© IWA Publishing 2017 Water Science & Technology: Water Supply | 17.4 | 2017

An approximate analytical method to size an air vessel in a water supply system

Di Miao, Jian Zhang, Sheng Chen and Dong-zhou Li

ABSTRACT

The size parameters of an air vessel, mainly including its initial gas volume and total volume, account for its performance in hydraulic transients. Traditionally, the size of an air vessel in a given water supply system is estimated by trial and error, usually aided by computers. To simplify the preliminary estimation of the air vessel's volume, in this paper, an approximate analytical method is proposed and derived on the basis of incompressible flow theory and simple harmonic vibration theory. With this method, a theoretically optimal size of an air vessel, which is able to meet the protective requirements in hydraulic transients, could easily be obtained in an analytical way. Besides, a real engineering case is introduced in this paper to illustrate that the proposed method could give guidance and reference in the preliminary work of designing an air vessel in practice.

Key words | air vessel, hydraulic transients, size parameters of air vessel, water supply system

Di Miao
Jian Zhang (corresponding author)
Sheng Chen
Dong-zhou Li
College of Water Conservancy and Hydropower Engineering,
Hohai University,
Nanjing 210098,
China
E-mail: *jzhang@hhu.edu.cn*

INTRODUCTION

Water supply systems are usually protected by protective devices such as surge tanks, air valves and air vessels that are capable of keeping the operating pressure within the limited value after any valve or pump operations. Among these devices, the air vessel generally alleviates negative pressures more effectively than others, and it can maintain a positive pressure in the line at all stages following a pump trip. Generally, the performance of an air vessel mainly depends on its size parameters, including its initial gas volume, total volume and inlet/outlet pipe diameter etc. (Stephenson 2002), and a better protective effect will be obtained with a larger volume of air vessel (Liang *et al.* 2005; Kim *et al.* 2015). However, an overlarge volume may lead to more cost. Therefore, an air vessel with optimal size is necessary, because it cannot only show its reliable protective effect on water hammer protection, but also avoid unnecessary cost by a large extent. Currently, more and more concerns have been emphasizing the sizing of an air vessel rationally and economically. A number of methods to size an air vessel have been proposed in recent years by numerous researchers (Thorley & Lastowiecki 1985; Stephenson 2002; Jung & Karney 2006; Zaki & Elansary 2011). However, most of these methods are either based on numerical computations or on various practical engineering experiences.

In this paper, an approximate analytical method to size an air vessel is introduced and derived. It allows designers to obtain a reasonable size of air vessel in an analytical and quick way at the preliminary stage of designing an air vessel. The verification of this method is also conducted by studying the practical case introduced in this paper.

Mathematical model

For a given water supply system equipped with an air vessel, shown in Figure 1, on the basis of the incompressibility theory of water, the mathematical model of the system could be obtained and is shown in the Appendix (available with the online version of this paper).

doi: 10.2166/ws.2016.201

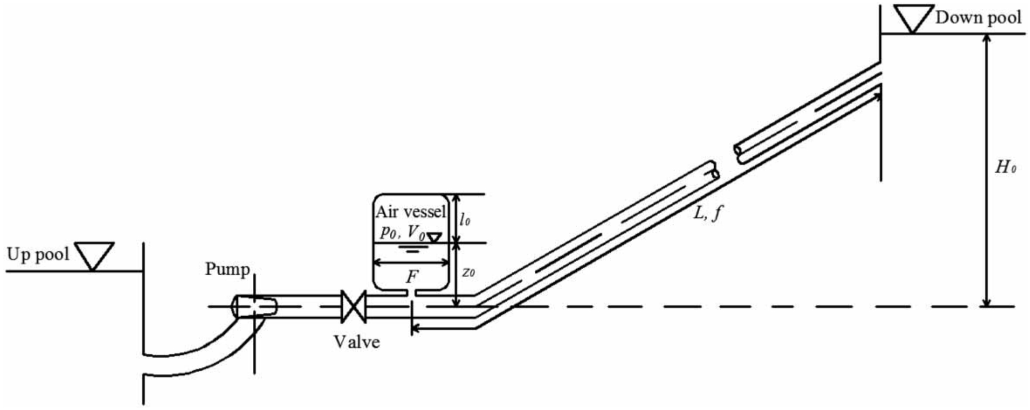

Figure 1 | Layout of a water supply system with an air vessel.

