The joint distribution of reference crop evapotranspiration and irrigation water in the irrigation district

Jinping Zhang, Xiaomin Lin and Yong Zhao

ABSTRACT

For the irrigation district, irrigation water is the manual water supply for the farmland while reference crop evapotranspiration (ET0) reflects water demand. Thus, the joint distribution of irrigation water and ET0 can reveal water shortage risk under the condition of the manual water supply. In order to understand their relationships and overcome the drawbacks of different marginal distributions of hydrological variables, Archimedean copulas are introduced. Based on the data series of ET0 and irrigation water in the Luhun irrigation district of China, the univariate marginal distributions of ET0 and irrigation water are first selected. Then, with the Gumbel–Hougaard copula in the Archimedean copulas, the joint distribution of ET0 and irrigation water is proposed. The results show that the best-fitting marginal distributions of ET0 and irrigation water are generalized extreme values and normal distributions, respectively, but for their joint distribution, the Gumbel–Hougaard copula is the best-fitting one. The water shortage risks with different encounter situations of ET0 and irrigation water are better revealed using the proposed copula-based joint distribution.

Key words | copula function, irrigation water, joint distribution, reference crop evapotranspiration, water shortage risk

INTRODUCTION

The features of the manual water supply and water demand in the irrigation district can be revealed based on the analysis of irrigation water and reference crop evapotranspiration (ET0). ET0 is a basic and key parameter used to estimate the crop water requirement (Kong et al. 2015). If rainfall in the irrigation district cannot meet the crop water requirement, irrigation water is needed. The crop water requirement occupies the majority of irrigation water and exhibits a positive correlation with it. Thus, there is some relevance between ET0 and irrigation water (Yamauchi 2014). Usually, irrigation water represents the manual water supply, while ET0 denotes water demand in the irrigation district.

Both ET0 and irrigation water can be influenced by a number of factors, such as precipitation, temperature, wind speed, relative humidity, solar radiation, etc. (Yu et al. 2002; Mojid et al. 2015), and can display complex spatial–temporal variations. These indicate that the necessity of studying the probability distributions of ET0 and irrigation water are based on their long-term data series. Now, research on the spatial–temporal variation characteristics of ET0 and irrigation water has found that the changing characteristics of ET0 and irrigation water are complicated and uncertain (Mo et al. 2004; Liu et al. 2007; Dinpashoh et al. 2011; Wang et al. 2014). In order to reveal these changing characteristics more effectively, the probability distributions of ET0 and irrigation water are studied based on their long time data series (Khanjani & Busch 1982; Zhao et al. 2015).

However, the analyses of the univariate distribution of ET0 and irrigation water without considering their
dependence structure are inadequate (Mishra et al. 2013; Oh et al. 2015; Feng et al. 2014; Zhao et al. 2014; Riediger et al. 2016), which may lead to incomplete or even erroneous conclusions. Therefore, it is necessary to apply a mathematical method that can conserve the dependence structure to capture the joint probability behaviors of ET0 and irrigation water. Ding et al. (2011) proposed that the joint distribution based on the dependence among multivariate distributions extracts more information.

The copula is a useful method for assessing multivariate distributions (Kao & Govindaraju 2010), which makes it the ideal method to study the joint probability of ET0 and irrigation water. Currently, the copula is widely used in hydrology and water resources studies, but there are only a few applications relevant to irrigation water. Copulas are functions that can combine univariate cumulative probability distributions to construct a multivariate joint distribution, as well as fully express the dependence structure among random variables (Joe 1997; Genest & Favre 2007), so they are flexible whatever the univariate marginal distribution is. In the field of hydrology and water resources, the copula method is widely used to describe the dependence structure among random variables. Its intelligibility and flexibility are evident from the related research on drought (Serinaldi et al. 2009; Kao & Govindaraju 2010; Yoo et al. 2014; Salvadori & De Michele 2015; Zhang et al. 2015), storm analysis (De Michele et al. 2007; Zheng et al. 2015), flood risk analysis (Svensson & Jones 2004; Grimaldi & Serinaldi 2006; Pinya et al. 2009; Ghizzoni et al. 2012; Zheng et al. 2014, 2015), tail dependence (Di Bernardino & Rullière 2016; Wang & Xie 2016), runoff and sediment (Zhang et al. 2014a, 2014b), and streamflow simulation (Chen et al. 2015; Jeong & Lee 2015). The detailed theoretical background and methodological descriptions of copula application in hydrology can be found in Klippelberg & Rootzén (2006) and Jaworski et al. (2009). Recently, except for the research by Ding et al. (2011) and Zhang et al. (2014a, 2014b), who analyzed the probabilistic behaviors of rainfall and ET0, few studies have been concerned with manual water supply and water demand in the irrigation region.

