Incorporating pollutants interaction with the environment and parameter uncertainty in water quality evaluation: a case of Lake Chauhan, China

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ABSTRACT

Water quality evaluation is a key task in water resource management and pollution control. Current evaluation methods are rooted in water quality index, which assesses the water quality based on the exact concentration of various pollutants. However, the interaction between the pollutants and the water environment should also be considered. This paper suggests a new approach, which integrates pollutant interaction with water environment and parameter uncertainty to water quality evaluation. The new approach is compared with traditional methods. Then, an inexact evaluation model, the integrated water quality evaluation model under uncertainty, is established in accordance with the proposed approach, in which catastrophe theory is used to deal with the ambiguous internal mechanism of the interaction between the pollutants and the water environment. As there are significant uncertainties in water quality evaluations, fuzzy random variables are employed to describe the inexact monitoring data. To solve the proposed model, a new algorithm is designed. The model is then applied to an actual case: Lake Chaohu, China. The results are compared between the proposed method and China’s current evaluation method (i.e. max-index method). Some brief analysis and discussion are given about the results, which could be helpful in guiding environmental management decision-making.

Key words | catastrophe theory, inexact model, uncertainty, water quality evaluation, water quality index

INTRODUCTION

Acceptable water quality is of utmost importance to public health and economic-social sustainable development (Grey & Sadoff 2007; Zeitoun 2011). Therefore, water quality evaluation plays a key role in water resources management. It has been reported that there are over 1.5 million people (90% are children under 5 years old) die of waterborne diseases every year (WHO 2004). In China, statistics show that, 70% of the national surface water is polluted, with 15% being seriously polluted (MEPPRC 2014). Degraded water quality poses a serious threat to public health and economic-social sustainability (Mondal et al. 2014). Therefore, research into new approaches to water quality evaluation is an urgent task for effective and efficient water resources management.

Water quality is identified in terms of its physical, chemical and biological parameters (Sargaonkar & Deshpande 2003). In 1965, Horton developed a water quality index (WQI) to identify the degree of water quality degradation (Horton 1965). Thereafter, based on Horton’s WQI, a range of other water quality indices were proposed to quantitatively measure water quality (House 1989; Lumb et al. 2011; Almeida et al. 2012) and many mathematical models were developed to evaluate the quality of various types of
water (du Plessis et al. 2014; Akomeah et al. 2015; Li et al. 2015a). When evaluating water quality, there are three key factors that should be examined: (1) the interaction between the pollutants and the water environment (González et al. 2014), (2) the ambiguity of the inner interaction mechanism (Chen et al. 2007; Wang et al. 2011; Yang et al. 2012), (3) the uncertainty in the water quality measurements (Akomeah et al. 2015; Li et al. 2015b; Zhang et al. 2015). To ensure the identified factors are included in water quality evaluations, a new evaluation approach is needed.

It is widely known that to determine water quality, the interaction between the physical, chemical and biological features of the water bodies and the local environmental factors must be considered (Ikonen et al. 2013; van der Linden et al. 2015). The features are made up of many factors, for example, total phosphorus (TP), total nitrogen (TN), and dissolved oxygen (DO), which together make up only some of the chemical features of a water body. When the physical, chemical, and biological features and the local environmental factors are expressed as only a simple combination of factors, evaluation methods based on the WQI can be deduced. Taking algae growth as an example: the growth of algae is influenced by an interaction of chemical (e.g. TP, TN, DO), physical (e.g. water temperature), and biological (e.g. algae species, algae density) features as well as local environmental factors (e.g. sunlight duration) (Imboden 1992; Ikonen et al. 2013; van der Linden et al. 2015). When the ratio of TN and TP (TN/TP), water temperature, algae density and sunlight duration continuously gradually change, an abrupt change in water quality may occur; that is, an algal bloom. Using a simple combination of WQI, it is difficult to express this kind of abrupt change in water quality. Furthermore, the inner interaction mechanism of TN/TP, water temperature, algal density and sunlight duration that causes algal bloom is unclear. Fortunately, using a potential function such as that in catastrophe theory, an abrupt change caused by several interacting factors can be described, without having to know the internal system mechanism. Using the advantage of catastrophe theory, De Sá & Berezin (1989) studied corrosion problems. Wang et al. (2014) assessed water quality under environmental changes. However, in these studies, the water body uncertainties were overlooked.

