Mixing at double-Tee junctions with unequal pipe sizes in water distribution systems
Tingchao Yu, Hongying Qiu, Jeffrey Yang, Yu Shao and Liang Tao

ABSTRACT
Pipe flow mixing with various solute concentrations and flow rates at pipe junctions is investigated. The degree of mixing affects contaminant spread in a water distribution system, and many studies have focused on mixing at the cross junctions; however, only a few have focused on double-Tee junctions of unequal pipe diameters. To investigate the solute mixing at such junctions, a series of experiments was conducted in a turbulent regime (Re = 12,500–50,000) with different Reynolds number ratios and connecting pipe lengths. Dimensionless outlet concentrations were found to depend on mixing mechanism at the impinging interface of junctions, where junctions with a larger pipe diameter ratio were associated with more complete mixing. Further, the inlet Reynolds number ratio affected mixing more strongly than the outlet Reynolds number ratio. Finally, the dimensionless connecting pipe length in a double-Tee played an important and complicated role in the flow mixing. The results were used to develop two-dimensional isopleth maps for the calculation of normalized north outlet concentrations.

Key words | double-Tee junctions, solute mixing, unequal pipe sizes, water distribution systems

INTRODUCTION
Contaminant transport in water distribution systems (WDSs) is a growing concern because of the potential occurrence of accidental or intentional contamination events that threaten the safety of WDSs (Ho 2008). Therefore, understanding the transport and mixing of either chemical or biological contaminants within WDSs is crucial to develop corresponding mitigation plans for contamination events (Romero-Gomez et al. 2011). EPANET software (Rossman 2000) is widely used to model the hydraulic and water quality variations in WDSs; however, it considers only cross-junctions, where it assumes that the mixing of solutes is complete and instantaneous. In contrast, recent research has shown incomplete mixing is widespread at pipe junctions (Fowler & Jones 1991; Ashgriz et al. 2001; Webb & van Bloemen Waanders 2006; Austin et al. 2008; Yu et al. 2014a), and thus the complete-mixing assumption is often a significant source of discrepancy between model predictions and field measurements of solute concentrations.

The assumption of instantaneous and complete mixing is unrealistic because of the limited interaction time between the two inlet flows at the cross junctions. Orear et al. (2005) concluded that transient instabilities at the impinging interface significantly affected the mixing at joints. Van Bloemen Waanders et al. (2005) found incomplete mixing as a result of the reflection of equal inlet flows at a cross junction. However, if the incoming flow rates are unequal, the degree of mixing increases but is still incomplete (Romero-Gomez et al. 2006). Several recent studies (Ho et al. 2006; McKenna et al. 2007; Webb 2007; Ho 2008; Romero-Gomez et al. 2008) have presented experimental and numerical results of single cross-junctions, which show...
incomplete mixing because of transient instabilities at the impinging interface. Computational and experimental investigations by Romero-Gomez et al. (2008), Austin et al. (2008), and Choi et al. (2008) identified the Reynolds number ratios of inlets and outlets as the significant mixing variables. More recently, Yu et al. (2014a) found that the Reynolds number ratio of inlets was the most important factor, followed by the pipe diameter ratio and the Reynolds number ratio of outlets.

For the Tee junctions that are common in a WDS, flow mixing has been investigated for decades – see for example Breidenthal (1981) and Holdeman (1993). More recently, Plesniak & Cusano (2005) found that flow mixing increased with turbulence at the Tee junction interface, while Webb & Van Bloemen Waanders (2006) used computational fluid dynamics models to simulate mixing behaviors at double-Tee junctions with 2.5 times the diameter of the connecting-length pipe. They found dimensionless concentrations for the two outlets of 0.59 and 0.41, respectively. Such results differed considerably from the experimental findings of Yu et al. (2014b). In Shao et al. (2014), the flow and solute mixing for double-Tee junctions were reasonably described by an analytical solution. Flow mixing for the unequal pipe diameters was not discussed.

The present study further extends the work of Yu et al. (2014b) to the mixing behaviors at double-Tee junctions of unequal pipe sizes. The aim is to examine the governing mechanisms that determine flow-mixing behavior under variable flow conditions, leading to the development of a generalized model for pipe junction mixing.