Dealing with the polytropic gas equation by Taylor expansion:

$$p = \frac{p_0 l_0^m}{(l_0 + z)^m} \approx p_0\left(1 - \frac{mz}{l_0}\right) \tag{1}$$

When the system is at steady state, we have:

$$z_0 + p_0 = H_0 + \alpha Q_0^2 + p_{\mathrm{a}} \tag{2}$$

where Q_0 is the initial flow discharge in the pipeline at the steady operating state.

When a pump trip occurs, the valve behind the pump will be closed rapidly to avoid reverse flow, and the discharge near the pump outlet can be neglected. Then the continuity equation can be written as:

$$fv = F\frac{dz}{dt} = Q = Q_{\mathrm{st}} \tag{3}$$

Taking Equations (1)–(3) into the motion equation of the system in the Appendix, we have:

$$\frac{d^2z}{dt^2} + \omega_0^2 z + \frac{fg}{FL}\left[(\alpha+\beta)F^2\left|\frac{dz}{dt}\right|\frac{dz}{dt} - \alpha Q_0^2\right] = 0 \tag{4}$$

where $\omega_0^2 = \frac{gf}{LF}\sigma$, $\sigma = \frac{mp_0}{l_0} - 1 \approx \frac{mp_0}{l_0}$.

Equation (4) describes the variation of water level z in the air vessel when the pump trip happens, and there is no explicit solution to it. At the preliminary stage of designing an air vessel in a given water system, we tend to initially estimate a proper size of the air vessel which could be regarded as guidance for the following further optimization of the vessel size. Then, in order to make the initial estimation more reasonable, an approximate analytical method is proposed in the following section of this paper.

Approximate analytical method of sizing an air vessel

Conditions for safe operation of a water supply system

In a water supply system equipped with an air vessel as a protective device, shown in Figure 2, in order to guarantee the safe operation of the system, the following three

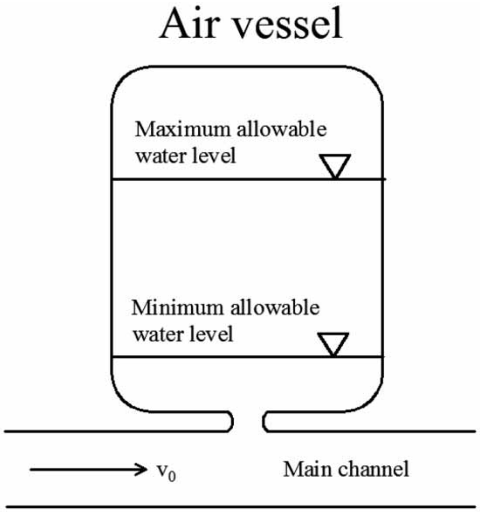

Figure 2 | Layout of air vessel.

conditions must be met: (1) the surge in the vessel must not go below the minimum limit so that the vessel would not be empty of water and the gas in the vessel would not run into the pipe; (2) the pressure in the system must not exceed the limit that brings damage to the pipeline; (3) the pressure in the system must remain positive and there should be some safety margin for it, otherwise negative pressure may cause water column separation resulting in overpressures. The above three conditions are closely associated with the air vessel's size parameters. Thus, it is critical to determine the size parameters properly to guarantee the safe operation of the system.

Approximate analytical method to size an air vessel

It should be noticed that the third term in Equation (4) stands for the impact of pipe friction and throttling. In order to find an analytical solution, we suggest neglecting pipe friction and throttling in the preliminary stage of designing an air vessel. In this way, we have:

$$\frac{d^2z}{dt^2}+\omega_0^2 z=0 \tag{5}$$

Equation (5) is obviously the simple harmonic vibration equation, through which we can respectively obtain the surge period $\triangle T$, the wave amplitude in the vessel $\triangle Z^*$ and the bottom pressure of the vessel $\triangle H$ in the Appendix.

The three conditions mentioned above that contribute to the safe operation of the system could be respectively described as the following:

$$\Delta Z^* \le \varphi_1 l_0 \tag{6}$$

$$\Delta H \le \varphi_2 H_0 \tag{7}$$

$$\Delta H \le h_{\min} \tag{8}$$

where φ_1 is the ratio of water to gas, generally $\varphi_1 \le 0.5$; φ_2 is the limited rate of pressure rise in the water system, generally $\varphi_2 \ge 0.3$; $h_{\min}$ is the lowest initial operating pressure along the pipeline at steady operating state, usually appearing at local high elevations.

