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Drought class transition analysis through different models: a case study in North China

Ting Zhang, Jianzhu Li, Rong Hu, Yixuan Wang and Ping Feng

ABSTRACT

The standardized precipitation index (SPI) and standardized runoff index (SRI) are computed for several gauge stations in Panjiakou Reservoir catchment of Luanhe Basin, a drought prone region of North China. Based on the SPI and SRI time series, two different models, a weighted Markov chain model and a Volterra adaptive filter model for chaotic time series, were established to predict drought classes and achieve both short- and long-term drought forecasting. These approaches were compared with a three-dimensional (3D) loglinear model, reported in our previous work. It was observed that all the three models have pros and cons when applied to drought prediction in Panjiakou Reservoir catchment. The 3D loglinear model is able to forecast drought class within 1 month. However, its predicting accuracy declines with the increase of prediction time scale, and this confines its application. The weighted Markov chain model is a useful tool for drought early warning. Its precision, which is significantly related to the stable condition of drought classes, is highest for Non-drought, followed by Moderate and Severe/Extreme drought, and lowest for Near-normal. The Volterra adaptive filter model for chaotic time series combined the phase space reconstruction technique, Volterra series expansion technique and adaptive filter optimization technique, and was for the first time used in a drought class transition study. This model is effective and highly precise in long-term drought prediction (for example, 12 months). It is able to provide reliable information for the medium- and long-term decisions and plans for water resources systems.

Key words | drought class prediction, Panjiakou Reservoir catchment, three-dimensional loglinear model, Volterra adaptive filter, weighted Markov chain

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INTRODUCTION

Droughts are recognized as one of the worst environmental disasters. It is for this reason that this issue has become the center of attention in various fields such as hydrology, meteorology and agriculture (Huang *et al.* 2015; Wang *et al.* 2015). Droughts may happen in all climatic zones, involving high and low rainfall areas. They are mostly related to the reduction in the amount of precipitation received over a certain period of time (Mishra & Singh 2010). According to the various forms and objects of influence, droughts are classified into meteorological drought, agricultural drought, hydrological drought, etc. (Council 1997). Meteorological drought can generally represent the others, whereas, the hydrological drought is the link between the other two types of droughts. In order to characterize hydrological drought, a variety of hydrological drought indicators have been proposed to reflect the water shortage caused by the reduction in the amount of surface runoff and groundwater level. Hydrological drought focuses on the changes of river runoff. The formation process of runoff involves precipitation, plant distribution, soil water evaporation, infiltration and lateral water movement. This represents the whole physical evolution process of the water over the underlying surface. Therefore, runoff-based hydrological drought indicators are often used to represent

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the water resources shortage in hydrological droughts. Shukla & Wood (2008) proposed the standardized runoff index (SRI) on the basis of the theory of computing standardized precipitation index (SPI) to characterize hydrological droughts. Many commonly used hydrological drought indices such as, cumulative flow distance (Keyantash & Dracup 2002) and surface water supply index (Shafer & Dezman 1982), require high-quality data and a large amount of calculation. They ignore the influence of seasonal runoff variation on droughts, and are unable to quantify the drought severity. On the contrary, SRI can well make up for these deficiencies, having the same advantages as SPI in practical applications.

The global climate change and growing social water demand aggravate the water shortage and drought problem of Luanhe Basin. Therefore, it is crucial to study the drought class prediction in Panjiakou Reservoir basin (a major part of Luanhe Basin) from the standpoints of both meteorological and hydrological droughts. This study established a weighted Markov chain model based on the SPI and SRI time series of Panjiakou Reservoir catchment. For the first time, Volterra adaptive filter model is adopted to predict the future SPI and SRI drought classes. The results of these two models are compared with a three-dimensional (3D) loglinear model, studied in our previous work (Li *et al.* 2015).

Amongst these three models, the loglinear model is easy to understand and calculate (Moreira *et al.* 2006). When applied in actual drought mitigation, it is able to provide reliable qualitative prediction results, e.g. mild, moderate and severe drought class etc. This assists in making satisfactory drought mitigation decisions. Paulo *et al.* (2005) adopted Markov chain and 3D loglinear prediction methods to analyze the SPI class transition process and pointed out that the latter applied in meteorological drought simulation and could provide early warning for droughts. Since meteorological drought is the fundamental cause of hydrological drought, and the SRI time series is in good consistency with SPI time sequence, the loglinear model is also applicable in hydrological drought prediction.

Markov Andrey first proposed the Markov chain method in the early 20th century. This method represented a discrete time stochastic process with Markov characteristics in mathematics. The weighted Markov chain method is an improvement on the basis of the Markov chain method. It has the advantage of being highly precise and has a strong scientific foundation. The Markov chain method has been applied in the research of drought prediction. Feng & Han (1999) first proposed this method in China and successfully applied it in the prediction of river runoff state. Chen & Yang (2012) predicted the drought classes by weighted Markov chain model based on SPI and computed the prediction accuracy for the SPI of different time scales. The results showed that this model is an effective tool for drought prediction and can provide decision-making for regional drought management. In the work of Avilés *et al.* (2015), two stochastic models, the Markov Chain First Order and the Markov Chain Second Order models, were applied to predict the frequency of monthly droughts. Their performance was checked using two skill scores. The results indicated that events with greater drought severity were more accurately predicted.

In this paper, a Volterra adaptive filter prediction model has been first proposed with a combination of phase space reconstruction technique, Volterra series expansion theory and adaptive filter optimization technique. The polynomial filter on the basis of Volterra series expansion is especially suitable for the adaptive filtering and prediction of a nonlinear system. The resulting output is still the linear combination of its expansion, making it possible to obtain a global optimal solution. Therefore, such an approach is applicable in the nonlinear prediction of hydrological time series. This study has adopted the method to forecast medium and long-term droughts in Panjiakou Reservoir catchment.