Uncertainty exists in water systems (Beck 1987; Warmink et al. 2010), which means that water quality evaluation modeling is beyond traditional mathematical methods (Xu et al. 2015). Stochastic and fuzzy approaches have been respectively proposed to deal with two separate uncertainties: random uncertainty and fuzzy uncertainty (Koklu et al. 2010; Lu et al. 2016; Yan et al. 2016). However, in a water body, there is another form of uncertainty that has both randomness and fuzziness simultaneously, such as the water pollutant concentration. This is especially evident in large shallow lakes, such as the target lake in our research, Lake Chaohu in Eastern China, which are influenced by wind direction, hydrology, pollutant dispersion, sampling methods and other objective factors, with the pollutant concentration in the lake water body having random uncertainty. Additionally, due to the lack of knowledge about lake hydrology and pollutant degradation, there is also fuzzy uncertainty. Therefore, the water pollutant concentration has the double uncertainties of both randomness and fuzziness. The fuzzy random variable proposed by Kwakernaak (1978) has been used to describe this kind of double uncertainty in previous research (Xu et al. 2015).

In this paper, different from previous evaluation methods (the weighted sum of WQI), a developed evaluation model, namely, an integrated water quality evaluation model under uncertainty (IWQEMU), is established to evaluate water quality, in which catastrophe theory is used to describe the pollutant interactions and the interaction between the pollutants and the water environment, and fuzzy random variables are employed to express the double uncertainty in the water body.

Key problem description

Currently, WQI are widely used to evaluate water quality around the world. There are two evaluation method categories based on WQI: single index methods and comprehensive index methods, some examples of which are listed in Table 1. IQBP, SFWQI and MWQI in Table 1 are respectively the abbreviations for the water quality evaluation models used in Canada, the USA and Malaysia (Hébert 1996, 2005; Cude 2001; Shuhaimi-Othman et al. 2007). It can be seen that in the WQI-based methods, only...
some water pollutants are selected as the evaluation index to evaluate the water quality singly or comprehensively. This method is able to evaluate the water pollutant content accurately, but is unable to determine the interactions between the identified pollutants or the interactions between these pollutants and the specific local water environment, interactions which could cause serious water quality destruction, such as algal blooms or black-odor.

‘Black-odor’ is a biochemical phenomenon in water bodies, and refers to a water body that appears black and/or emits a repulsive odor. Black-odor can be caused by organic pollutants, suspended particles, environmental conditions and sediment pollution, amongst others.

Conceptual framework for IWQEMU

Referring to the National Quality Standards of China: environmental quality standards for surface water (GB 3838-2002) (MEPPRC 2003), nine major water pollutants in three categories and three water environmental factors were chosen as the WQI, the conceptual structure of the developed approach for describing water quality is shown in Figure 1. In Figure 1, a new approach to describe water quality can be written as

\[ WQ = F(f_{PF}(EC, TSS, Tur), f_{CF}(COD, BOD, TP, TN), f_{BF}(Mor, Phy), f_{WEF}(Sd, Temp, Fr, Hrt)) \]  

(1)

where \( WQ \), \( f_{PF}, f_{CF}, f_{BF}, f_{WEF} \), respectively, denote the water quality, and the physical, chemical, biological features and the local water environmental factors. Compared with the evaluation methods listed in Table 1, evaluation methods based on WQI are special forms of the developed approach described in Formula (1). For example, when \( f_{WEF} = 0 \), and \( F = \max \), \( f_{PF}, f_{CF}, f_{BF} \) are vector ID functions, i.e. \( f_{PF}(EC, TSS, Tur) = (EC, TSS, Tur) \), \( f_{CF} \) and \( f_{BF} \) similarly. Formula (1) is reduced to the max-index method, i.e. \( \max \{P_1, P_2, \ldots, P_n\} \) (MEPPRC 2003). When \( F = \min \) and \( f_{WEF} = 0 \), \( f_{PF}, f_{CF} \) and \( f_{BF} \) are refined into seven features, the IQBP (Hèbert 1996, 2005) is obtained. When \( f_{PF}, f_{CF} \) and \( f_{BF} \) are vector ID functions, \( f_{WEF} = 0 \), \( F \) is respectively a weighted average, weighted geometric average, weighted square average, and harmonic square average, then, four forms of the multi-index mean (Deininger & Maciuinas 1971; SRDD 1976; Tyson & House 1989; Cude 2001) can be derived. Other WQI-based evaluation methods are approached in a similar way.