Therefore, this study aims to construct the joint distribution of ET0 and irrigation water with the copula method based on the annual observed data series of ET0 and irrigation water from 1970 to 2013 of the Luhun irrigation district in China. The results from this study allow us to explore the statistical changing characteristics of the manual water supply and water demand in the irrigation district, and then to capture their joint probability behaviors.

The subsequent sections of this paper are organized as follows: A brief introduction of copula is described in ‘Methodology,’ including the identification and goodness-of-fit evaluation of a univariate marginal distribution and copula-based joint distribution. The used ET0 and irrigation water data series of the Luhun irrigation district in China are introduced in the ‘Case study.’ The results and applications of statistical analysis on the marginal distributions and the joint distribution of ET0 and irrigation water are provided in ‘Results and applications’. Finally, conclusions are stated in ‘Conclusions’.

**METHODOLOGY**

An overview of the methodology in this study is: (1) select the appropriate univariate marginal distributions of ET0 and irrigation water according to their statistical characteristics; (2) estimate the parameter of the copula function; (3) determine the best-fitting copula function with the identification and goodness-of-fit evaluation method; (4) construct the joint distribution of ET0 and irrigation water.

**Selection of univariate marginal distribution**

Hydrological variables have been generally assumed to be subject to a P-III-type distribution in China (Huang 2005). However, in order to better reflect the statistical probability characteristics of ET0 and irrigation water, seven commonly used marginal distributions in hydrology and water resources (Chen 2013) were applied to select the most appropriate one, including two-parameter gamma, two-parameter lognormal, generalized Pareto, exponential, the classic P-III-type, normal and generalized extreme value (GEV) distributions.

For ET0 and irrigation water, the parameters of these marginal distributions were estimated by the maximum likelihood estimation (MLE) method. The identifications of the marginal distribution were derived by the Kolmogorov–Smirnov (KS) test with a 5% significant level. Then, the
root mean square error (RMSE) was calculated to select the best-fitting marginal distribution. RMSE is given by:

\[
RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (P_i - P_{ei})^2}
\]

(1)

where \(P_i\) and \(P_{ei}\) are, respectively, the theoretical and empirical frequency of the marginal distribution, and \(n\) is the number of observations.

\(P_{ei}\) can be expressed as:

\[
P_{ei}(x_i) = P(X \leq x_i) = \frac{n_i}{n + 1}
\]

(2)

where \(n_i\) is the number of \(x \leq x_i\).

**Copula method**

Copulas are functions combining univariate cumulative distributions to constitute multidimensional probability distributions. Given two continuous random correlated variables, \(X\) and \(Y\), assume that the univariate marginal distribution functions are \(F_X(x)\) and \(F_Y(y)\), respectively. Meanwhile, let \(u = F_X(x)\) and \(v = F_Y(y)\). According to Sklar’s theorem (Sklar 1959), there exists a unique copula \(C\) to link these two marginal distributions. The copula function is generally expressed as follows:

\[
F_{X,Y}(x,y) = C(u,v)
\]

(3)

where \(F_{X,Y}(x,y)\) is the joint distribution of pairs \((X \leq x, Y \leq y)\) with its marginal distribution submitted to uniform distributions on \([0, 1]\). \(\theta\) is the parameter of copula \(C\).

If the marginal distributions \(F_X(x)\) and \(F_Y(y)\) are continuous (which means their corresponding probability density functions \(f_X(x)\) and \(f_Y(y)\) exist), the copula \(C\) is unique with the following joint probability density function \(f_{X,Y}(x,y)\):

\[
f_{X,Y}(x,y) = c_{\theta}(F_X(x), F_Y(y))f_X(x)f_Y(y)
\]

(4)

where \(c\) is the joint probability density function of copula \(C\) expressed as:

\[
c(u,v) = \frac{\partial^2 C(u,v)}{\partial u \partial v}
\]

(5)

where \(u\) and \(v\) denote, respectively, a specific value of \(F_X(x)\) and \(F_Y(y)\).