Through these methods, the forecast of short-, medium- and long-term droughts has been achieved and the predicting ability of these models has been analyzed. The specific objectives of the study are as follows:

(1) Computation of SPI and SRI time series based on the monthly rainfall and runoff data of 21 rainfall gauge stations and 9 hydrological stations, and the derivation of meteorological (SPI-based) and hydrological (SRI-based) drought class time series according to the classification criteria.
(2) One month lead drought prediction through a weighted Markov chain modeling approach with the correlation coefficient of each order as weight.
(3) Establishment of a Volterra adaptive filter model, in which the SPI and SRI time series will be extended to

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multi-dimensions through the phase reconstruction method of chaotic theory and then expanded to Volterra functional series, in which 12 months' lead drought class forecast has been achieved.

(4) Comparison with the 3D loglinear model in our previous work, in which a drought forecast with 2-month foreseeable period was achieved.

DATA

Luanhe Basin covers an area of 44,750 km^2, where rainfall is concentrated into the summer, and rare during the spring and winter due to the climate, topography and other factors. Panjiakou Reservoir was built at the end of the flood period in 1979. It is located in the Yanshan Mountain area at the junction of Tangshan and Chengde in Hebei Province, North China. The main function of the reservoir is to supply water, followed by flood control and power generation. The multi-year average runoff of Panjiakou Reservoir catchment is 2.45 billion m^3. This accounts for more than 50% of the total water amount. The control area reaches 33,700 km^2, which is 75.3% of Luanhe Basin. The global climate change and increase of demand for water have led to acute water shortage. This has resulted in droughts intensifying day by day in Luanhe Basin. Therefore, it is crucial to study the drought class prediction in Panjikou Reservoir catchment, as this issue has real-world significance.

At present, there are 14 hydrological stations and 107 rainfall gauge stations in Panjiakou Reservoir catchment. They are mainly distributed along the Luanhe River and its tributaries. In this study, 21 gauge stations with relatively complete observation data are chosen (detailed information of the stations can be seen in Table 1 and the locations are shown in Figure 1). Monthly precipitation data of 21 stations and monthly runoff data of 9 stations out of the 21 (Sandaohezi, Goutaizi, Boluonuo, Hanjiaying, Xiahenan, Chengde, Liying, Pingquan, Xiabancheng, etc.) of the period 1958–2009 have been considered.

METHODS

SPI and SRI time series are the main focus of this study. The two models, a weighted Markov chain model and a Volterra adaptive filter model, are established to predict the drought class based on them and compared against a 3D loglinear model (Li *et al.* 2015).

SPI and SRI

SPI

Edwards & McKee (1997) proposed the meteorological drought index SPI. SPI has multi-timescale features and only requires a relatively long sequence of precipitation data (more than 30 years) for calculation.

Table 1 | Information of the selected gauge stations in Panjiakou Reservoir catchment

Station	River	Station type	Station	River	Station type
Sandaohezi	Luanhe	Rainfall, runoff	Xuanjiangyingzi	Luanhe	Rainfall
Goutaizi	Xiaoluanhe	Rainfall, runoff	Miaogong Reservoir	Yinxunhe	Rainfall
Boluonuo	Xingzhouhe	Rainfall, runoff	Zhangsanying	Yinxunhe	Rainfall
Hanjiaying	Yixunhe	Rainfall, runoff	Banjieta	Yimatuhe	Rainfall
Xiahenan	Yimatuhe	Rainfall, runoff	Baihugou	Yimatuhe	Rainfall
Chengde	Wuliehe	Rainfall, runoff	Qijia	Wuliehe	Rainfall
Liying	Liuhe	Rainfall, runoff	Xiaoxishan	Wuliehe	Rainfall
Pingquan	Puhe	Rainfall, runoff	Sangou	Laoniuhe	Rainfall
Xiabancheng	Laoniuhe	Rainfall, runoff	Xinglong	Liuhe	Rainfall
Yudaokou	Xiaoluanhe	Rainfall, runoff	Kuancheng	Puhe	Rainfall
Jiutun	Luanhe	Rainfall, runoff			

