Storage targets optimization embedded with analytical hedging rule for reservoir water supply operation
Jing Wang, Tiesong Hu, Xiang Zeng and Muhammad Yasir

ABSTRACT
A two-period model is widely used to derive optimal hedging rules for reservoir water supply operation, often with storage targets as the goal to conserve water for future use. However, the predetermined storage targets adopted in the two-period model result in shortsighted decisions without considering the control of long-term reservoir operation. The purpose of this paper is to propose a new model to seek a more promising water supply operation policy by embedding the hedging rule derived from the two-period model in an optimization program for storage targets. Two modules are incorporated in the new model: the two-period model for optimizing water release decisions in each period with given storage targets and the optimization module to determine the optimal values of storage targets for connecting different periods. The Xujiahe water supply system is taken as a case study to verify the effectiveness of the proposed model. The results demonstrate that the new model is superior to others based on standard operation policy or rule curves during droughts and reduces the maximum water shortage.

Key words | hedging rule, optimization, storage targets, water supply operation

INTRODUCTION
As severe droughts may occur in the future, reservoir operation for water supply is not always rational to satisfy the full current demand (You & Cai 2008). Hedging rules are designed for conserving more water for future use during droughts by curtailing delivery even when there is enough water to meet current demand (Maass et al. 1962). A good hedging rule can effectively reduce a very high-percentage single-period shortage. However, unnecessary hedging increases more frequent small shortages and thus decreases the reliability of the water supply (Shiau & Lee 2005). Therefore a lot of effort has been expended in investigating the effects of hedging on water supply performance and suggesting an optimal hedging rule (Srinivasan & Philipose 1998; Soltanjalili et al. 2015; Lian et al. 2016).

Recently, several studies have presented some analytical work to derive an optimal hedging rule based on the two-period model (Draper & Lund 2004; Zhao et al. 2011; Zeng et al. 2015). You & Cai (2008) expanded a theoretical analysis and developed a conceptual two-period (the current period and the next future period) model for reservoir operation with hedging. For a two-period model, hedging cannot be initiated earlier for an impending severe shortage based only on water availability in the current two periods, which is termed the shortsighted decision limitation of the two-period model (You & Cai 2008).

Based on the former general derivation of the two-period model with hedging, Shiau (2011) proposed the operation goal expressed as an explicit incorporation of reservoir release and storage targets, so the derived optimal hedging rule is directly related to water availability and storage targets. A storage target is the desired carryover storage target at the end of a period, which can be an ideal operational level, such as flood control or water conservation (Johnson et al. 1991; Lund & Guzman 1999; Li et al. 2010). In the
two-period model, storage targets can be used as the prede-
termined goals to allocate water between the current two
periods and the further future. For the calculation of the
value of storage targets it affects the performance of water
supply during the overall long-term operation, hence, it
may be improper for Shiau (2014) to take a constant value
or the upper rule curve as storage targets.

The optimal value of storage targets is difficult to derive
because the impact of storage targets during long-term op-
eration depends on uncertain future inflows and the
adaptation of reservoir operation to respond. Fortunately,a
simulation model is the representation of a scheme that
can be used to predict the behavior of the water supply
system under a given set of conditions. The simulation
model combined with an optimization algorithm directly
optimizes performance measures by searching for optimal
values of decision variables (Tu et al. 2008; Chu et al. 2016;
Maiolo & Pantusa 2016). Therefore an optimization model
can be used to find the optimal value of storage targets
which is known as a parameter in the operation of the
two-period model.

Following the derivation of optimal hedging presented
by Shiau (2011), this paper proposes a new water supply op-
eration model, which embeds the hedging rule derivation
from a two-period model in the optimization program to de-
termine the storage targets systematically for seeking a more
promising policy. In the two-period module, analytical opti-
mal hedging is derived from the objective function based on
given storage targets by Karush–Kuhn–Tucker (KKT) con-
ditions (Bazaraa et al. 2006). In the optimization module,
storage targets are optimized through simulation of long-
term operation. The two modules are connected through
the optimal value of storage targets as a given parameter
in the two-period module while a decision variable in the
optimization module.