**Bivariate Archimedean copula functions**

Bivariate Archimedean copula functions with only one parameter are extensively applied in hydrology and water resources (Cherubini et al. 2004; Grimaldi et al. 2005). In this study, three one-parameter Archimedean copulas are employed (as shown in Table 1) (Genest & Mackay 1986).

**Parameter estimation**

The parameter \(\theta\) of the copula functions in Table 1 exhibits the association between \(u\) and \(v\), and it can be estimated by the relations between \(\theta\) and Kendall’s rank coefficient \(\tau\). \(\tau\) can measure the nonlinear dependence between random variables and is calculated as:

\[
\tau = \frac{1}{n(n-1)} \sum_{i<j} \text{sign}[(x_i - x_j)(y_i - y_j)]
\]

(6)

where \((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\) are observations of the random variables \(X\) and \(Y\) for \(i, j = 1, 2, \ldots, n\):

\[
\text{sign}[(x_i - x_j)(y_i - y_j)] = \begin{cases} 
1 & (x_i - x_j)(y_i - y_j) > 0 \\
0 & (x_i - x_j)(y_i - y_j) = 0 \\
-1 & (x_i - x_j)(y_i - y_j) < 0
\end{cases}
\]

(7)

As shown in Table 1, it is obviously seen that the three Archimedean copulas are suitable to describe the positive dependence, but for the negative dependence, only the Frank copula can be used.

**Identification and goodness-of-fit evaluation of copula function**

The adaptive copula functions are identified by the KS test, and the corresponding statistical magnitude \(D\) is written as:

\[
D = \max_{1 \leq k \leq n} \left\{ \left| C_k - \frac{m_k}{n} \right|, \left| C_k - \frac{m_k - 1}{n} \right| \right\}
\]

(8)
where $C_k$ represents the value of observation $(x_k, y_k)$ of the copula function; $m_k$ is the number of $(x_k, y_k)$ such that $x \leq x_k$ and $y \leq y_k$, and $n$ is the number of observations. When the statistical magnitude $D$ is less than its critical value, the assumed distribution passes the KS test.

Let $(x_i, y_i)$ for $i = 1, 2, \ldots, n$ be observations of a two-dimensional distribution, the empirical distribution functions of $X$ and $Y$ are exhibited as $F_n(x)$ and $H_n(y)$, respectively. Thus, the empirical copula $C_0(u, v)$ is defined as:

$$C_0(u, v) = \frac{1}{n} \sum_{i=1}^{n} I_{[F_n(x_i) \leq u]} I_{[H_n(y_i) \leq v]}$$

(9)

where $n$ is the sample size, $I(A)$ is the indicator function of set $A$ satisfying $I = 1$ if $A$ is true, otherwise $I = 0$, and $u$ and $v$ are two variables between 0 and 1.

The ordinary least square (OLS) is employed to evaluate the goodness-of-fit of the copula function:

$$\text{OLS} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (C(i) - C_0(i))^2}$$

(10)

where $C(i)$ is the calculated values of the theoretical copula, $C_0(i)$ is derived from the corresponding empirical copula, and $n$ is the sample size. If the value of OLS is close to zero, the copula function performs more efficiently. If few differences exist in the values of OLS, the Akaike information criterion (AIC) is applied for the selection of the best-fitting copula function. AIC can be given by:

$$AIC = n \ln \left( \frac{1}{n} \sum_{i=1}^{n} (C(i) - C_0(i))^2 \right) + 2k$$

(11)

where $k$ is the number of parameters of the copula function, and $n$ is the sample size.