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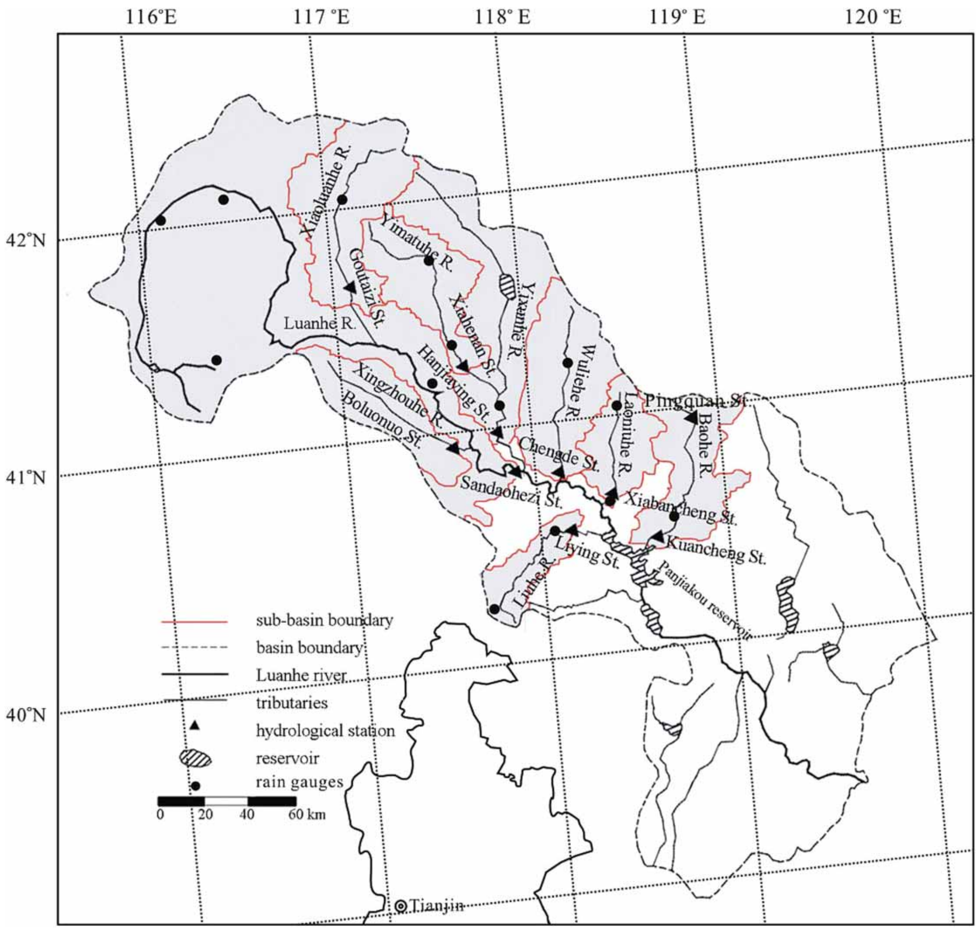

Figure 1 | Distribution of gauge stations and tributaries of Panjiakou Reservoir catchment.

The precipitation shows a skewed distribution rather than a normal distribution. As a consequence of this, a gamma probability distribution is used to describe the change of precipitation distribution. Cumulative probability of the given time scale is computed and then transformed into standard normal distribution function, based on which the drought classification is conducted. Therefore, there is no variation in temporal or spatial distribution when employing SPI for characterizing drought severity.

Assuming Γ to represent the rainfall amount of a given time scale, the probability density function of Γ is defined as follows:

$$g(x) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}, \tag{1}$$

where, $\alpha > 0$, $\beta > 0$ are the shape parameter and scale parameter respectively, $\Gamma(\alpha)$ is a gamma function. The best estimate of α, β can be obtained by the maximum likelihood estimation method:

$$\hat{\alpha} = \frac{1 + \sqrt{1 + 4A/3}}{4A},$$

$$\hat{\beta} = \bar{x}/\hat{\alpha}, \tag{2}$$

where, $A = \ln(\bar{x}) - \sum \ln(x)/n$, and n is the length of rainfall time series.

G(x) is the distribution function of the rainfall amount x, and defined as:

$$G(x) = \frac{1}{\Gamma\hat{\alpha}} \int_0^x t^{\alpha-1} e^{-t} dt, \tag{3}$$

where, $t = x/\hat{\beta}$.

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Considering a situation of zero rainfall amount, gives the cumulative probability of a given time scale as shown below:

$$H(x) = u + (1 - u)G(x), \tag{4}$$

where, u is the probability of rainfall amount 0, $u = m/n$, m is the number of 0s in the rainfall time series.

Upon converting the cumulative probability $H(x)$ to standard normal distribution function, the SPI can be defined as the following.

When $0 < H(x) \leq 0.5$,

$$SPI = -\left(k - \frac{2.52 + 0.8k + 0.01k^2}{1 + 1.43k + 0.19k^2 + 0.001k^3}\right),$$

$$k = \sqrt{\ln\left(1/H(x)^2\right)}; \tag{5}$$

When $0.5 < H(x) \leq 1.0$,

$$SPI = k - \frac{2.52 + 0.8k + 0.01k^2}{1 + 1.43k + 0.19k^2 + 0.001k^3},$$

$$k = \sqrt{\ln\left(1/(1 - H(x)^2)\right)}. \tag{6}$$

SRI

The SRI is a hydrological drought index proposed by Shukla & Wood (2008). The calculation principle is the same as that of SPI, only requiring relatively long time series of runoff data (more than 30 years). Similar to SPI, SRI can be used under different time scales (e.g. 1 month, 3 months, 6 months, 12 months, etc.).

According to drought classification standards (Moreira *et al.* 2008), droughts are categorized into four classes: non-drought, near-normal drought, moderate drought and severe/extreme drought (see Table 2).

Weighted Markov chain model

A Markov chain (Çinlar 1975) is a stochastic sequence in a probability space. The status space satisfies such a property when given X_t, X_{t+1} is conditionally independent of $X_1, X_2, \ldots, X_{t-1}$. Also, the probability that X_{t+1} takes a particular value i relies on the previous value only through X_t, known as the Markov property, which is expressed as the following:

$$P(X_{t+1} = x | X_1 = x_1, X_2 = x_2, \ldots, X_t = x_t) = P(X_{t+1} = x | X_t = x_t) \tag{7}$$

Table 2 | Drought classifications of SPI and SRI (from Moreira *et al.* 2008)

Code	Drought classes	SPI or SRI values
1	Non-drought	SPI ≥ 0
2	Near-normal	$-1 <$ SPI < 0
3	Moderate	$-1.5 <$ SPI $\leq$ -1
4	Severe/extreme	SPI $\leq$ -1.5

The characteristics of a Markov chain are described by a set of states E and the transition probability P_{ij} between states. The transition probability P_{ij} signifies the probability that the sequence transfers from state i at the current time point (t) to state j at the next time point ($t + 1$). The method of computing SPI and SRI, from moving precipitation and runoff observation data, and the assignment of each SPI or SRI value to a drought class makes it suitable for Markov chain modeling.