METHODOLOGY

Model framework

The proposed model consists of two modules: in the two-
period module, the analytical hedging rule is derived with
the storage targets predetermined for each period; while
in the optimization module, the storage targets are used as
the decision variables for connecting the optimal operation
of each two-period module. Therefore, the optimization
module plays a ‘control’ role and the two-period module
plays a ‘feedback’ role.

The framework of the model is described in Figure 1 and
the steps are as follows:

1. Storage targets are initially set in the optimization
module and given as input to the two-period module
for describing current water supply operation.
2. The analytical hedging rule with the storage targets as
given parameter is derived from the two-period
module and the optimal release is determined
concomitantly.
3. The optimal release of each period is transferred to the
optimization module for the simulation of long-term
water supply operation.
4. The storage targets are updated as the decision variables
in the optimization module based on the evaluation of
long-term water supply performance.
5. The above procedure is repeated until the optimal value
of storage targets is found out.

Figure 1 | Framework of storage targets optimization embedded with an analytical hedging rule.
Two-period reservoir operation model

Mathematical formulation

The operation of a water supply reservoir in the current period is either to release water to meet the immediate demand or to retain water for future utilization. So the objective function of the two-period model consists of two value functions, one for current release and the other for carryover storage. The objective function which minimizes the weighted sum of squared deviations from the storage targets and the water demands is usually adopted to derive the optimal operation rule (Jothiprakash & Shanthi 2006; Shiau 2011). Because a very severe single-period water deficit is not accepted in a water supply system, vulnerability limitation (defined by Bayazit & Ünal 1990) is taken as the minimum release constraint in this model. The two-period model for reservoir operation is formulated by Equation (1) with the objective function of shortage loss and the constraints on release, storage and mass balance:

\[
\min_{R_t} f = B(R_t) + C(S_{t+1})
\]

\[
= w \left( \frac{D_t - R_t}{D_t} \right)^m + (1 - w) \left( \frac{S^T_{t+1} - S_{t+1}}{S^T_{t+1}} \right)^m
\]

\[
\text{s.t.} \quad \begin{cases} 
R_t + S_{t+1} = W_{At} \\
aD_t \leq R_t \leq D_t \\
0 \leq S_{t+1} \leq K
\end{cases}
\]

where \( R_t \) is release at time \( t \); \( S_{t+1} \) is the water stored in the reservoir at the end of time \( t \); \( B(R_t) \) is the current release value function; \( C(S_{t+1}) \) is the carryover storage value function, and both the value functions are assumed to be convex; \( W_{At} \) is water availability at time \( t \); \( D_t \) is water demand at time \( t \); \( a \) is vulnerability limitation, \( aD_t \) is the minimum release at time \( t \); \( K \) is reservoir capacity; \( S^T_{t+1} \) is storage target at time \( t \); \( w \) is the weighting factor assigned to the two value functions; \( m \) is an exponent of the value functions; and \( m > 1 \) is the convex function requirement. The KKT conditions derived in Appendix A (available with the online version of this paper) are adopted to solve this convex programming.

Analytical optimal hedging

The optimal hedging is achieved when the marginal benefit of release is equal to the marginal benefit of storage (Draper & Lund 2004). So the optimal release is derived as:

\[
R_t = \frac{S^T_{t+1} + \eta_t^{1/m-1} (W_{At} - S^T_{t+1})}{S^T_{t+1} + \eta_t^{1/m-1}}
\]

where \( W_{At} \) equals the sum of \( I_t \), inflow at time \( t \), plus \( S_t \), the water stored in the reservoir at the beginning of time \( t \), minus \( L_t \), water loss (evapotranspiration and leakage from the reservoir) at time \( t \), as Equation (3); \( \eta_t \) is a factor defined by Equation (4):

\[
W_{At} = S_t + I_t - L_t
\]

\[
\eta_t = \left( 1 - \frac{w}{w} \right) \left( \frac{D_t}{S^T_{t+1}} \right)
\]

The starting water availability (SWA) and the ending water availability (EWA) used to implement the derived optimal release are given in the discussion of other cases shown in Appendix B (available with the online version of this paper) and the derived optimal hedging rule is described in Figure 2.

