An optimization procedure for the sustainable management of water resources
Mario Maiolo and Daniela Pantusa

ABSTRACT
The sustainable management of water resources requires the identification of procedures to optimize the use and management of resources that are able to deal adequately with the problems of an integrated water service. Taking cue from the classic problem of transport of operations research, this paper proposes a procedure to optimize the allocation of resources for water supply in an area. The transportation problem has been particularized to the case of water resources management considering water intake structures present in a territory as source nodes and utilities in the area as destination nodes. The variables to be determined are the quantity of water resources transferred from each source node to each destination node, in order to achieve a balance between the needs and availability, minimizing the costs of supply consequently necessary. The problem that is obtained, is a constrained non-linear type problem that can be used, for example, to solve local problems of supply, through the redefinition of the distribution system. An application of the model to the province of Croton in southern Italy is shown. The results obtained from the application of the model show the reliability of the methodology in applications similar to that of the case study.

Key words | allocation of water resources, operations research, optimization of water resource management, sustainability

INTRODUCTION
The need to promote sustainability has started a number of activities aimed at the implementation of procedures and methods for estimation and quantification of sustainability itself. Sustainability aims to improve the long-term well-being of society and this improvement over time cannot happen without sustainable management of water systems. Proper management of water resources requires the identification of appropriate methodological and operational tools for overcoming the environmental, social and economic issues related to the use of these resources. In this direction, there are several examples of studies and models aimed at identifying sustainability indices for the management of water resources (Baan 1994; Loucks 2002; Nachtnebel 2002; Maiolo et al. 2006).

The sustainability of water resources, however, implies also the fulfillment of the objectives of optimization, inherent, for example, in the overcoming of conflict between different uses of water, the proper allocation of resources, water quality, planning and operation of water systems; other operational tools, therefore, are needed to support decision-makers in the various technical and managerial choices.

With reference to integrated water systems, the pursuit of sustainable management of these systems, requires the improvement of water systems by reducing structural and management deficiencies, the analysis of the risk associated with the vulnerability of drinking water systems (Maiolo & Pantusa 2015) and the proper allocation of available water resources.

The proper allocation of resources is an important type of optimization problem, and there are different studies, including recent ones, on this issue. Yamout & Fadel
have developed and applied a water resources allocation model to serve as a multi-sectoral decision for water resources management; the model uses a linear programming formulation with a framework of dynamic optimization, to determine the optimal water allocation pattern. Zhanping & Juncan (2012) have proposed an optimization model for the optimal planning of complex water systems with multiple supply sources and multiple users, taking into account environmental considerations. The proposed model analyzes the water resources demand in different periods and areas and formulates a sustainable development water resources allocation. As the model includes social, economic and environmental objectives, genetic algorithm (GA) is employed to optimize the allocation. Sun & Zeng (2012) have adopted the optimization theory of dynamic programming principle to build the Weinan city water resources optimization allocation model. Bai et al. (2013) have developed an optimal model for water allocation and water distribution network management in which cost and water conservation are regarded as objectives. Ni et al. (2014) have investigated the optimal allocation of water resources for an urban water management system through a water resource optimal allocation model based on multiagent modeling technology in which different optimization objectives are abstracted into various properties of different agents.

In this context, this paper proposes an optimization procedure relative to proper allocation of water resources for drinking water purposes; the proposed optimization procedure does not require the collection and processing of a large and complex number of data and, compared to other models, does not require excessive computational costs; this model, therefore, can be an easy to use tool to identify the proper allocation of resources within integrated water systems.

**OPTIMIZATION AND OPERATIONS RESEARCH**

Operations research is a discipline that deals with the development and application of scientific methods to the solution of many decision problems. The typical phases of a study of operations research include definition of the problem and collection of data, formulation of the mathematical model representing the problem, development of suitable software tools for the determination of the solutions, testing of the model and possible refinements, development of a system to support the practical application of the model and implementation of the system (Hillier & Lieberman 2005).