In the long run, the drought class probabilities are independent of the initial state of the Markov chain i. Therefore, they are referred to as steady-state probabilities and are often defined as the following:

$$\lim_{n\to\infty} = P_{ij}(m) = \pi(j), \tag{8}$$

where, $P_{ij}(m)$ is the probability with m steps transition, $\{\pi(j), j \in E\}$ is the only stationary distribution of the Markov chain.

Passing the Markov property test is the precondition for using the Markov chain model. For a drought index time sequence that has passed the Markov property test, the weighted Markov chain from the correlated lag time of each order is required for predicting the index value of a certain future period. The auto-correlated coefficient of each order represents the correlation level of the index at each order of time lag. Therefore, the drought class of the present period should first be predicted on the basis of the corresponding classes of the index values of previous periods. Subsequently, the correlation level between the present period and previous periods

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should be used to calculate the weighted sum of the corresponding transition probability. Finally, the drought class of the current period can be predicted according to the weighted sum. Hence, following these steps in the weighted Markov chain model makes it capable of forecasting drought class.

SPI and SRI time series are dependent random variables. Considering their continuous multi-order interaction effects, the procedures for drought class prediction using weighted Markov chain model are as follows:

(1) Determine the state space of the Markov chain, namely, the classification standard of the SPI (SRI) index. Hereon, the standard proposed by McKee *et al.* (1993) is adopted (see Table 2) and the state space $E = \{1, 2, 3, 4\}$.
(2) Calculate the SPI (SRI) index, and classify the drought level according to the classification standard. Compute the transition probability matrix of the Markov chain at each time step. Subsequently, through the Markov property test determine whether the SPI (SRI) index time series possesses Markov properties or not.
(3) Calculate the auto-correlation coefficient of each order:

$$r_k = \sum_{t=1}^{n-k}(x_t - \bar{x})(x_{t+k} - \bar{x}) / \sum_{t=1}^{n}(x_t - \bar{x})^2, \tag{9}$$

where, k is the step size, r_k is the autocorrelation coefficient of the k^{th} order time lag, x_t is the SPI (SRI) value of the t^{th} time step, $\bar{x}$ is the mean value, and n the length of the time series. In this paper, $k = 1, 2, 3, 4$, namely, the step sizes are 1 month, 2 months, 3 months and 4 months.

Compute the weight of the Markov chain with each step size:

$$w_k = |r_k| / \sum_{k=1}^{4} r_k. \tag{10}$$

(4) Use the SPI (SRI) class of each step size before the predicted month as the initial value, and predict the occurrence probability $p_t^{(k)}(t \in E)$ of the SPI (SRI) class of the present month through the corresponding transition probability matrix of each step size.
(5) Calculate the weighted sum of the occurrence probability $p_t^{(k)}$ of the same class in the predictions and obtain the prediction probability P_t of each class:

$$P_t = \sum_{k=1}^{4} w_k p_t^{(k)}, \ (t \in E). \tag{11}$$

(6) Find the maximum value of the prediction probability of each class computed in step (5), i.e. $\max\{P_t, t \in E\}$, where the corresponding t is the prediction class of the SPI (SRI) in the current period. Once the SPI (SRI) class is determined, it is added to the original sequence and steps (2)–(6) are repeated for the prediction of the next period.
(7) Analyze the characteristics of the Markov chain (ergodicity and stationary distribution, etc.).

Volterra adaptive filter model for chaotic time series

This paper for the first time combined the theory of phase space reconstruction and the Volterra functional series method to predict the medium- and long-term droughts. Through the phase reconstruction, the one-dimensional SPI and SRI time series were expanded to multi-dimensional, and then a Volterra functional series expansion was conducted. In this way, a Volterra adaptive filter model was set up for 12 months' lead drought class prediction. This model predicts drought class through the following steps:

Determine the chaotic characteristics of the hydrological time series, reconstruct the phase space, and determine the embedding dimensions m and delay time τ. The input vector is expressed as:

$$(x(n), x(n-\tau), \ldots, x(n-(m-1)\tau)), \tag{12}$$

with expected response $x(n+1)$.

Determine the order of Volterra adaptive filter, the second order in this paper, and expand the second order Volterra series variables:

$$x(n+1) = \sum^{m-1} h_1(i_1)x(n - i_1\tau) + \sum^{m-1} h_2(i_1, i_2)x(n - i_1\tau)x(n - i_2\tau). \tag{13}$$

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Based on the expanded expression of the second order Volterra series variables, determine the input signal of the filter:

$$U(n) = [1, x(n), x(n-\tau), \ldots, x(n-(m-1)\tau), \\ x^2(n), x(n)x(n-\tau), \ldots, x^2(n-(m-1)\tau)]^T, \tag{14}$$

and the filter coefficient vector $H(n)$.

Determine the adaptive parameters of the filter (convergence factor μ, and constant φ).

Iteratively compute the second order Volterra filter prediction model through the NLMS approach with an estimation error:

$$e(n) = x(n+1) - U^T(n)\hat{H}(n), \tag{15}$$

and the iterative formula:

$$\hat{H}(n+1) = \hat{H}(n) + \frac{\mu}{\varphi + U^T(n)U(n)} U(n)e(n). \tag{16}$$

Subsequently, calculate the root-mean-square error.

Use the established model for single and multi-step predictions.