If \( W_{At} < aD_t \), it is assumed that water is released as SOP (standard operation policy) because hedging cannot be derived from the two-period model in this case. In Figure 2, O-A-B-C stands for SOP, SWA1-EWA stands for the hedging rule and O-SWA2-EWA stands for full hedging.

The two different SWA derived from the lower release bound or the lower storage bound represent the two types of hedging rule distinguished by the parameter \( \eta_t \). Correspondingly, when \( \eta_t < 1 \) (line I in Figure 2), the type I
hedging gives priority to the current release; while $\eta > 1$ (line II), the type II hedging is preferred to achieve storage targets; and when $\eta = 1$ (line III), the type III hedging is a special case, termed one-point hedging.

Synthesized from the derivation of the two-period model by the KKT conditions, the analytical optimal hedging rule is presented as follows:

$$R_t = \begin{cases} \frac{WA_t}{DT_t} < aD_t \\ R'_t, aD_t \leq WA_t < SWA_t \\ R_t, SWA_t \leq WA_t \leq EWA_t \\ D_t, WA_t > EWA_t \end{cases} \tag{5}$$

where $R'$ is chosen with the factor $\eta$:

$$\eta \leq 1, R' = \frac{WA_t}{DT_t}, aD_t \leq WA_t < SWA_t \tag{6}$$

$$\eta \geq 1, R' = \begin{cases} aD_t, aD_t \leq WA_t < aD_t + S_{t+1} \\ WA_t - S_{t+1}, aD_t + S_{t+1} \leq WA_t \leq SWA_t \end{cases}$$

Storage targets optimization model

Because the analytical optimal hedging rule derived from the two-period model depends on the storage target, it can be taken as the control variable to connect the release in the current period and that in the future. So the optimization model is proposed to optimize the storage targets, using the derived analytical optimal hedging rule as input to describe the current operation.

The optimization model can be formulated by Equation (7) with the objective function of minimum water shortage under the constraints on derived optimal release, mass balance and storage constraints:

$$\min_{S_{t+1}} f = \sum_{t=1}^{n} \left( \frac{D_t - R_t}{D_t} \right)^m$$

s.t. $R_t = \begin{cases} \frac{WA_t}{DT_t} < aD_t \\ R'_t, aD_t \leq WA_t < SWA_t \\ R_t, SWA_t \leq WA_t \leq EWA_t \\ D_t, WA_t > EWA_t \end{cases}$

$$S_{t+1} = S_t + I_t - R_t - L_t - SU_t$$

$$SU_t \leq SU_{max}$$

$$0 \leq S_t \leq K$$

where $R_t$ is release at time $t$ according to the analytical optimal hedging rule derived from the two-period model; $SU_t$ is spill at time $t$; $SU_{max}$ is the maximum of reservoir spill; and $n$ is the total period amount of long-term operation.

For solving the proposed optimization module, an improved particle swarm optimization (IPSO) by Jiang et al. (2007) is adopted in this paper.

CASE STUDY

Overview of the Xujiahe water supply system

The Xujiahe water supply system, located in Hubei Province of central China, is used as an example to illustrate the effectiveness of the proposed model. The Xujiahe Reservoir is the main water source for irrigation of the Xujiahe Irrigation District, with a total storage capacity of $440 \times 10^6 \text{m}^3$, and a dead storage of $141 \times 10^6 \text{m}^3$. The mean annual inflow for the period 1973 to 2000 for the Xujiahe Reservoir was $252 \times 10^6 \text{m}^3$ while the mean water demand was $167 \times 10^6 \text{m}^3$.