The initial air volume $[V_a]$ can be determined by the simultaneous Equations (6)–(8):

$$[V_a] \ge \max\left[\frac{v_0^2 Lf}{\varphi_1\varphi_2 g H_0}, \frac{v_0^2 Lf}{\varphi_1 g h_{\min}}\right] \tag{9}$$

When equality is valid in Equations (6)–(8), the following equations are valid:

$$\sigma=\frac{\varphi_2 H_0}{\varphi_1 l_0} \text{ or } \sigma=\frac{h_{\min}}{\varphi_1 l_0} \tag{10}$$

Through Equation (10), the approximate value of φ_1 can be obtained:

$$\varphi_1 \approx \frac{\varphi_2 H_0}{m p_0} \text{ or } \varphi_1 \approx \frac{h_{\min}}{m p_0} \tag{11}$$

Taking Equation (11) into Equation (9), we have:

$$[V_a]_{\min}=\max\left[\frac{v_0^2 Lfmp_0}{\varphi_2^2 g H_0^2}, \frac{v_0^2 Lfmp_0}{g h_{\min}^2}\right] \tag{12}$$

where $p_0 \approx H_0 + p_a$.

Equation (12) describes the theoretical minimum air volume $[V_a]_{\min}$ which is required for the safe operation of the pipeline system. For a pipeline system that rises uniformly in profile, $v_0^2 Lfmp_0/\varphi_2^2 g H_0^2$ dominates the value of $[V_a]_{\min}$, and it can make sure that the positive water hammer pressure due to pump failure is within the maximum limit. Whereas, when it comes to a pipeline with convex points in profile, then $v_0^2 Lfmp_0/g h_{\min}^2$ determines the value of $[V_a]_{\min}$, and it can guarantee that no negative pressures appear along the pipeline.

The approximate analytical size of the air vessel can eventually be described as:

$$V_T = V_a(1+\varphi_1) = k|V_a|_{\min}(1+\varphi_1) \tag{13}$$

where k is the correction coefficient to make up for the approximation in the derivation; its value is 1.1. V_T and V_a are respectively the total volume and the air volume of the air vessel. As long as the air height l_0 is determined, the cross-sectional area of the vessel F can be obtained by $F = V_a/l_0$.

Case study

The engineering parameters of a pump-station water supply system located in China are shown in Table 1, and the schematic diagram of the pipeline system is shown in Figure 3. The pipeline rises uniformly in profile. According to some particular engineering principles in China, the limited rate of pressure rise in the pipeline system $\varphi_2 = 30\%$. The polytropic exponent $m = 1.2$. Since this water supply project is still under construction currently, no measured data about the pipeline system could be presented for now. Therefore, the simulation numerical computation, which is on the basis of the method of characteristics (Streeter & Wylie 1963; Wylie *et al.* 1993; Chaudhry 2014), was given instead of measured data to make a comparison with the theoretical result.

Table 1 | Parameters of pipeline system

Upstream water level (m)	110	Wave speed (m/s)	1,000
Downstream water level (m)	170	Roughness coefficient	0.012
Installation elevation of air vessel (m)	100	Pump speed (r/min)	1,450
Pipe cross-sectional area (m^2)	1	Motor power (kW)	710
Pipe length (m)	3,000	Pump head (m)	60
Rated flow discharge (m^3/s)	1	Moment of inertia (kg m^2)	374

RESULTS AND DISCUSSION

Since the pipeline in this case rises uniformly in profile, $[V_a]_{\min} = v_0^2 Lfmp_0/\varphi_2^2 \gamma g H_0^2$ was adopted to calculate the analytical size of the air vessel, and Table 2 shows the corresponding size parameters of the vessel. In addition, in Figures 4 and 5, the theoretical result for the amplitudes of the bottom pressure of the vessel and amplitudes of the water depth of the vessel are given. In the numerical computation, the analytical size of the air vessel is adopted, and the area of the impedance orifice is 0.5 m, as large as 20% of the pipeline's (Gong *et al.* 2013), and the orifice-metering coefficient is 0.6. Pipe friction and throttling were considered during the numerical computation, and the numerical result is shown in Figures 4 and 5.

Through the comparison between the theoretical result and the numerical result, we can see that the surge periods of these two results can match well with each other, whereas the amplitudes of the bottom pressure are different in varying degrees in these two results, as well as the amplitudes of the water depth. In addition, the differences between the two results grow increasingly as time goes on.