RESULTS AND DISCUSSION

Weighted Markov chain model

The monthly SPI and SRI class sequences from January 1959 to December 2008 are considered in order to establish a weighted Markov chain model for predicting the drought class of the 12 months of 2009. The foreseeable period is 1 month, which helps to demonstrate the 1-month lead drought class prediction ability of weighted Markov chain prediction approach. Meteorological and hydrological drought class prediction results of 4 (21 in total) stations are shown in Figures 2 and 3, respectively. It can be observed that there is a close agreement between most of the predicted drought classes with the observation. Despite a few inconsistencies, the predicted classes align to the observed levels. For example, in Figure 2 the observed drought class of June 2009 is '3' while the predicted class is '4'. In this case, despite the drought class not being accurately predicted, the occurrence of drought has been given and the predicted class is close to the actual level. Therefore, this case is considered as an accurate prediction in the statistics. Tables 3 and 4 respectively show the statistical rate of prediction accuracy of meteorological and hydrological drought class.

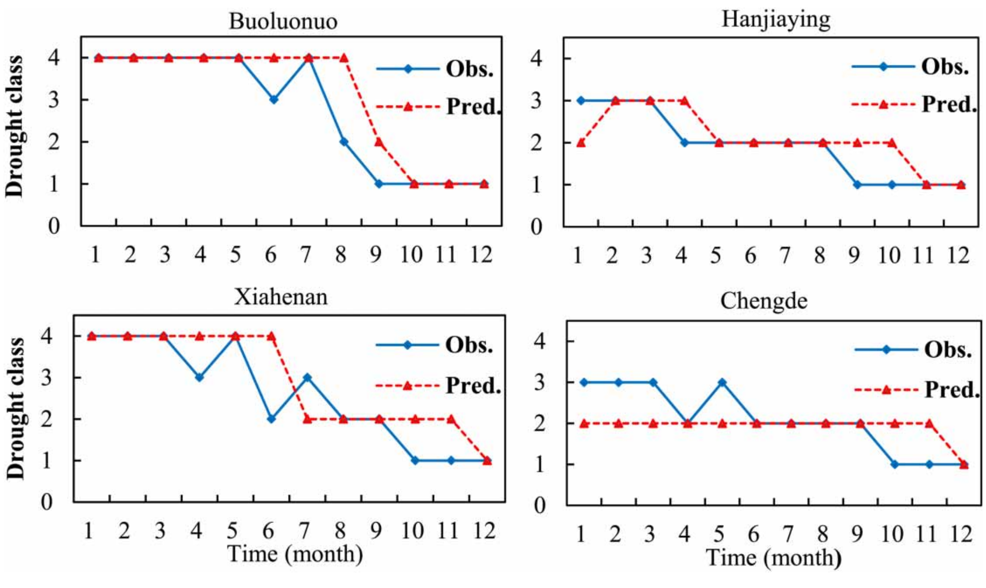

Figure 2 | Observed and predicted meteorological drought class results of 2009 through weighted Markov chain model (SPI-based).

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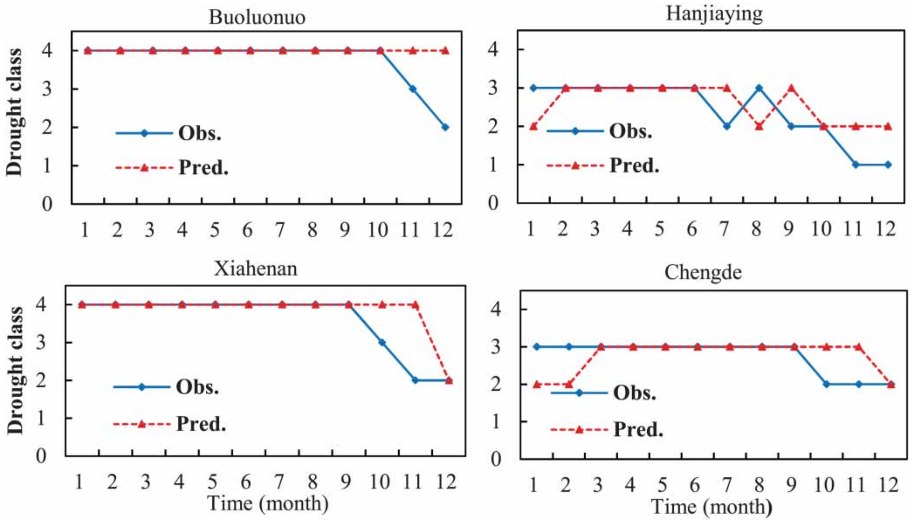

Figure 3 | Observed and predicted hydrological drought class results of 2009 through weighted Markov chain model (SRI-based).

Table 3 | The rates of accuracy of predicted meteorological drought classes based on the SPI of 21 stations through weighted Markov chain model

Count	Non-drought	Near-normal	Moderate	Severe/ Extreme	Total
Obs.	92	81	50	29	252
Accurate Pred.	91	53	45	22	211
Rate of accuracy (%)	98.9	65.4	90.0	75.9	83.7

Table 4 | The rates of accuracy of predicted hydrological drought classes based on the SRI of 9 hydrological stations through weighted Markov chain model

Count	Non-drought	Near-normal	Moderate	Severe/ Extreme	Total
Obs.	2	13	42	51	108
Accurate Pred.	2	11	42	50	105
Rate of accuracy (%)	100	84.6	100.0	98.0	97.2