This discipline uses different techniques such as, linear programming, non-linear programming, integer programming, dynamic programming, sequencing theory, Markov process, game theory; this list, of course, is not exhaustive (Havinal 2012).

Optimization methods have been widely applied in water resources problems; since the 1970s, complex problems of water resources distribution and quality management were considered from the point of view of systems analysis and general optimization techniques (VV.AA. 1975). Several studies were carried out in the last several decades in order to solve various types of water resources problems; the state-of-the-art of these models has been reviewed by several authors in time (Loucks et al. 1981; Yeh 1985; Mays & Tung, 1992; Wurbs 1993; Labadie 2004).

Loucks et al. (1981) has described various quantitative methods for evaluating and comparing alternative water resources projects and plans with reference to, for example, surface water quantity management, water quantity aspects of irrigation planning and operation, and surface water quality planning. Yeh (1985) has provided a general overview of mathematical models developed for reservoir operations, with a strong emphasis on optimization techniques. Algorithms and methods surveyed include linear programming (LP), dynamic programming (DP), nonlinear programming (NLP), and simulation. Mays & Tung (1992) have provided a framework for hydrosystems modelling in engineering and management, which includes the analysis of surface water systems, of groundwater systems, of distribution systems. Wurbs (1993) has reviewed numerous reservoir-system analysis models, to assess the most useful models for various types of decision-support situations. Models are inventoried and compared from a general overview perspective, with an emphasis on practical applications. Labadie (2004) has provided a state-of-the-art review of the optimization of reservoir system management and operations. Optimization methods designed to prevail over the high-dimensional, dynamic, nonlinear, and stochastic characteristics of reservoir systems have been scrutinized, as well as extensions into multiobjective optimization. Application of
heuristic programming methods using evolutionary and genetic algorithms have been described, along with application of neural networks and fuzzy rule-based systems.

TRANSPORTATION PROBLEM

Among the various models of operational research, what historically has been the most widely used is linear programming, which not only applies to many real problems that inherently have a linear structure, but is also an indispensable tool of support, technical and conceptual, for more complex models of a discrete type, such as combinatorial optimization or integer linear programming. The most common type of application of linear programming concerns the general problem of allocating scarce resources among competing activities in the best way possible. This problem deals with the selection of the level for activities that compete for the use of scarce resources, which are essential to the realization of the assets. The choice of the level of activity determines, then, how much of each resource will be used for each activity (Hillier & Lieberman 2005). One of the particularly important problems of linear programming is the transportation problem, so named because many of its applications include how to transport goods in an optimal way.

The transportation problem is a classic problem of optimal management of goods that must be transferred from sources to destinations with the objective of minimizing the cost (time) of transport.

The transportation problem is bound, therefore, to the distribution of any kind of goods from whatever group of distribution centers, called source nodes, to any group of receiving centers, called destination nodes.

Each source node has a fixed offering that must be sent entirely to the destination nodes (\(a_i\) indicates the number of units that are available at the source node \(i\), for \(i = 1, 2, \ldots, m\)). Similarly, each destination node has a fixed demand that must be satisfied by the source nodes (\(b_j\) indicates the number of units required by the destination node \(j\), for \(j = 1, 2, \ldots, n\)). The cost of transporting one unit between source \(i\) and destination \(j\) is \(c_{ij}\); \(x_{ij}\) indicates the quantity transported from source \(i\) to destination \(j\) and the cost associated with this transport is \(c_{ij}x_{ij}\).

In order to minimize the transportation costs, the following problem must be solved:

\[
\begin{align*}
\text{Minimize} & \quad z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \cdot x_{ij} \\
\text{subject to} & \quad \sum_{j=1}^{n} x_{ij} \leq a_i & \text{for } i = 1, 2, \ldots, m \\
& \quad \sum_{i=1}^{m} x_{ij} \geq b_j & \text{for } j = 1, 2, \ldots, n \\
& \quad x_{ij} \geq 0 & \text{for all } i \text{ and } j
\end{align*}
\]

The constraints specify that the sum of all transfers from a source cannot exceed the number of units that are available at the source node and that the sum of all transfers to a destination node must be at least as large as the demand.

