non-linear models. The provided codes have been run in a five core computer with central process unit of 2.30 gigahertz.

After solving the first model the highest number of days of plowing and transplanting for the first group is 18, the second group is 27, the third group is 28, the fourth group is 26 and the fifth group is 25 (Table 3). As the irrigation for each group is done successively during 35 days, it implies that a decrease in the number of days means a decrease in the volume of water.

By performing the second model, the need for successive irrigation during plowing and transplanting is eliminated and the optimal timing of opening and closing of valves is specified in Table 4.

The maximum time interval for valves to be open is 12 for the first group, 12 for the second group, 14 for the third group, 18 for the fourth group and 18 for the fifth group (Table 5).

During 120 days of cultivation, the number of irrigation days in current conditions is at least 75 days. The volume of water which is used for irrigation of farms in Foumanat network is about 818.6 B m³. From the 75 days of irrigation, at least 35 days is used for plowing and transplanting; so the amount of used water in this period is 382 B m³. Considering the results from the first model, the minimum saving is 76.4 B m³ which means 20% decrease in water amount (Figure 5).

Table 5 | The result obtained from conducting the second model

<table>
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<th>Irrigation days</th>
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<tr>
<td></td>
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</tbody>
</table>

Figure 5 | The amount of saving resulting from conducting the first model.
By applying the second model and determining the optimal timing of opening and closing of the valves, the volume of water which would be saved was at least 185.6 B m³. Figure 6 shows the percentage of saving resulting from applying the second model in each section. This amount, in addition to expanding the water-based products’ cultivation area, helps to facilitate the economic growth of the other industries.

CONCLUSION AND SUGGESTIONS

In this article, two mathematical models were proposed with the aim of reducing the duration of plowing and transplanting as well as determining the optimal time for opening and closing the outlet valve, using linear and nonlinear programming. The method was such that after planning the labor and machinery and specifying the optimal time for these two operations, the obtained period along with the other parameters were considered as the input of the second model. The outcome of the presented models, in comparison with the real situation, showed substantial reduction in the amount of the consumed water. As suggested models consider the input parameters in a complete certainty, applying fuzzy logic for increasing the accuracy of the results and also using meta-heuristic methods to plan the whole irrigation network (due to the inability of the offered model for the large-scale problems) are suggested for future investigation. Also, for future research projects in this field, formulating a model to determine the right amount of evaporation and transpiration using fuzzy regression and presenting an integrated model from the applied models to decrease the calculation time is proposed.

REFERENCES


*First received 15 January 2015; accepted in revised form 30 April 2015. Available online 19 May 2015*