**Motivation for using catastrophe theory**

However, the inner mechanisms of the physical, chemical and biological actions between the pollutants and the local water environment are currently still ambiguous. That is, the function relationship of \( F, f_{PF}, f_{CF}, f_{BF} \) and \( f_{WEF} \) in Formula (1) remains unknown. Catastrophe theory can be applied to deal with these sorts of inexact phenomena.
Poston and Stewart expounded the mathematical structures of catastrophe theory and then illustrated its various applications (Poston & Stewart 2014). Catastrophe theory can be widely applied to various fields because it has the following advantages: (1) it is not necessary to know the internal mechanism of the system; (2) it simultaneously deals with random and fuzzy uncertainty; (3) it gives a mathematical description and a quantitative processing of the system.

Flay summarized the five essential features of catastrophe theory using cusp catastrophe as an example (Flay 1978). When a system meets these five features, catastrophe theory adapts. The surface water system has these five essential features. (1) Mutability – the dynamic system balance of the water quality can suddenly change from one level to another, e.g. blue-green algae blooms. (2) Multimodal – the system has multiple steady states; for instance, the water quality states of clean, polluted, black-odor and algal bloom. (3) Hysteresis – when the support conditions change, the system state continues for some time, e.g. black-odor. (4) Unreachability – the system has an unstable state, which is unreachable in practice; for example, the critical point between being polluted and being clean. (5) Divergent – in some situations, a slight change in the control variables induces a system state change; for example, the effect of temperature on algal blooms. Therefore, catastrophe theory can be employed to model the water quality evaluation.

**Conceptual model framework**

When a system has the foregoing five features, satisfies only one state variable and has no more than four control variables, four of the seven basic catastrophe models proposed by Zeeman can be used to describe this system (Zeeman 1976). Using $x$ to denote the state variable, $a$, $b$, $c$, $d$ to denote the control variables and, $f(x)$ as a potential function of $x$, the four models can be written as follows.

The fold catastrophe model:

$$f(x) = x^3 + ax$$  \hspace{1cm} (2)

The cusp catastrophe model:

$$f(x) = x^4 + ax^2 + bx$$  \hspace{1cm} (3)

The swallowtail catastrophe model:

$$f(x) = x^5 + ax^3 + bx^2 + cx$$  \hspace{1cm} (4)
The buttery catastrophe model:

\[ f(x) = x^6 + ax^4 + bx^3 + cx^2 + dx \]  

(5)

From the conceptual structure of the integrated evaluation method (Figure 1), it can be seen that the integrated evaluation method proposed in this paper is a multi-level catastrophe model. In every level of the model, there is only one state variable and no more than four control variables. That is, Formulas (2)–(5) can be selected as the evaluation submodel for each level. Further, the important control variables are generally written in front of the less important control variables. Starting from the top level (i.e. water quality), the current and subsequent levels are treated, respectively, as the state variable and the control variables. In this way, the conceptual framework in Figure 1 was converted into the conceptual model framework in Figure 2.

**Development of IWQEMU approach**

**Uncertainty processing**

There is significant uncertainty in the water environment (Beck 1987; Krysanova et al. 2007). Impacted by microorganisms, algae, flow rate, temperature and pollutant emissions, the concentration of pollutants in the water is not a constant, but an inexact value. Therefore, real-time monitoring of pollution data such as time series data is unsuitable for directly evaluating water quality.

For convenience, the NH$_3$-N concentration was examined as an example to explain the uncertainty processing method. Real-time NH$_3$-N monitoring data were obtained from the environmental monitoring center (http://58.68.130.147), which reported that the levels were respectively 0.32 and 0.33 mg/L at 8 am on October 25 and October 30, 2014. Obviously, these were not definite values. A weekly value for the NH$_3$-N concentration in 2011 was extracted for observing the characteristics of the data, and was expressed using the most likely value (the sum of the product of the monitoring value and its frequency divided by the number of monitoring times) and a range. For example, the NH$_3$-N concentration in the first week of 2011 can be expressed as: the most likely value: 0.29 mg/L, Range: 0.26–0.35 mg/L. Using $b_i$, $r_i$, and $R_i$ to represent the most likely value and the lower and upper bounds of the NH$_3$-N concentration in the $i$th week of 2011, the weekly NH$_3$-N concentration value in 2011 can be expressed as $b_i$, $[r_i, R_i]$, $i = 1, 2, \ldots, 52$. It was found that the maximum value for the upper bound and the minimum value of the lower bounds were 0.47 and 0.2 mg/L, and the most likely value was a random variable (denotes as $\tilde{b}$). Comparing