A transportation problem is said to be ‘balanced’ if the total supply from all source nodes equals the total demand in the destinations. In this case, the constraints are defined as follows:

\[
\begin{align*}
\sum_{j=1}^{n} x_{ij} = a_i & \quad \text{for } i = 1, 2, \ldots, m \\
\sum_{i=1}^{m} x_{ij} = b_j & \quad \text{for } j = 1, 2, \ldots, n \\
x_{ij} \geq 0 & \quad \text{for all } i \text{ and } j
\end{align*}
\]

A transportation problem is said, instead, to be ‘not balanced’ when the total supply from all source nodes is not equal to the total demand of the destination nodes. In this case, it is possible to balance an unbalanced transportation problem by introducing a dummy destination or a dummy source.

In the case where supply exceeds demand, it is necessary to introduce a dummy destination to which will be assigned a demand equal to:

\[
b_{\text{dummy destination}} = \sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j
\]

Since no transport takes place, the unit transportation costs can be set to zero.
In the case where demand exceeds supply, it is necessary to introduce a dummy source to which will be assigned a supply equal to:

$$a_{\text{dummy source}} = \sum_{j=1}^{n} b_j - \sum_{i=1}^{m} a_i$$  \hspace{1cm} (9)

Since the source does not really exist, no transportation from the source will occur, so the unit transportation cost can be set to zero.

To formulate and solve transportation problems software such as Excel, LINGO/LINDO, MPL/CPLEX are now available.

**OPTIMIZATION PROCEDURE FOR THE SUSTAINABLE MANAGEMENT OF WATER RESOURCES**

The optimization procedure proposed involves the analysis of water resources present in a territory in relation to the demand of the users in order to determine the possible transfer of water resources in the territory by different schemes. A first disclosure of this problem has already been made, at a more general level of analysis, considering the volumes available and in demand in an area with the costs of transferring the resource evaluated in a simplified manner (Maiolo et al. 2008). This paper proposes, however, a further level of detail directly considering the water intake structures and a more detailed cost of transfer of the resource. On the basis of these new assumptions, a more complex optimization problem has been achieved with a non-linear objective function.

To describe the problem correctly, we define:

- **source nodes:**
  - water intake structures present in a territory (springs, wells, derivations);
- **destination nodes:**
  - utilities in the area (municipalities);
- **$$a_i$$:**
  - water availability of the source node $$i$$;
- **$$b_j$$:**
  - demand of the user $$j$$;
- **$$c_{ij}$$:**
  - cost of transfer of the water resources from the $$i$$-th origin to the $$j$$-th destination.

With the problem defined in this manner, it is necessary to identify the data needed to be collected for the formulation of the problem itself. As for the source nodes, it is necessary to identify all water intake structures present in the area, collect all the information about the availability of water, estimate the quotas, and identify their geographic location through coordinates. As for the destination nodes, considering the reference time horizon for modeling, it is necessary to estimate the resident and fluctuating population and the corresponding demands of water. For each municipality, it is also necessary to calculate the centroid of the polygon representing the administrative boundaries, which is the position of the destination node.

Regarding the cost, at present the retail price of the water is obtained by applying a tariff, often in agreement with the Price Cap Regulation, which depends on several factors; to obtain an expeditious evaluation, and, to avoid referring to all the variables of the tariff, the method mainly refers to the cost of building $$C_u$$.

As for this cost, it was assumed to consider only possible transfers due to gravity. In this case, the annual cost per unit length, $$C_a$$, can be expressed as:

$$[C_a = r \cdot C_u] \hspace{1cm} (10)$$

where

- $$r$$ = rate of cost for amortization and operating expenses
- $$C_u$$ = cost of construction of the pipeline.