### Table 1 | Three one-parameter Archimedean copulas

<table>
<thead>
<tr>
<th>Archimedean copulas</th>
<th>$C_k(u, v)$</th>
<th>Relation between $\tau$ and $\theta$</th>
<th>Range of $\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clayton</td>
<td>$(u^{-\theta} + v^{-\theta} - 1)^{-1/\theta}$</td>
<td>$\tau = \frac{\theta}{\theta + 2}$</td>
<td>$\theta &gt; 0$</td>
</tr>
<tr>
<td>Gumbel–Hougaard</td>
<td>$\exp \left[ -\left( (-\ln u)^\theta + (-\ln v)^\theta \right)^{1/\theta} \right]$</td>
<td>$\tau = 1 - \frac{1}{\theta}$</td>
<td>$\theta \geq 1$</td>
</tr>
<tr>
<td>Frank</td>
<td>$-\frac{1}{\theta} \ln \left[ 1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{e^{-\theta} - 1} \right]$</td>
<td>$\tau = 1 - \frac{4}{\theta} \left[ -\frac{1}{\theta} \int_0^t \exp(-t) - 1 , dt - 1 \right]$</td>
<td>$\theta \in R$</td>
</tr>
</tbody>
</table>

### CASE STUDY

The Luhun irrigation district (34°10′28″–34°45′34″N, 112°5′06″–112°5′24″E) is located in the west of Henan province in China (Figure 1). As a larger irrigation district, it crosses the Yellow River basin and Huai River basin with a total area of 1,838.48 km². Dominated by the Siberia winter monsoon and the East Asian summer monsoon, the climate in the Luhun irrigation district is typically characterized by hot and rainy summers, and cold and dry winters. The average annual rainfall is about 600 mm, but 60% of it occurs from June to September. The average annual evaporation is above 1,400 mm, and the average annual drought duration is 112 days. It is evident that the Luhun irrigation district is prone to meteorological drought. The average annual irrigation water is 1.81×10⁸ m³, and it is mainly from the Luhun reservoir located on the Yi River (a tributary of the Yellow River) with a total storage capacity of 13.20×10⁸ m³.

The ET₀ is estimated by the Penman–Monteith formula recommended by the Food and Agriculture Organization (FAO) in 1998 (Allen et al. 1998), and the FAO Penman–Monteith model requires the input of meteorological variables. Thus, it is necessary to obtain data for these meteorological variables. The data used in this study from 1970 to 2013 were obtained from the Irrigation Administration Bureau of the Luhun irrigation district, and included irrigation water, daily rainfall, average wind velocity, mean relative humidity, maximum temperature, minimum temperature, and sunshine duration. Figure 2 shows the data series of annual ET₀ and irrigation water from 1970 to 2013. Table 2 represents the statistical characteristics of annual ET₀ and irrigation water.
RESULTS AND APPLICATIONS

Marginal distributions of ET₀ and irrigation water

The marginal distributions of ET₀ and irrigation water were fitted by two-parameter gamma, two-parameter lognormal, generalized Pareto, exponential, classic P-III-type, normal and GEV distributions, respectively. The parameters of marginal distributions were estimated by the MLE method. According to Massey (1951), when the sample size is 44, the KS statistic $D$ with the 5% significance level is 0.201. If $D$ of a marginal distribution is less than 0.201, it passes the KS test. Table 3 shows the statistic $D$ of these seven marginal distributions. The results show that for ET₀, the
marginal distributions of gamma, lognormal, P-III, normal, and GEV all pass the KS test, while for irrigation water, the marginal distributions of gamma, lognormal, Pareto, P-III, normal, and GEV all pass. Table 4 shows the RMSE of these marginal distributions. It can be seen that due to the smallest value of RMSE, the normal distribution is the preferred marginal distribution for irrigation water and the GEV distribution for ET0. Thus, the cumulative distribution function (CDF) of the GEV distribution is applied to ET0, and it can be presented as:

$$F(x) = \exp\left\{-\left(\frac{x - 1017}{80.3}\right)^{1/0.5}\right\},$$

$$1 - 0.5 \times \left(\frac{x - 1017}{80.3}\right) > 0$$

(12)

The CDF of the normal distribution is applied to irrigation water, and it can be expressed as:

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{(x - 181)^2}{0.92}\right) dx$$

(13)

### Joint distribution of ET0 and irrigation water

Based on the Archimedean copula functions, the joint distribution construction procedure of ET0 and irrigation water involves: (1) calculation of the association between ET0 and irrigation water with Kendall's $\tau$; (2) estimation of the parameter $\theta$ according to the relations between $\theta$ and Kendall's $\tau$; (3) identification of the Archimedean copula function by KS test; and (4) determination of the best-fitted copula by employing OLS and AIC.

As noted in Table 5, the proposed Archimedean copulas all pass the KS test (for their $D < 0.201$) obviously. But the Gumbel–Hougaard copula function gives a better fit than the other two copulas because of its smallest OLS value as well as AIC value.