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**Figure 2 | Conceptual model framework for IWQEMU.**

\[ NB: BI=Biological indicators; BOD=Biological oxygen demand; CI=Chemical indicators; COD=Chemical oxygen demand; EC=Electrical conductivity; Fr=Flow rate; Hrt=Hydraulic residence time; Mor=Microorganism; Phy=Phycophyta; PI=Physical indicators; RI=Reaction indicators; Sd=Sunlight duration; Temp=Temperature; TN=Total nitrogen; TP=Total phosphorus; TN/TP=Ratio of TN to TP; TOD=total oxygen demand; TSS=Total suspended solids; Tur=Turbidity. \]
It was found that \( \hat{b} \) approximately followed a normal distribution with a mean value of 0.33 and a variance of 0.01 (i.e., \( \hat{b} \sim N(0.33, 0.01) \)), which can be estimated using a maximum likelihood method and justified using a chi-square goodness-of-fit test.

In this way, the \( NH_3-N \) concentration can be converted into a Kwakernaak’s fuzzy random variable (Kwakernaak 1978) and written as \( (0.2, \hat{b}, 0.47) \), where \( \hat{b} \sim N(0.33, 0.01) \). Therefore, the \( NH_3-N \) concentration uncertainty can be described as a fuzzy random variable. The conversion process is shown in Figure 3. The uncertainty of other indicators can be calculated in a similar way.

**IWQEMU formulation**

According to Formulas (2)–(5), the conceptual model framework in Figure 2 can be translated into a mathematical form. Taking the chemical indicators (CI) as an example, the conversion process is as follows. As seen in Figure 2, there are four chemical indicators, i.e. chemical oxygen demand (COD), biochemical oxygen demand (BOD), TP and TN. Taking CI as the state variable, and the four chemical indicators as the control variables, the chemical features of the water body can be described by 1 state variable and 4 control variables. As mentioned above, if there is only one state variable and four control variables, the buttery catastrophe model, i.e. Formula (5), is suitable. In addition, for large shallow lakes, the chemical characteristics of the pollution are mainly characterized by eutrophication, and TN and TP are the main causes of eutrophication. Therefore, for these four chemical indicators, the most important is TN, followed by TP, then COD and BOD. According to the principle, the important control variable is written at the front, so the mathematical form for the chemical indicators is

\[
\phi(CI) = CI^6 + TN \cdot CI^4 + TP \cdot CI^3 + COD \cdot CI^2 + BOD \cdot CI
\]  

In Formula (6), the order TN, TP, BOD, and COD can be replaced with the actual situation in different water bodies. Formula (6) describes the combined effect of the control variables (i.e. COD, BOD, TP and TN) on the state variable (i.e. CI) by using a potential function \( \phi \). If the system is at an equilibrium, then \( \phi'(CI) = 6CI^5 + 4TN \cdot CI^3 + 5TP \cdot CI^2 + 2COD \cdot CI + BOD = 0 \), thus, the interactive relationship between the control variables is revealed.

The other indicators are similarly converted. Let \( x \) be the state variable for water quality, \( f(x) \) be the potential function of \( x \), so the whole model can be deduced as follows.