The cost, $$C_u$$, can be regarded as a function of the diameter $$D$$:

$$[C_u = C_u(D)] \hspace{1cm} (11)$$

It could be expressed in monomial form:

$$[C_u = KD^\alpha] \hspace{1cm} (12)$$

where the constants $$K$$ and $$\alpha$$ depend on the type of material, on its class, on laying conditions, etc.

Consider the monomial formula, Darcy type (Milano 1996):

$$J = \beta Q^n / D^m$$

where $$\beta$$ is a coefficient which is generally related to the type of pipe and to the roughness, the coefficient $$n$$ has a value
generally around 2 and the coefficient \( m \) has value generally around 5.

For the case study, \( n = 2 \) and \( m = 5 \):

\[
J = \beta Q^2 / D^5
\]  

(13)

Now, it is possible to express the unit cost as:

\[
[C_a = \frac{K\beta^{0.5}}{Y^{0.5}} \cdot Q^{2\alpha}] 
\]  

(14)

Thus, the annual cost is equal to:

\[
[C_a = r \cdot \frac{K\beta^{0.5}}{Y^{0.5}} \cdot Q^{2\alpha}] 
\]  

(15)

Denoting by \( L \) the length of the pipeline, \( J = Y/L \) and the total cost will be:

\[
[C_a \cdot L = r \cdot \frac{K\beta^{0.5}}{Y^{0.5}} \cdot L^{(1+\alpha/5)} \cdot Q^{2\alpha}] 
\]  

(16)

Now it is possible to proceed with the mathematical formulation of the problem, where the variables to be determined are the amount of water resources transferred from origin to destination. The variable is, therefore, the flow \( Q_{ij} \) that the \( i \)-th source node (spring, well, derivation) must provide to the \( j \)-th municipality.

In cases where the availability of water exceeds the demand of the users, it is necessary to introduce a dummy destination; in the case, instead, in which the availability is less than the demand, it is necessary to introduce a dummy origin. The objective function and the constraints will be expressed as follows:

[Minimize \( z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}(Q_{ij}) \)]

(17)

[subject to \( \sum_{j=1}^{n} Q_{ij} \leq a_i \) for \( i = 1, 2, \ldots, m \)]

(18)

[and \( \sum_{i=1}^{m} Q_{ij} \geq b_j \) for \( j = 1, 2, \ldots, n \)]

(19)

[where \( Q_{ij} \geq 0 \) for all \( i \) and \( j \)]

(20)

where \( c_{ij} (Q_{ij}) \) is equal to:

\[
[r \cdot \frac{K\beta^{0.5}}{Y^{0.5}} \cdot L^{(1+\alpha/5)} \cdot Q_{ij}^{2\alpha}] 
\]  

(21)

The \( \alpha \) parameter can assume the value of 1; the product \( r \cdot K\beta^{0.5} / Y^{0.5} \) can be expressed in parametric form as it is constant (it may be indicated by the symbol \( A \)), and as mathematically it has no influence on the identification of the transfer of resource between source nodes and destination nodes. The solution that we want to be obtained through the modeling is, in fact, the identification of a set of links source nodes – destination nodes, which represent the proper distribution of drinking water supplies in the area under consideration. The effects of economies, and the impact of the procedure over the years, are evaluated through the tariff that includes the interest rate and the inflation rate.

The problem that is obtained, is no longer linear but a constrained non-linear type problem.

To solve the problem, it is necessary to translate the problem through the means of a suitable modeling language for mathematical programming. The results obtained by solving this model are the variables \( Q_{ij} \), which allow us to determine how much resource the generic source \( i \) can provide to the generic destination \( j \). Regarding the dummy destination, the flow transferred from the generic source node to that destination is to be considered a surplus of water that remains at the source node itself and that, therefore, it could, if necessary, be made available for other uses, while in the case of the dummy source, the flow represents a resource that must be integrated into the system.