MATERIALS AND METHODS

Mixing parameters

The Tee-junction joint configuration used in the experiments below comprised two adjacent inlets, a connected pipe, and two outlets, as shown in Figure 1. Pipes were labelled using geographic notations as W (west inlet, tracer water with high concentration $C_W$), S (south inlet, clean water with low concentration $C_S$), E (east outlet, concentration $C_E$), and N (north outlet, concentration $C_N$). Sodium chloride (NaCl) was used as the tracer. The main pipe had a diameter of $D = D_S = D_N$ and the branches had a diameter of $D_W = D_E$.

To describe the degree of mixing, the dimensionless concentration ($C_N$) for the north outlet was introduced (Ho 2008) as:

$$C_N = \frac{C_N - C_S}{C_W - C_S}.$$  

Experimental method

Branch pipes are usually smaller than the main pipe, with a minimum of half the diameter of the main pipe at most
junctons. Initially, three pential diameter ratios \((D_W/D = 25:25, 25:32\) and \(25:50\)) were considered for an investigation of the effect of pipe diameters on mixing, and because equal pipe diameters were studied in Yu et al. (2014b), two unequal diameter ratios \((D_W/D = 25:32\) and \(25:50\)) were ultimately selected. In terms of pipe length, the minimum of the dimensionless connecting pipe length \((L/D)\) is 2.5 to allow sufficient space between the two branch pipes for the installing operation. Further, because a dimensionless connecting pipe length \((L/D)\) greater than 10.0 should produce complete mixing (Shao et al. 2014), four dimensionless connecting pipe lengths \((L/D = 2.5, 5.0, 7.5, \) and \(10.0\)) were included in the study. Finally, the hydraulic parameters \(Re_{W/S}\) and \(Re_{N/E}\) were chosen. All experimental cases are listed in Table 1. Note that the experimental setup and approach were based on the work of Yu et al. (2014b), and every experiment was repeated three times to obtain average results. The percent mass fraction error, as defined by McKenna et al. (2007), ranged from \(-1.0\) to \(3.0\%\) for most experimental runs, indicating well-controlled experimental conditions and measuring apparatuses.

**RESULTS AND DISCUSSION**

The experimental results at double-Tee junctions with different \(L/D\) and \(D_W/D\) were used to determine dimensionless concentrations, \(C_n\), under varying flow conditions. \(C_n = 1\) indicates equal concentrations at the west inlet and at the north outlet for a perfect flow bifurcation, while \(C_n = 0\) means that the concentrations at the south inlet and north outlet are equal.

**Effect of flows on mixing**

The experimental results show the significant effect of relative flow rates on solute mixing at pipe junctions. The \(C_n\) values were significantly affected by \(Re_{W/S}\) when \(Re_{N/E} = 1\) (Figure 2(a) and 2(b)). As \(Re_{W/S} \rightarrow 0\), the \(C_n\) value at double-Tee junctions tended toward that of cross junctions: specifically, a small to insignificant flow from the west inlet was able to penetrate and mix with the large flow from the south inlet. Conversely, the \(C_n\) value indicated complete mixing because the west inlet flow increased relative to south inlet flow under an increasing \(Re_{W/S}\). When the flow velocity at the west inlet was twice as large as the velocity at the south inlet (i.e., \(Re_{W/S} \geq 1.50\) for \(D_W/D = 25/32\) or \(Re_{W/S} \geq 1.0\) for \(D_W/D = 25/50\)), the \(C_n\) value coincided with that of complete mixing for double-Tee junctions with \(L/D \geq 5.0\).