Thus, the Gumbel–Hougaard copula function is used to describe the joint distribution of ET0 and irrigation water. Let $u$ and $v$ denote the marginal distribution of annual ET0 and irrigation water of the Luhun irrigation district. Their joint distribution $F(x, y)$ is expressed as:

$$F(x, y) = \exp\left[(-\ln u)^{1.3399} + (-\ln v)^{1.3399}\right]^{1/1.3399}$$

(14)

where $x$ and $y$ denote ET0 and irrigation water, respectively.

Figure 3 gives the goodness-of-fit evaluation of $F(x, y)$, which indicates that the correlation coefficient between the theoretical copula and the empirical copula is 0.97. It means the selected Gumbel–Hougaard copula is reasonable. Also, the joint distribution $F(x, y)$ is plotted in Figure 4.

### Analysis of the joint distribution

For IR, representing irrigation water, with the constructed copula-based joint distribution of ET0 and IR, two different joint probability distributions can be presented as follows

<table>
<thead>
<tr>
<th>Table 2</th>
<th>The statistical characteristics of annual ET0 and irrigation water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>Minimum</td>
</tr>
<tr>
<td>ET0 (mm)</td>
<td>724.59</td>
</tr>
<tr>
<td>Irrigation water ($10^8$ m$^3$)</td>
<td>0.60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3</th>
<th>KS test for the different marginal distributions of ET0 and irrigation water</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>Two-parameter gamma</td>
</tr>
<tr>
<td>ET0 (mm)</td>
<td>0.169</td>
</tr>
<tr>
<td>Irrigation water ($10^8$ m$^3$)</td>
<td>0.107</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 4</th>
<th>RMSE values for different fitted marginal distributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>Two-parameter gamma</td>
</tr>
<tr>
<td>ET0 (mm)</td>
<td>0.082</td>
</tr>
<tr>
<td>Irrigation water ($10^8$ m$^3$)</td>
<td>0.082</td>
</tr>
</tbody>
</table>
to reveal the possible encounter situations of the manual water supply and water demand:

\[
G_{X,Y}(x, y) = P(X \geq x \text{ or } Y \leq y) = 1 - F(x) + F(x, y) \tag{15}
\]

\[
G_{X,Y}^0(x, y) = P(X \geq x, Y \leq y) = F(y) - F(x, y) \tag{16}
\]

where \(X\) and \(Y\) denote ET\(_0\) and IR, \(x\) and \(y\) are their specific values, \(F(x)\) and \(F(y)\) are the marginal distributions of ET\(_0\) and IR, respectively. \(F(x, y)\) is the copula-based joint distribution of ET\(_0\) and IR. \(G_{X,Y}(x, y)\) is joint probability distribution \(I\) given \(X \geq x\) or \(Y \leq y\). \(G_{X,Y}^0(x, y)\) indicates joint probability distribution \(II\) given \(X \geq x\) and \(Y \leq y\). The contour plots of \(G_{X,Y}(x, y)\) and \(G_{X,Y}^0(x, y)\) are described in Figures 5 and 6. Additionally, the exceeding probability of ET\(_0\) and marginal probability distribution of IR are presented in Figures 7 and 8, respectively.

From Figures 5 and 6, it can be observed that the counter plot of \(G_{X,Y}(x, y)\) shows the inverse direction to that of

Figure 3 | The goodness-of-fit evaluation of the joint distribution of ET\(_0\) and irrigation water \((F(x, y))\).

Figure 4 | The contour surface of the joint distribution of ET\(_0\) and irrigation water \((F(x, y))\).

Figure 5 | The contour plot of the joint probability distribution \(I\) given \(X \geq x\) or \(Y \leq y\) \((G_{X,Y}(x, y))\) (the different lines and the numbers represent probabilities).