\[
f(x) = x^8 + CI \cdot x^4 + PI \cdot x^3 + RI \cdot x^2 + BI \cdot x
\]

where CI, PI, RI, BI solve from

\[
\begin{align*}
\varphi_1(CI) &= CI^6 + TN \cdot CI^4 + TP \cdot CI^3 + COD \cdot CI^2 + BOD \cdot CI \\
\varphi_2(PI) &= PI^5 + EC \cdot PI^3 + TSS \cdot PI^2 + Tur \cdot PI \\
\varphi_3(RI) &= RI^4 + Ab \cdot RI^2 + Bo \cdot RI \\
\varphi_4(BI) &= RI^4 + Phy \cdot BI^2 + Mor \cdot BI
\end{align*}
\]

where Ab, Bo solve from

\[
\begin{align*}
\varphi_1(\text{Ab}) &= Ab^6 + NP \cdot Ab^4 + Phy \cdot Ab^3 + Temp \cdot Ab^2 + Sd \cdot Ab \\
\varphi_2(\text{Bo}) &= Bo^6 + TOD \cdot Bo^4 + Wb \cdot Bo^3 + Mor \cdot Bo^2 + Temp \cdot Bo
\end{align*}
\]

\[
Wb = \begin{cases} 
Fr, & \text{for river} \\
Hrt, & \text{for lake}
\end{cases}
\]  

**Figure 3** | The process of converting \( NH_3-N \) concentration into a fuzzy random variable. Data derived from Binghu monitoring point, Hefei, Anhui Province.
where $Ab$, $NP$, $Bo$ and $Wb$ denote: algal bloom, ratio of TN to TP, black-odor and water balance, respectively; $\varphi_1$, $\varphi_2$, $\varphi_3$, $\varphi_4$, $\psi_1$, $\psi_2$ are potential functions.

However, as described in subsection **Uncertainty processing**, the actual monitoring data for the model parameters are fuzzy random numbers. Therefore, the model should be rewritten as Formula (8).

$$f(x) = x^6 + \overline{CI}x^4 + \overline{PI}x^3 + \overline{RI}x^2 + \overline{BI}x$$

where $\overline{CI}$, $\overline{PI}$, $\overline{RI}$, $\overline{BI}$ solve from

$$\begin{align*}
\varphi_1(\overline{CI}) &= CI^6 + \overline{TN}CI^4 + \overline{TP}CI^3 + \overline{COD}CI^2 + \overline{BOD}CI \\
\varphi_2(\overline{PI}) &= PI^5 + \overline{EC}PI^3 + \overline{TSS}PI^2 + \overline{Tur}PI \\
\varphi_3(\overline{RI}) &= RI^4 + \overline{Ab}RI^2 + \overline{Bo}RI \\
\varphi_4(\overline{BI}) &= BI^4 + \overline{Mor}BI^2 + \overline{Ab}BI
\end{align*}$$

where $\overline{Ab}$, $\overline{Bo}$ solve from

$$\begin{align*}
\varphi_1(\overline{Ab}) &= Ab^6 + \overline{NP}Ab^4 + \overline{Phy}Ab^3 + \overline{Temp}Ab^2 + \overline{Sd}Ab \\
\varphi_2(\overline{Bo}) &= Bo^6 + \overline{TOD}Bo^4 + \overline{Wb}Bo^3 + \overline{Mor}Bo^2 + \overline{Temp}Bo
\end{align*}$$

$Wb = \{Fr, \text{for river}\}$

$$Wb = \{Hrt, \text{for lake}\}$$

(8)

**Algorithm design**

To solve the model (i.e. Formula (8)), the normalized formula development can be obtained as follows. Taking potential function $f(x)$ as an example, the process is similar to the potential function

$$\varphi_1(CI), \varphi_2(PI), \varphi_3(RI), \varphi_4(BI), \psi_1(\overline{Ab}), \psi_2(\overline{Bo})$$. The balance point set and singular point set for function $f(x)$ can be obtained by solving equations $f'(x) = 0$ and $f''(x) = 0$, respectively. The bifurcation set for function $f(x)$ is obtained by canceling the state variable $x$ in equations $f(x) = 0$ and $f'(x) = 0$. This indicates that a catastrophe happens when all the control variables satisfy the bifurcation point set equation. In this way, the normalized formula for the model can be deduced as follows.