Flow mixing at double-Tee junctions with equal pipe diameters is a special condition, with \(C_n\) values larger than that of complete mixing, but smaller than that of cross-mixing (Yu et al. 2014b). In such a configuration, connecting pipe segments offer more space and time for flow mixing than the cross junctions. However, Figure 2(b) shows that the \(C_n\) value at double-Tee junctions with \(D_W/D = 25/50\) exceeded that of cross mixing, when \(Re_{W/S} = 2.0\) and \(L/D \geq 5.0\), or \(Re_{W/S} = 4.0\) and \(L/D \geq 2.5\), where the high-speed flow from the west inlet penetrated the main pipe

### Table 1 | List of the experimental conditions

<table>
<thead>
<tr>
<th>Group</th>
<th>(D_W/D)</th>
<th>(L/D)</th>
<th>(Re_{W/S})</th>
<th>(Re_{N/E})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25/32</td>
<td>2.5</td>
<td>0.25,0.5,0.67,1,1.5,2,0.4,0</td>
<td>0.25,0.5,0.67,1,1.5,2,0.4</td>
</tr>
<tr>
<td>2</td>
<td>25/32</td>
<td>5.0</td>
<td>0.25,0.5,0.67,1,1.5,2,0.4,0</td>
<td>0.25,0.5,0.67,1,1.5,2,0.4</td>
</tr>
<tr>
<td>3</td>
<td>25/32</td>
<td>7.5</td>
<td>0.25,0.5,0.67,1,1.5,2,0.4,0</td>
<td>0.25,0.5,0.67,1,1.5,2,0.4</td>
</tr>
<tr>
<td>4</td>
<td>25/32</td>
<td>10.0</td>
<td>0.25,0.5,0.67,1,1.5,2,0.4,0</td>
<td>0.25,0.5,0.67,1,1.5,2,0.4</td>
</tr>
<tr>
<td>5</td>
<td>25/50</td>
<td>2.5</td>
<td>0.25,0.5,0.67,1,1.5,2,0.4,0</td>
<td>0.25,0.5,0.67,1,1.5,2,0.4</td>
</tr>
<tr>
<td>6</td>
<td>25/50</td>
<td>5.0</td>
<td>0.25,0.5,0.67,1,1.5,2,0.4,0</td>
<td>0.25,0.5,0.67,1,1.5,2,0.4</td>
</tr>
<tr>
<td>7</td>
<td>25/50</td>
<td>7.5</td>
<td>0.25,0.5,0.67,1,1.5,2,0.4,0</td>
<td>0.25,0.5,0.67,1,1.5,2,0.4</td>
</tr>
<tr>
<td>8</td>
<td>25/50</td>
<td>10.0</td>
<td>0.25,0.5,0.67,1,1.5,2,0.4,0</td>
<td>0.25,0.5,0.67,1,1.5,2,0.4</td>
</tr>
</tbody>
</table>

*Note: Diameters of main pipes are 32 mm and 50 mm, diameter of the branch pipe size is 25 mm.*
flow. For the cross junctions, the flow may directly enter the opposite east outlet. However, for double-Tee junctions with certain connecting pipe segments (i.e., L/D ≥ 5.0), the flow may be impeded from opposite pipe wall, allowing more mixing in the collection pipe segment (Shao et al. 2014); as a result, less tracer water flows into the east outlet as compared with the cross-mixing conditions.

Where the \( C_N \) values lay between complete mixing and cross mixing (Figures 2(c) and 2(d)), the general \( C_N \) variations showed little change with varying \( Re_{N/E} \), which indicates that mixing at double-Tee junctions is insensitive to the Reynolds number ratio of the outlets. For the junction of \( D_{W/D} = 25/32 \), the mixing was closer to cross mixing than complete mixing for \( L/D = 2.5 \), while complete mixing was approached when \( L/D ≥ 5.0 \). For the junction of \( D_{W/D} = 25/50 \), the \( C_N \) value matched that for complete mixing.

**Effect of pipe sizes on mixing**

The pipe diameter ratio (\( D_{W/D} \)) can affect the degree of mixing at double-Tee junctions because it changes the junction geometry. Figure 3 shows normalized north outlet concentrations for \( L/D = 5.0 \) and \( D_{W/D} = 25/25, 25/32, \) and \( 25/50 \), when \( Re_{W/S} \) is made to equal \( Re_{E/N} \) through adjustment of the flow rates. Note that the \( D_{W/D} = 25/25 \) results are from Yu et al. (2014b). The different pipe diameters resulted in different flow velocities for a given Reynolds number or flow rate, and these velocity differences then caused intense turbulence during mixing. In addition, inlet flows from smaller pipes were encircled by the flow in the large pipe (e.g., Choi et al. 2008), which increased the contact surface of the two flows. Therefore, larger diameter differences between the branch and main pipes led to improved mixing at the double-Tee junctions.
The pipe-diameter ratio had a similar effect on mixing for other dimensionless connecting pipe lengths of $L/D = 2.5, 7.5, \text{ and } 10.0$.