Alternative operation policies for comparison

Two alternative operation policies, namely, SOP and rule curves, are selected for comparison with the hedging rule derived from the proposed model. The optimization of rule curves is formulated with the same equation (Equation (7)) and optimized by the IPSO. In the optimization program for storage targets, the end of a prolonged wet season when the reservoir is full is taken as the beginning and end of the optimization period and a monthly time-step is employed. In addition, the value of $m$ is taken as 2, as widely suggested (Jothiprakash & Shanthi 2006), and the value of $\alpha$ equals 0.8 according to the regulation of Xujiahe Reservoir operation, while the weighting factor is optimized as the decision variable in the optimization module.

The following three indices are employed to numerically measure the water supply performance of each policy: shortage index (SI) (Hydrologic 1975), maximum shortage ratio of one period (MSR), and water supply reliability (P):

$$SI = \frac{100}{n} \sum_{t=1}^{n} \left( \frac{D_t - R_t}{D_t} \right)^2 \tag{8}$$
MSR = \max \left\{ \frac{\min[R_t - D_t, 0]}{D_t} \times 100\% \right\} \quad (9)

\[ P = \left( 1 - \frac{\sum_{t=1}^{n} \left\{ \begin{array}{l}
1, \text{if}(R_t < D_t) \\
0, \text{otherwise}
\end{array} \right\}}{n} \right) \times 100\% \quad (10) \]

where SI is the index of total water shortage during the entire long-term operation; MSR is defined as the maximum value of the single-month shortage ratio over the operation period; and P is the water supply reliability that measures the probability that the reservoir can supply the required volume of water.

Results and discussion

The optimized rule curves and storage targets of the Xujiahe Reservoir in each month are shown in Figure 3. The water supply performance of SOP, rule curves and the derived hedging rule are presented in Table 1. The reservoir storage processes under these policies are shown in Figure 4.

As shown in Table 1, the least water shortage is obtained by the derived hedging rule. SOP is more effective when reservoir inflow is plentiful. However, it neglects the potential shortage vulnerability in the later period. So SOP is not valid during prolonged droughts, leading to more severe water shortage where the shortage index is 0.534. Without rationing water supply, the level of reservoir storage with SOP is too low to release water during prolonged drought, and 100% shortage vulnerability occurred in 1979. Release is restricted with a ration ratio under rule curves, so both the shortage index of 0.321 and maximum shortage ratio of 20% are less than SOP. But the water supply ratio in the rule curves is constant while the water supply ratio, or as termed the hedging factor, derived from the analytical hedging rule, is time-varying. Therefore the shortage index of the hedging rule is improved to 0.088. Furthermore, the water supply reliability of 81% and the maximum shortage ratio of 19.9% from the hedging rule both achieve the required values (80% and 20% respectively) of the Xujiahe water supply system. Thus, it can be concluded that the derived hedging rule based on storage targets is effective for water supply operation.

As shown in Figure 4, hedging is triggered at the beginning of drought, so that some water is stored in the reservoir before prolonged drought, with both the hedging rule and rule curves, while SOP leads to severe single-period shortage without hedging in advance. The hedging factor of the rule curves is higher than that of the derived hedging rule, and the higher reservoir storage level of the rule curves indicates that unnecessary water is stored rather than released due to the unnecessary hedging.