The model can require several iterations; for example, some links between sources and destinations, as predicted by the model, may not be feasible due to problems orographic, structural, or other, making e/o impossible and the creation of the necessary work for the transfer of the resource inconvenient. The base solution, through possible further iterations, is susceptible to improvements, by imposing additional constraints in the links to be improved.
APPLICATION OF OPTIMIZATION PROCEDURE IN THE PROVINCE OF CROTON

In this section are shown the results of the first case study, relevant to the province of Croton in southern Italy. The province of Croton covers an area of about 1,700 km² and is the smallest province of the region Calabria, in Italy. The territory includes areas of archaeological interest with respect to the period of Magna Graecia, areas of geological interest and natural beauty. Important for the local economy is seaside tourism; the territory has, in fact, many tourist resorts and a marine protected area.

Regarding the water supply in the province of Croton, it is obtained from springs, derivations and wells and the total resource availability is sufficient compared to the overall needs of the users. However, the distribution of the resource through the various water supply schemes is not balanced and there are several municipalities that are affected by the shortage of water resource, especially in summer. For this reason the province of Croton is well suited to test and apply the optimization procedure.

For this application, it was necessary to collect all the data relating to the springs, the derivations and wells in the area of the province of Croton and all the data related to the demands of users within the territory itself.

Regarding the total availability of water, there are 29 springs, 3 derivations and 7 wells, while for the users there are 27 municipalities with a total demand (residents and fluctuating) of about 21 Mm³/year.

The data were organized into tables as shown schematically in Tables 1 and 2.

At this point, knowing the quotas of the source nodes and of the destination nodes it is possible to calculate the values of Yij, while on the basis of geographical coordinates it is possible to determine distances Lij as described schematically in Tables 3 and 4.

Regarding costs, it equals zero in the case of a source node already connected to a destination node and it is a very high amount in the case of negative values of Yij, that is, in the case in which transfer of the resource is not of the gravity type.

As already mentioned, the variables xij to be determined are the quantity of water resources transferred from each source node present in the territory of the province to the municipalities of the province itself.

In this case, we will have that the summation of availability is equal to:

\[ \sum_{i=1}^{39} a_i = 1534.8 \, \text{l/s} \]  

Table 1 | Schematic example of the data on the source nodes of the province of Croton

<table>
<thead>
<tr>
<th>Num.</th>
<th>Cod_ID</th>
<th>Coordinates</th>
<th>Flow (l/s)</th>
<th>Elevation (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>180300G0002S0001</td>
<td>653022.56 4344542.09</td>
<td>3</td>
<td>700</td>
</tr>
<tr>
<td>2</td>
<td>180300G0005S0001</td>
<td>648885.7 4344929.56</td>
<td>0.5</td>
<td>1000</td>
</tr>
<tr>
<td>3</td>
<td>180300G0005S0002</td>
<td>648868.99 4346073.09</td>
<td>0.5</td>
<td>1000</td>
</tr>
<tr>
<td>20</td>
<td>180200G1001S0009</td>
<td>645471.71 4332579.69</td>
<td>40</td>
<td>1672</td>
</tr>
<tr>
<td>21</td>
<td>180200G1001S0010</td>
<td>644306.91 4332419.14</td>
<td>15</td>
<td>1554</td>
</tr>
<tr>
<td>22</td>
<td>180200G1001S0011</td>
<td>645376.2 4329756.58</td>
<td>2.5</td>
<td>1469</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>39</td>
<td>180300G3029L0001</td>
<td>678395.11 4319318.23</td>
<td>203.9</td>
<td>160</td>
</tr>
</tbody>
</table>
The summation of demand is equal to:

\[ \sum_{j=1}^{n} b_j = \sum_{j=1}^{27} b_j = 922.8 \text{ l/s} \]  \hspace{1cm} (23)

In this case, the availability exceeds the demand, for which it is necessary to formulate the problem by introducing a dummy destination. To this destination will be assigned a demand equal to:

\[ b_{\text{dummy destination}} = \sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j = 611.99 \text{ l/s} \]  \hspace{1cm} (24)

The problem is characterized, therefore, by:

- 39 source nodes, \( m \);
- 28 destination nodes, \( n \);
- 1092 decision variables to be determined, \( x_{ij} \).