The reason that could explain the differences between the above two results is that pipe friction and throttling are neglected in the theoretical result, while they are considered in the numerical result. In the numerical result, water hammer due to pump trip will be increasingly relieved by pipe friction and throttling as time goes on, whereas water hammer in the theoretical result will not. As a result, the analytical size of air vessel is relatively larger

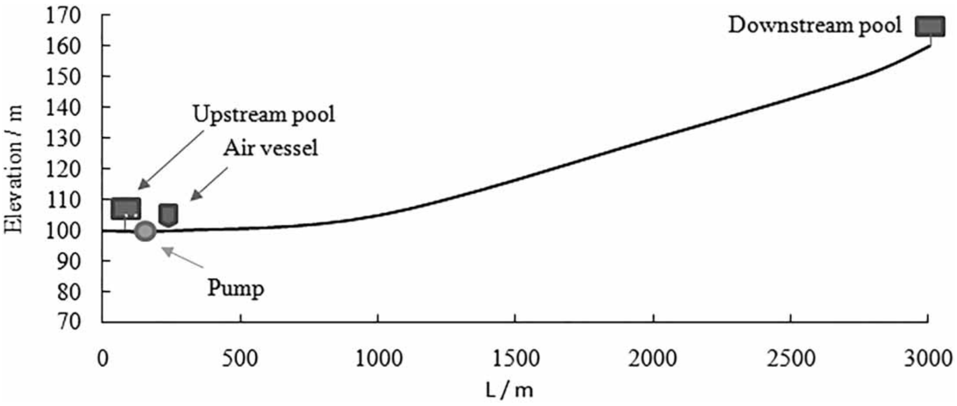

Figure 3 | Schematic diagram of the pipeline system.

Table 2 | Size parameters of the air vessel

Total volume (m^3)	Water volume (m^3)	Gas volume (m^3)	Cross-sectional area (m^2)	Water height (m)	Gas height (m)	Water–gas ratio
89	16	73	14.6	1.1	5.0	0.22

than expected in practical cases, and it can leave some safety margin to the pipeline system. Table 3 shows that the analytical size of the air vessel left a margin of about 7 m for the system pressure and a margin of 0.15 m for the water depth in the vessel. Moreover, it could be implied that, as the length of the pipeline increases, the impact of pipe friction on relieving water hammer becomes larger, and that the analytical size of the air vessel obtained by this analytical method will be increasingly larger than the one actually required in the practical case.

Despite the fact that this analytical method may make the air vessel volume relatively larger than actually needed in practical cases, it is able to provide a reference to size an air vessel in a safe and conservative way. At the preliminary stage of designing an air vessel, we can use this analytical method to approximately estimate the size of the air vessel, giving guidance for the following optimization of the air vessel. In addition, it must be pointed out that, in

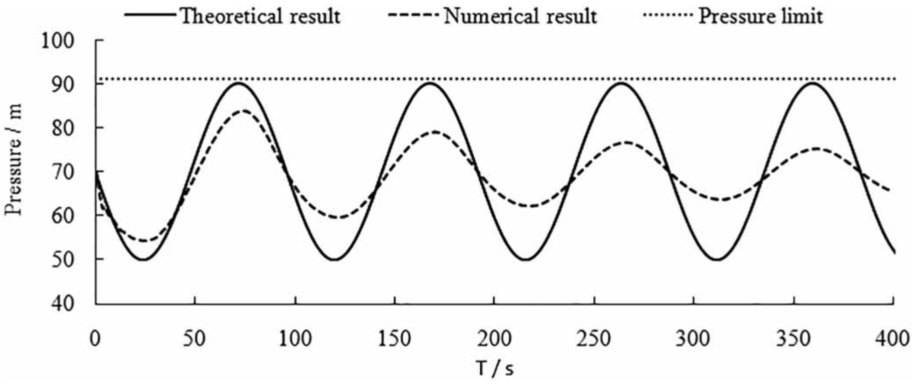

Figure 4 | Variation of bottom pressure of air vessel.