Table 3 shows the rates of accuracy of predicted meteorological drought classes based on SPI through the weighted Markov chain model in all the 21 stations. Table 4 presents those of the predicted hydrological drought classes based on SRI in 9 hydrological stations. From Tables 3 and 4, it can be observed that the prediction accuracy of the weighted Markov chain model is highest for non-drought, followed by moderate, severe/extreme and lowest for near-normal. The total prediction accuracy is greater than 80%. In the case of inconsistency between the prediction and observation, the predicted drought class is lower than the observed class. In addition to this, inaccurate prediction usually occurs, when the drought class transition happens. On the contrary, the prediction is better when the drought level is relatively steady. The longer the steady state is, the better the prediction result will be. The main reason for this is that due to the Markov transition probability matrix the more recent drought conditions are more likely to reappear. However, in reality, the level of drought in a given month is likely to change in the near future. Therefore, the accuracy of this method has an important relationship with the stable state of the drought level. When the drought degree is relatively stable, it has a strong prediction ability. This predicting ability becomes weaker when there is a change in drought level. It can also be observed that the rates of accuracy of hydrological droughts prediction (SRI) are higher than that of meteorological droughts prediction (SRI), owing to the weighted Markov chain model.

Volterra adaptive filter model for chaotic time series

The monthly SPI and SRI class sequences from January 1959 to December 2008 are considered as an example in order to reconstruct the phase space and establish the

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Volterra adaptive filter model for predicting the drought class of the 12 months of 2009. The foreseeable period is 12 months, which assists in demonstrating the 12-month lead drought class prediction ability of the Volterra adaptive filter modeling method. A 12-month lead drought class prediction of 21 rainfall gauges and 9 hydrological stations has been carried out. Prediction results of two stations (Boluonuo and Xiahenan) are given in Figure 4.

In Figure 4, the prediction accuracy is high when the change process of the drought index time series is relatively steady. However, the accuracy of the prediction is not as satisfactory when the values of drought index change significantly. The causes include the existence of noise in the time series, the suitability of the selected delay time and the embedding dimensions. As well as the great variability of the peak value of the drought indices. The time series contains noise due to observation error. Therefore, it is necessary to study the influence of the error in the observation time series on the chaos analysis method and prediction model. Noise reduction and an effective combination of the chaos analysis with hydrological physical mechanism can assist in improving the precision of the prediction.

A comparison between the meteorological, hydrological predicted drought class and that evaluated from the observation data of four stations (Boluonuo, Hanjiaying, Xiahenan and Chengde) are respectively shown in Figures 5 and 6.

The rates of prediction accuracy have been statistically analyzed using the same method as that described in the Weighted Markov chain model section above, and are shown in Table 5 (meteorological) and Table 6 (hydrological). For the second time, results demonstrate that the rates of accuracy of hydrological drought prediction (SRI) are higher than that of meteorological drought prediction (SPI) except for the non-drought class in Table 6, consisting of only two observed samples.

In Table 5, it is observed that for Volterra adaptive filter model displays a similar pattern in accuracy to the weighted Markov chain method for forecasting non-drought. The prediction accuracy from high to low in turns are: non-drought, moderate, severe/extreme and near-normal. Table 6 presents a similar pattern except in the case of non-drought. Since there are only two observed and one predicted non-drought, therefore, these are too small to be accounted for when comparing with the total number of 108. For this reason, it has been excluded in the analysis, which does not affect the conclusion. The total prediction accuracy is greater than 80%, which is relatively high.

Comparison among different models

Since a 3D loglinear model has been applied in the drought prediction of Panjiakou Reservoir in our previous work

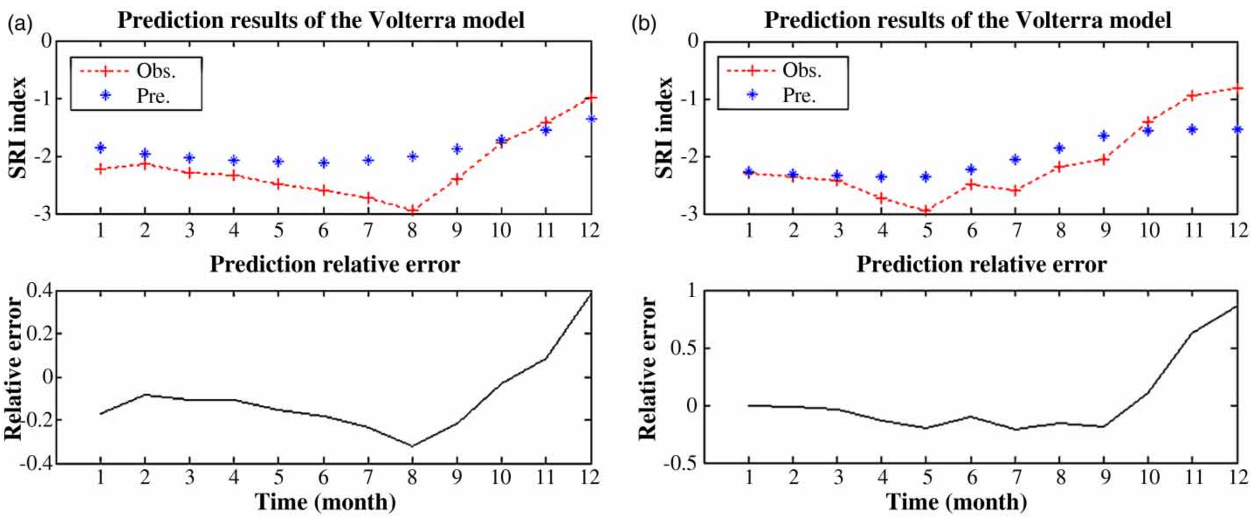

Figure 4 | Results of two stations predicted by the Volterra adaptive filter model. (a) Boluonuo Station. (b) Xiahenan Station.