Figure 6 | The contour plot of the joint probability distribution \(II\) given \(X \geq x\) and \(Y \leq y\) \((G_{X,Y}^0(x, y))\) (the different lines and the numbers represent probabilities).
If $x = y$, $G'_{X,Y}(x, y) < G_{X,Y}(x, y)$. Meanwhile, the contour plots of $G_{X,Y}(x, y)$ and $G'_{X,Y}(x, y)$ exhibit various encounter situations of ET0 and irrigation water, so the joint probability with given combinations of pairs (ET0, IR) can be achieved. Take for example that the probability is 0.89 for encounter situation pairs (ET0 ≥ 1,000 mm or IR ≤ 1.8 × 10^8 m³) and 0.29 for pairs (ET0 ≥ 1,000 mm and IR ≤ 1.8 × 10^8 m³). Similarly, the different pairs (ET0, IR) with a given joint probability are also obtained from the contour plots of $G_{X,Y}(x, y)$ and $G'_{X,Y}(x, y)$. Typically, the encounter situations with the probability of 0.7 include pairs (ET0 ≥ 1,100 mm or IR ≤ 1.9 × 10^8 m³), (ET0 ≥ 1,050 mm or IR ≤ 1.55 × 10^8 m³), (ET0 ≥ 850 mm and IR ≤ 2.25 × 10^8 m³) and (ET0 ≥ 950 mm and IR ≤ 2.45 × 10^8 m³), etc. Moreover, this bivariate joint probability of pairs (ET0, IR) is obviously less than the exceeding probability of ET0 and marginal probability of IR. For instance, the exceeding probability of ET0 ≥ 1,000 mm is 0.66 (noted in Figure 7), and the marginal probability of IR ≤ 2.0 × 10^8 m³ is 0.61 (shown in Figure 8). They are all larger than the bivariate joint probability of pairs (ET0 ≥ 1,000 mm and IR ≤ 2.0 × 10^8 m³) with the value of 0.38. Thus, if the univariate marginal distribution of ET0 and IR is considered only in irrigation activities, the statistical characteristics of the manual water supply and water demand can be explained incorrectly. Especially for irrigation planning and drought resistance, ET0 and IR need to be involved simultaneously to reflect the actual water shortage risk with the manual water supply and water demand.

In practice, the larger ET0 and the less IR are of more concern in irrigation planning and management. According to Table 2, the encounter situations of the maximum of ET0 and the minimum of IR from 1970 to 2013 in the Luhun irrigation district are considered as pairs (ET0 ≥ 1,054 mm or IR ≤ 1.8 × 10^8 m³) and pairs (ET0 ≥ 1,054 mm and IR ≤ 1.8 × 10^8 m³). This reveals the water shortage risk under the condition of manual water supply and water demand. From Figures 5 and 6, it can be seen that $G_{X,Y}(x, y) = P(X ≥ 1034$ or $Y ≤ 1.8) = 0.82$ and $G'_{X,Y}(x, y) = P(X ≥ 1034$ and $Y ≤ 1.8) = 0.19$, which shows the irrigation water in the Luhun irrigation district can usually satisfy water demand, but the water shortage risk still exists and should be paid more attention.

**CONCLUSIONS**

The study employed seven CDFs to describe the marginal probabilistic behaviors of ET0 and irrigation water and found that the GEV and normal distributions are the best-fitted ones for them respectively. The Gumbel–Hougaard copula among the Archimedean copulas was employed to construct the bivariate joint distribution of ET0 and irrigation water. The empirical distribution was in close agreement with the theoretical distribution by which the statistical characteristics of the manual water supply and water demand in Luhun irrigation district were explored.
With the copula-based joint distribution of ET₀ and irrigation water, two different probabilities of pairs (ET₀, IR) could be estimated effectively to reflect water shortage risk with the manual water supply in the Luhun irrigation district. In fact, the water shortage risk of extreme encountered situations of the maximum of ET₀ and the minimum of IR in the Luhun irrigation district are more concerned. Compared with the univariate marginal distribution, the bivariate joint distribution of ET₀ and irrigation water involves these two variables simultaneously, so the water shortage risk can be revealed more comprehensively, and the results are naturally more reasonable. It can be concluded that the irrigation water in the Luhun irrigation district can basically satisfy water demand, but water shortage risk still exists.

Moreover, the bivariate joint distribution of ET₀ and irrigation water has a broader application. In practice, if an appropriate distribution process of ET₀ and irrigation water of a typical year is known, using the same frequency amplification method, the several groups of ET₀ and irrigation water with the same occurrence probability can be obtained. Thus, if a certain occurrence probability is given in the future, the corresponding encounter situation of ET₀ and irrigation water can be found. This will provide technology support for irrigation management and drought resistance in the Luhun irrigation district.

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