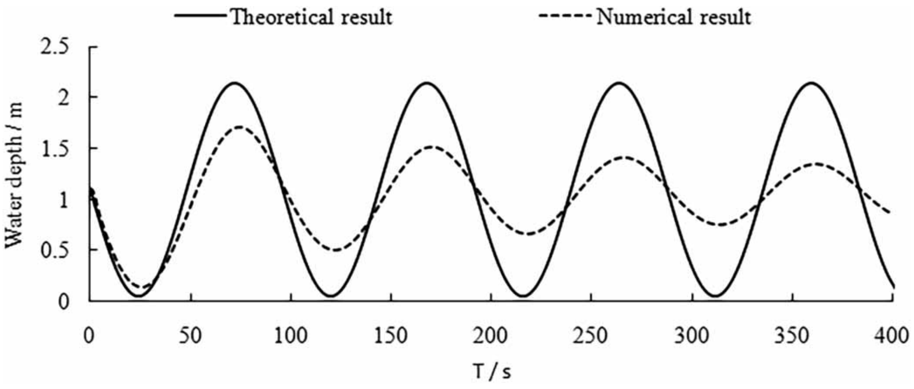

Figure 5 | Variation of water depth in air vessel.

Table 3 | Statistics of results

Results	Initial pressure (m)	Maximum pressure (m)	Minimum pressure (m)	Initial water depth (m)	Maximum water depth (m)	Minimum water depth (m)
Theoretical	70.0	90.0	50.0	1.1	2.1	0.1
Numerical	70.0	83.6	54.2	1.1	1.7	0.2

this paper, little safety margin for the water level in the vessel has been taken into consideration during the derivation of this method, because we wanted to seek a theoretical minimum volume of the air vessel by this method, not a practical one. Therefore, in practical engineering cases, the analytical size of the air vessel should be properly increased by some extent for enough safety margin.

CONCLUSIONS

The following conclusions can be drawn in this paper:

1. An analytical method to size an air vessel, which allows designers to size an air vessel in an analytical way, is proposed and derived on the basis of incompressible flow theory and simple vibration theory.
2. This analytical method is verified by studying the comparison between the theoretical and numerical results of a practical case introduced in this paper. The results showed that the analytical method is capable of providing a reasonable size of air vessel which can be regarded as guidance for the subsequent optimization of the air vessel. Additionally, since no safety margin was considered in the derivation of the analytical method, the analytical volume of air vessel obtained by this method, especially the initial volume that contains water, should be increased by some extent for enough safety margin in practical cases.
3. Currently, the practical project introduced in this paper is still under construction. Therefore, no measured data are available to make comparison with the theoretical and numerical results. Further research will be carried out to improve this analytical method after the measured data are next obtained in the future.

ACKNOWLEDGEMENTS

The present project is financially supported by the National Natural Science Foundation of China (No. 51379064), the Fundamental Research Funds for the Central Universities (Grant No. 2015B00414 and No. 2016B10814) and the Priority Academic Program Development of Jiangsu Higher Education Institutions.

REFERENCES

Chaudhry, M. 2014 *Applied Hydraulic Transients*, 3rd edn. Springer, New York, USA.

Gong, J., Zhang, J. & Yu, X. D. 2013 Optimization of impedance orifice of air vessel in high-lifted water supply system. *Water Resource and Power* **31** (5), 166–169.

Jung, B. & Karney, B. 2006 Hydraulic optimization of transient protection devices using GA and PSO approaches. *J. Water Resour. Plann. Manage.* **132** (1), 44–52.

Kim, S. G., Lee, K. B. & Kim, K. Y. 2015 Water hammer in the pump-rising pipeline system with an air chamber. *Journal of Hydrodynamics* **26** (6), 960–964.

Liang, X., Liu, M. Q., Zhang, J. G., Shen, Y. Z. & Lin, Q. 2005 Protection study on water hammer of pipelines by air vessel. *Drainage and Irrigation Machinery* **23** (6), 16–18.

Stephenson, D. 2002 Simple guide for design of air vessels for water hammer protection of pumping lines. *Journal of Hydraulic Engineering* **128** (8), 792–797.

Streeter, V. L. & Wylie, E. B. 1967 *Hydraulic Transients*. McGraw-Hill Book Company, New York, USA.

Thorley, A. R. D. & Lastowiecki, P. 1985 Air vessel design for rising mains. In: *Proceedings of the International Conference on Pumping Stations*, BHRA, pp. 89–98.

Wylie, E. B., Streeter, V. L. & Suo, L. S. 1993 *Fluid Transients in System*. Prentice-Hall, Inc., Englewood Cliffs, NJ, USA.

Zaki, K. O. & Elansary, A. S. 2011 Optimal design of air vessel for water hammer protection in water distribution network. *Journal of Engineering and Applied Science* **58** (3), 219–236.

First received 22 July 2016; accepted in revised form 5 December 2016. Available online 20 December 2016