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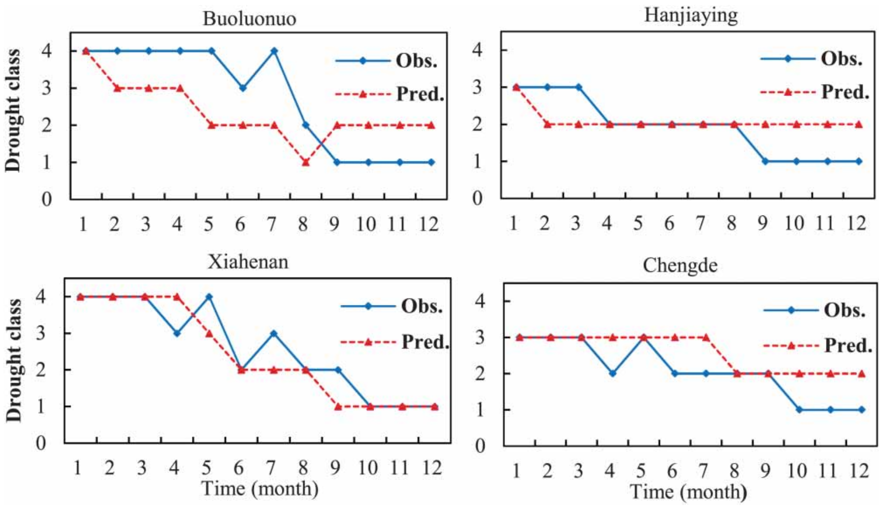

Figure 5 | Observed and SPI-based meteorological drought class prediction results of 2009 through Volterra adaptive filter model.

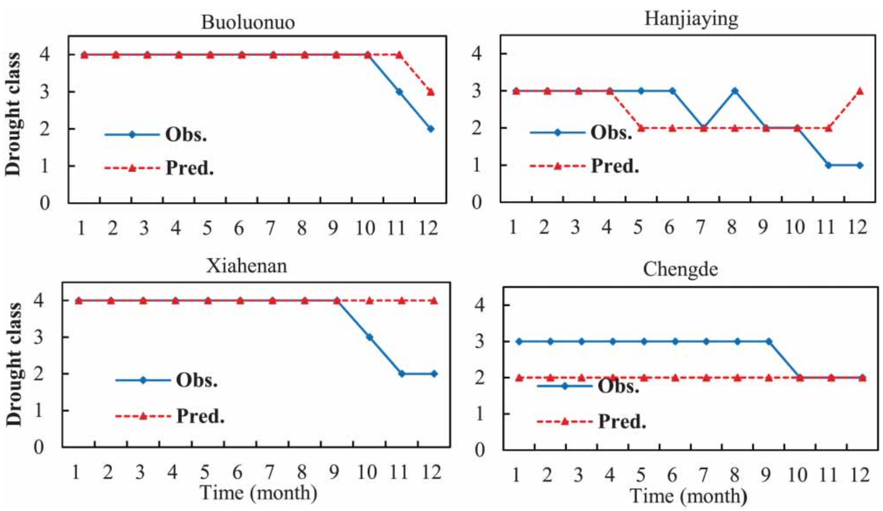

Figure 6 | Observed and SRI-based meteorological drought class prediction results of 2009 through Volterra adaptive filter model.

Table 5 | The rates of accuracy of predicted meteorological drought classes of 2009 based on the SPI of 21 stations through Volterra adaptive filter model

Count	Non-drought	Near-normal	Moderate	Severe/Extreme	Total
Obs.	92	81	50	29	252
Accurate Pred.	91	60	47	16	214
Rate of accuracy (%)	98.9	65.4	90.0	75.9	84.9

Table 6 | The rates of accuracy of predicted hydrological drought classes of 2009 based on the SRI of 9 hydrological stations through Volterra adaptive filter model

Count	Non-drought	Near-normal	Moderate	Severe/Extreme	Total
Obs.	2	13	42	51	108
Accurate Pred.	1	11	41	50	103
Rate of accuracy (%)	50.0	84.6	97.6	98.0	95.4

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(Li *et al.* 2015), it is of interest to compare this model with the two models considered in this paper. Figures 7 and 8 compare the meteorological and hydrological drought class prediction results of 2009 in the four selected sub-basins considered in this work, with those from the 3D loglinear model. As can be seen, all of the models have achieved satisfactory prediction results. When statistically analyzing all the considered stations, the 3D loglinear model performs best in meteorological drought class prediction amongst the three models with the accuracy rate of 87.3% (83.7% for weighted Markov chain model and 84.9% for Volterra adaptive filter model). However, when it comes to the hydrological drought class prediction, the weighted Markov chain model and Volterra adaptive filter model have much higher accuracy rates of 97.2% and 95.4% respectively (88% for 3D loglinear model).

As discovered in our previous work, the 3D loglinear prediction method is suitable for 1-month lead drought class prediction. However, its accuracy decreases with the increase of prediction step size. Therefore, it is not applicable for long-term prediction. It should be noted that it is not advised to elongate the foreseeable period through augmenting the dimensions of the contingency table.