Effects of storage targets on hedging rule

As discussed in the methodology section, water supply performance is closely affected by the storage targets. The oversize degree and range of hedging may lead to water shortage by unnecessary hedging when the values of the storage targets are too high; otherwise reservoir operation guided by the derived hedging rule fails to store enough water to mitigate future deficits. In order to analyze the relationship between storage targets and the hedging rule, the upper rule curve is taken as the storage targets proposed by Shiau (2011) to compare with the optimized storage targets in this paper. The operation results of these two methods are presented in Table 2 and the reservoir storage processes are compared in Figure 5. As presented in

![Figure 3 | Optimized storage targets and rule curves.](https://iwaponline.com/ws/article-pdf/18/2/622/206878/ws018020622.pdf)
Table 2, the water supply performance with the optimized storage targets beats that with the upper rule curve targets according to the three indicators. Moreover, the operation results guided by the upper rule curve targets fail to meet the acceptable maximum shortage ratio and required water supply reliability from the Xujiahe water supply system due to the improper level of storage targets. In the comparison of storage processes, the higher reservoir storage level of the upper rule curve indicates that unnecessary hedging is triggered for a high storage target value.

Table 2 | Water supply performance of the upper rule curve and optimized storage targets

<table>
<thead>
<tr>
<th>Hedging rule</th>
<th>The upper rule curve</th>
<th>Optimized storage targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shortage index</td>
<td>0.160</td>
<td>0.088</td>
</tr>
<tr>
<td>Maximum shortage ratio (%)</td>
<td>27.8</td>
<td>19.9</td>
</tr>
<tr>
<td>Water supply reliability (%)</td>
<td>70.9</td>
<td>81.0</td>
</tr>
</tbody>
</table>

In order to analyze the sensitivity of water supply performance with the value of storage targets, the water storage between the upper and lower rule curve is divided into ten copies (scenario 1 corresponds to taking the lower rule curve for the storage targets, while the storage targets in scenario 10 are the upper rule curve). As shown in Figure 6, both maximum shortage ratio and water supply reliability decrease with increasing storage targets: the maximum shortage ratio decreases from 57.5% to 27.8%, while water supply reliability decreases from 94.7% to 70.9%, which indicates that more hedging is triggered during long-term operation. However, the shortage index first increases to the value of 0.31 and after that decreases with increasing storage targets. Moreover, water supply performance of all the ten scenarios is not better than the scenario with optimized storage targets. Therefore, storage targets must be selected at the proper level to measure the shortage degree of the future period so that the water supply performance in long-term operation can be improved.
As shown in Figure 2, there are three types of hedging: hedging I, hedging II and hedging III. The triggering frequency for hedging I, hedging II and SOP (hedging III does not appear in the operation simulation of Xujiahe Reservoir) is presented in Figure 7. SOP is triggered most during long-term operation while the hedging rule is only applied to a few periods due to the release and storage constraints. This indicates that during Xujiahe water supply system operation, the marginal benefit of current release is always more than the marginal benefit of carryover storage, even during prolonged drought. So water rarely stored first and then released is consistent with the actual operation of Xujiahe water supply system. The triggering frequency of hedging I increases from 15 to 109, while the triggering frequency of hedging II decreases from 6 to 0 and the triggering frequency of SOP decreases from 315 to 227 with increasing storage targets. The results indicate that water release in the current period decreases with increasing storage target value based on the hedging rule.

CONCLUSION

This paper proposes a new model of storage target optimization embedded with an analytical hedging rule for reservoir water supply operation. Compared with other models based on SOP or rule curves in the case study of the Xujiahe water supply system, the operation performance of the new model is improved both in shortage index and maximum single-period shortage with lower water supply reliability by taking appropriate hedging before severe droughts.

The sensitivity analysis of the storage targets with hedging indicates that the triggering frequency of the hedging rule increases while the triggering frequency of SOP decreases as the storage targets increase. When the value of the storage targets is set too low, reservoir operation fails to store enough water to mitigate future deficits. On the other hand, a high level of storage target would result in shortage by unnecessary hedging. Optimization of storage targets through the proposed optimization program can overcome the limitation of the shortsighted decisions of the two-period model and reduce the maximum single-period water shortage.

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