### Table 2 | Schematic example of the data relating to municipalities in the province of Croton

<table>
<thead>
<tr>
<th>Num.</th>
<th>Municipality</th>
<th>E(m)</th>
<th>N(m)</th>
<th>Demand (l/s)</th>
<th>Elevation (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Belvedere S.</td>
<td>664939.04</td>
<td>4341875.63</td>
<td>8.5</td>
<td>330</td>
</tr>
<tr>
<td>2</td>
<td>Caccuri</td>
<td>656779.15</td>
<td>4342649.43</td>
<td>7.4</td>
<td>646</td>
</tr>
<tr>
<td>3</td>
<td>Carfizzi</td>
<td>671649.36</td>
<td>4354986.53</td>
<td>3.5</td>
<td>512</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Isola di Capo Rizzuto</td>
<td>681158.5</td>
<td>4314488.13</td>
<td>158</td>
<td>90</td>
</tr>
<tr>
<td>14</td>
<td>Melissa</td>
<td>677232.58</td>
<td>4352632.43</td>
<td>21.3</td>
<td>256</td>
</tr>
<tr>
<td>15</td>
<td>Mesoraca</td>
<td>661869.6</td>
<td>4325605.67</td>
<td>37.9</td>
<td>415</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>Verzino</td>
<td>659264.21</td>
<td>4352371.89</td>
<td>8.2</td>
<td>550</td>
</tr>
</tbody>
</table>

### Table 3 | Schematic example of values of \( Y_{ij} \)

<table>
<thead>
<tr>
<th>Belvedere di Spinello</th>
<th>Caccuri</th>
<th>Carfizzi</th>
<th>–</th>
<th>–</th>
<th>Isola di Capo Rizzuto</th>
<th>Melissa</th>
<th>Mesoraca</th>
<th>–</th>
<th>–</th>
<th>Verzino</th>
</tr>
</thead>
<tbody>
<tr>
<td>180300G0002S0001</td>
<td>370</td>
<td>54</td>
<td>188</td>
<td>–</td>
<td>–</td>
<td>610</td>
<td>444</td>
<td>285</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>180300G0005S0001</td>
<td>670</td>
<td>354</td>
<td>488</td>
<td>–</td>
<td>–</td>
<td>910</td>
<td>744</td>
<td>585</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>180300G0005S0002</td>
<td>670</td>
<td>354</td>
<td>488</td>
<td>–</td>
<td>–</td>
<td>910</td>
<td>744</td>
<td>585</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>180200G1001S0009</td>
<td>1,342</td>
<td>1,026</td>
<td>1,160</td>
<td>–</td>
<td>–</td>
<td>1,582</td>
<td>1,416</td>
<td>1,257</td>
<td>1,122</td>
<td></td>
</tr>
<tr>
<td>180200G1001S0010</td>
<td>1,224</td>
<td>908</td>
<td>1,042</td>
<td>–</td>
<td>–</td>
<td>1,464</td>
<td>1,298</td>
<td>1,139</td>
<td>1,004</td>
<td></td>
</tr>
<tr>
<td>180200G1001S0011</td>
<td>1,139</td>
<td>823</td>
<td>957</td>
<td>–</td>
<td>–</td>
<td>1,379</td>
<td>1,213</td>
<td>1,054</td>
<td>919</td>
<td></td>
</tr>
<tr>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

The summation of demand is equal to:

\[ \sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j = 611.99 \text{ l/s} \]  \hspace{1cm} (24)
The objective function and the constraints will be expressed as follows:

\[
\text{minimize } z = \sum_{i=1}^{39} \sum_{j=1}^{28} c_{ij}(Q_{ij})
\] (25)

\[
\text{the following constraints } \sum_{j=1}^{28} Q_{ij} = a_i \quad \forall \ i = 1, 2, \ldots, 39
\] (26)

\[
\text{and } \sum_{i=1}^{39} Q_{ij} = b_j \quad \forall \ j = 1, 2, \ldots, 28
\] (27)

\[
\text{where } Q_{ij} \geq 0 \quad \forall i, j
\] (28)

where \( c_{ij}(Q_{ij}) \) is equal to:

\[
c_{ij}(Q_{ij}) = A \cdot \frac{L_y^{0.5}}{Y_q^{1.5}} \cdot Q_{ij}^5
\] (29)

For the solution of this model, a LINGO solver was used (LINGO Systems Inc.), which is a software package that allows us to formulate and solve linear and non-linear optimization problems while allowing the analysis of the results.