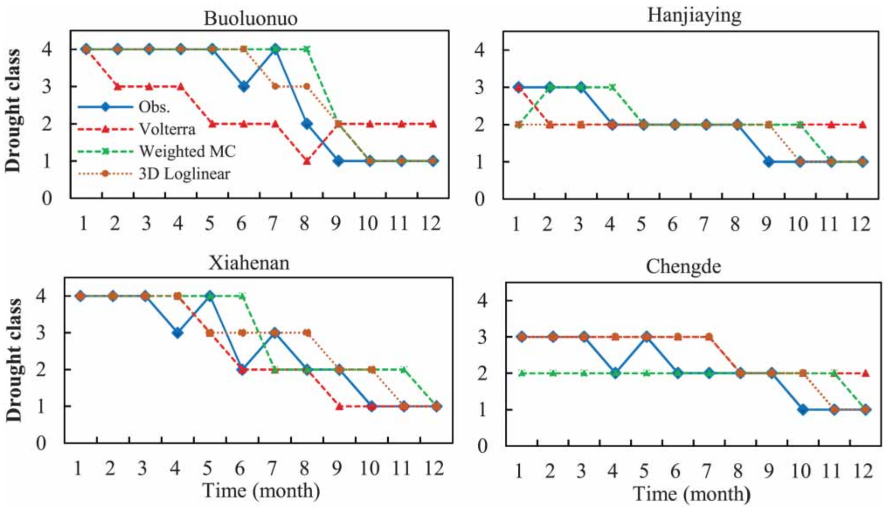

Figure 7 | A comparison of the SPI-based meteorological drought class prediction results of 2009 through three different models.

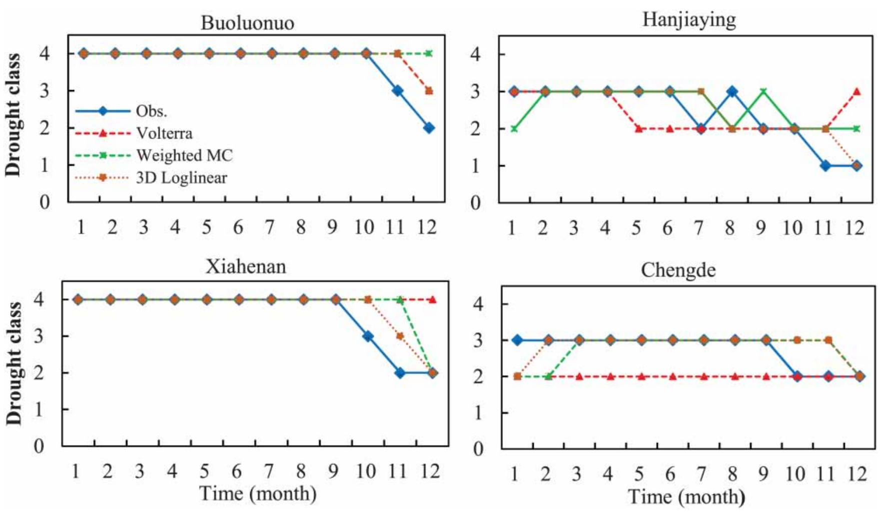

Figure 8 | A comparison of the SRI-based hydrological drought class prediction results of 2009 through three different models.

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The weighted Markov chain model adopted in this paper also applies in 1-month lead drought prediction. However, its prediction accuracy is related to the steady state of drought severity. In comparison with the 3D loglinear model and weighted Markov chain model, the prediction accuracy of the Volterra adaptive filter model has not significantly declined with substantial increase in prediction step (12-month lead in V-model, while 2 months in 3D L-model and 1 month in weighted M-model). This is because the V-model always uses SPI/ SRI index time series as input and output variables. This does not require any transformation to drought class until the analysis of prediction results.

Since, the 3D L-model and weighted M-model are the most commonly used models in drought prediction, the V-model (first proposed in this paper) is able to extend the foreseeable period to a large extent. This is very important in the medium- and long-term water resources planning for decision makers.

CONCLUSION

Based on the SPI and SRI time series of Panjiakou Reservoir catchment, this study established two different models, a weighted Markov chain model and a Volterra adaptive filter model. The two methods were used to predict both short- and long-term drought prediction and their results were then compared with a 3D loglinear model reported by Li *et al.* (2015).

The prediction accuracy rates of both the weighted Markov chain model and Voterra adaptive filter model are greater than 80%, which are satisfactory. When compared with the 3D loglinear model, their prediction accuracy rates of SPI based meteorological drought classes are a little lower (87.3% for 3D loglinear model, 83.7% for weighted Markov chain model and 84.9% for Volterra adaptive filter model). However, for the SRI based hydrological drought class prediction, their accuracy rates are much higher (97.2% for weighted Markov chain model and 95.4% for Volterra adaptive filter model) than the 3D loglinear model (88%). Therefore, the weighted Markov chain model and Volterra adaptive filter model are effective tools, respectively, for the short- and long-term drought class prediction, in particular for the SRI based hydrological drought.

The weighted Markov chain prediction model is suitable for 1-month lead drought class prediction and has highest predicting accuracy in non-drought, followed by moderate, severe/extreme, and near-normal in order. Significant relevance has been found between the predicting accuracy and the steady state of drought severity. Consequently, the prediction result is better when the development process of the drought class is relatively stable as opposed to when massive changes occur.

The Volterra adaptive filter prediction model combines the phase space reconstruction technique, Volterra series expansion theory and adaptive filter optimization technique. To our best knowledge, this is the first time of application of a Volterra adaptive filter prediction model in the study of drought class prediction. This model produces results with high prediction accuracy in the application of long-term drought class prediction for Panjiakou Reservoir Basin. Compared with the weighted Markov chain model and 3D loglinear model, the prediction accuracy of the Volterra adaptive filter model has not significantly declined with substantial increase in prediction step. Its foreseeable period can be extended to 12 months to provide more detailed information in order to support medium- and long-term decisions, as well as, planning of the water resources system.

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