Contour graphs of $C_N$ and its usage

For $D_W/D = 25/32$ and $25/50$, mixing experiments were conducted under 196 ($7 \times 7 \times 4$) flow conditions (Table 1), with each condition tested three times. The experimentally-determined $C_N$ values are shown in logarithmic plots of $Re_{W/S}$ and $Re_{N/E}$. The $C_N$ contour graphs for each dimensionless connecting pipe length are shown in Figures 4 and 5, while the $C_N$ values for equal pipe diameters ($D_W/D = 25/25$) can be found in Shao et al. (2014) and Yu.
et al. (2014b). For junctions of pipe diameters that satisfy the following flow conditions, \(-1.0 \geq \frac{D_W}{D} \geq 0.5\), \(10 \geq \frac{L}{D} \geq 0\), \(4.0 \geq \frac{Re_{W/S}}{C_21} \geq 0.25\), and \(4.0 \geq \frac{Re_{N/E}}{C_21} \geq 0.25\), linear interpolation can be used to obtain the predicted \(C_N\) value from the available contour graphs.

Figures 4 and 5 show clearly that \(C_N\) values decrease with an increase in \(L/D\), for given \(Re_{S/W}\) and \(Re_{N/E}\), which demonstrates that the connecting pipe segments promoted mixing at the junctions. Further, the \(C_N\) value increased as \(Re_{W/S}\) increased, but changed only slightly with \(Re_{N/E}\) and nearly paralleled the \(Re_{N/E}\) axes—especially when \(L/D = 7.5\) and 10.0.

The top-left corner of the contour map shows an extreme case of a very small inlet Reynolds number ratio \((Re_{W/S} < 0.25)\) coupled with a large outlet Reynolds number ratio \((Re_{N/E} > 1)\). This situation indicates that momentum in the main pipe was much larger than for the branch pipe, which caused nearly all of the tracer water to flow directly north and resulted in a bulk mixing. The bottom right corner of the contour map also shows an extreme case of a large inlet Reynolds number ratio \((Re_{W/S} > 1)\) and an extremely small outlet Reynolds number ratio \((Re_{N/E} < 0.25)\). Here, the momentum in the main pipe was smaller than that in the branch pipe, and the tracer water penetrated through the water in the main pipe and resulted in complete mixing. Under general conditions when \(0.25 \leq \frac{Re_{W/S}}{C_20} \leq 1\) and \(0.25 \leq \frac{Re_{N/E}}{C_20} \leq 1\), the hydraulic conditions and the geometric characteristics of junctions \((D_W/D \text{ and } L/D)\) had a more complicated effect on mixing. Additional experiments and computational studies were necessary to determine the mixing characteristics at double-Tee junctions of flows ranging from laminar to transitional flows.

Figure 5 | Contour graphs of \(C_N\) with \(D_W/D = 25/50\): (a) \(L/D = 2.5\); (b) \(L/D = 5.0\); (c) \(L/D = 7.5\); and (d) \(L/D = 10.0\).
CONCLUSIONS

Solute mixing phenomena at double-Tee junctions with unequal pipe diameters were investigated. The results should aid in predicting solute concentrations and quantifying water quality variations in a distribution pipe. The effect of the inlet and outlet Reynolds number ratios and the connecting pipe length on mixing were found to increase in complexity when unequal pipe diameters were considered. The larger the difference in the pipe diameters, the more complete the mixing. In addition, the inlet Reynolds number ratio exerted a greater impact on mixing than the outlet Reynolds number ratio, while the connecting pipe also promoted mixing at the connecting segments. These observations were generalized using two-dimensional contour graphs for a normalized north outlet concentration, based on which linear interpolation can be used to obtain predicted outlet solute concentrations for other flow and junction conditions.

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