The results obtained have provided accurate indications on the redefinition of the distribution system in order to obtain a new, sustainable and optimal allocation of resources (Table 5); each municipality is satisfied in terms of resource availability; as the total availability, as already

| Table 4 | Schematic example of distances \( L_y \) between the source nodes and destination nodes |
|------------------|------------------|------------------|------------------|------------------|
| Belvedere di Spinello | Caccuri | Carfizzi | – | – | Isola di Capo Rizzuto | Melissa | Mesoraca | – | – | Verzino |
| 180300G0002S0001 | 12,211 | 4,206 | 21,355 | – | – | 41,169 | 25,526 | 22,729 | – | – | 10,013 |
| 180300G0005S0001 | 16,341 | 8,216 | 24,886 | – | – | 44,365 | 29,365 | 24,966 | – | – | 12,771 |
| 180300G0005S0002 | 16,609 | 8,619 | 24,462 | – | – | 45,169 | 29,112 | 25,958 | – | – | 12,155 |
| – | – | – | – | – | – | – | – | – | – | – |
| – | – | – | – | – | – | – | – | – | – | – |
| – | – | – | – | – | – | – | – | – | – | – |
| 180200G1001S0009 | 21,573 | 15,141 | 34,458 | – | – | 40,011 | 37,561 | 18,693 | – | – | 24,124 |
| 180200G1001S0011 | 23,013 | 17,212 | 36,426 | – | – | 38,904 | 39,219 | 17,603 | – | – | 26,539 |
| – | – | – | – | – | – | – | – | – | – | – |
| – | – | – | – | – | – | – | – | – | – | – |
| – | – | – | – | – | – | – | – | – | – | – |

| Table 5 | Schematic example of results |
|------------------|------------------|------------------|
| Source node (water intake structures) | Destination node (municipality) | Flow to transfer (l/s) |
| 180300G0002S0001 | Caccuri | 3 |
| 180300G0005S0001 | Castelsilano | 0.5 |
| 180300G0005S0002 | Castelsilano | 0.5 |
| – | – | – |
| 180300G1001S0009 | Isola di Capo Rizzuto | 37.56 |
| 180300G1001S0010 | Mesoraca | 2.44 |
| 180300G1001S0011 | Mesoraca | 15 |
| – | – | – |
| – | – | – |
| 180300G3029L0001 | Isola di Capo Rizzuto | 116.08 |

For the solution of this model, a LINGO solver was used (LINGO Systems Inc.), which is a software package that allows us to formulate and solve linear and non-linear optimization problems while allowing the analysis of the results.
stated exceeds the total needs, the application of the model also has identified the distribution systems that have a surplus of resource that can possibly be used, for other purposes and/or activities.

**CONCLUSIONS**

The rationalization of water resources can be obtained through a review of the distribution systems and the identification of optimal solutions to resource allocation. In this direction, this paper proposes an optimization model that has these characteristics:

- the formulation of the model is not complex in terms of type and number of input data, and in terms of mathematical modeling;
- it is well applicable to different territorial scales and can also be easily adapted to different types of distribution systems, other than drinking water distribution.

By virtue of the previous considerations, the most interesting perspectives are certainly related to the possibility that the managers of integrated water systems can use the model as a decision support tool and, therefore, as a tool able to improve the level of management efficiency of the service, and solve any specific and local supply problems.

**REFERENCES**


Milano, V. 1996 Acquedotti. Hoepli, Milano, Italy.