$$x_{CI} = (\overline{CI})^{1/2}, x_{PT} = (\overline{PI})^{1/3}, x_{RI} = (\overline{RI})^{1/4}, x_{BI} = (\overline{BI})^{1/5},$$

where $\overline{CI}$, $\overline{PI}$, $\overline{RI}$, $\overline{BI}$ solve from

$$\begin{align*}
CI_{TN} &= (\overline{TN})^{1/2}, CI_{TP} = (\overline{TP})^{1/3}, CI_{COD} = (\overline{COD})^{1/4}, \\
CI_{BOD} &= (\overline{BOD})^{1/5} \\
PI_{EC} &= (\overline{EC})^{1/2}, PI_{TSS} = (\overline{TSS})^{1/3}, PI_{Tur} = (\overline{Tur})^{1/4} \\
RI_{Ab} &= (\overline{Ab})^{1/2}, RI_{Bo} = (\overline{Bo})^{1/3} \\
BI_{Phy} &= (\overline{Phy})^{1/2}, BI_{Mor} = (\overline{Mor})^{1/3}
\end{align*}$$

where $\overline{Ab}$, $\overline{Bo}$ solve from

$$\begin{align*}
Ab_{NP} &= (\overline{NP})^{1/2}, Ab_{Phy} = (\overline{Phy})^{1/3}, Ab_{Temp} = (\overline{Temp})^{1/4}, \\
Ab_{Sd} &= (\overline{Sd})^{1/5} \\
Bo_{TOD} &= (\overline{TOD})^{1/2}, Bo_{Wb} = (\overline{Wb})^{1/3}, Bo_{Mor} = (\overline{Mor})^{1/4}, \\
Bo_{Temp} &= (\overline{Temp})^{1/5}
\end{align*}$$

$Wb = \{Fr, \text{for river}\}$

$$Wb = \{Hrt, \text{for lake}\}$$

(9)

Two principles named, the ‘complementation principle’ and the ‘non-complementation principle’, need to be complied with when operating within the catastrophe model. The complementation principle should be followed when the control variables are obviously interrelated. In this case, the state variable $x$ takes the mean of the control variable values. The non-complementation principle should be followed when the control variables are not obviously interrelated. In this case, the state variable $x$ takes the minimum value of the control variable values.

By combining Formula (9) and the two principles, a common calculation structure for the model can be obtained (Figure 4).
Case study

Case study area

Lake Chaohu, one of the five largest freshwater lakes in China, is located at 31 N and 117 E. The Lake Chaohu watershed had an area of 13,486 km², and a catchment population of 9.7 million at the end of 2010. According to the statistics in 2010, the total amount of surface water and groundwater was respectively 6.96 billion and 1.86 billion m³, and the amount of water per capita was 909 m³, which indicates that there is a severe water shortage area accordance with UN standards.

With rapid economic development and urban construction, urban sewage and industrial wastewater, emissions have substantially increased in the past three decades, most of which has been discharged unprocessed into the nearby rivers, which flow into Lake Chaohu. Increasing pollution pressure has become a serious threat to the regional economic and ecological coordinated development.

Data acquisition

In China, surface water has been divided into five categories according to WQI values. When the index value exceeds the maximum limits, the corresponding surface water is called inferior fifth category water. The surface water categories and the interval of the index value limits were derived from the National Quality Standards of People’s Republic of China (MEPPRC 2003), as listed in Table 2.

The monitoring data of all the water quality indices from Lake Chaohu from 2009 to 2013 were taken from two real-time monitoring points at the National Environmental Monitoring Center of China: Binghu and Yuxikou monitoring points (Hefei, Anhui Province, http://58.68.130.147). Using the method for processing uncertainty proposed in subsection Uncertainty processing, the monitoring data were converted into fuzzy random numbers, as shown in Table 3.

RESULTS AND ANALYSIS

Results of the model

A dimensional difference exists in the multi-index input system. Therefore, to do the calculation, a dimensionless analysis is first required. In this research, the upper limits of the index value for the fifth category and the lower limit of the index value for the first category were used as benchmarks for the nondimensionalization. The standardization formulas used were as follows:

\[ X_{i,\text{std}} = \frac{X_i}{X_{V_{-sup}}} \quad i = 1, 2, \ldots, n \]  

Table 2 | Surface water index limits and categories

<table>
<thead>
<tr>
<th>Index</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>V bad</th>
</tr>
</thead>
<tbody>
<tr>
<td>COD (mg/L)</td>
<td>&lt;15</td>
<td>&lt;15</td>
<td>15–20</td>
<td>20–30</td>
<td>30–40</td>
<td>&gt;40</td>
</tr>
<tr>
<td>BOD (mg/L)</td>
<td>&lt;3</td>
<td>&lt;3</td>
<td>3–4</td>
<td>4–6</td>
<td>6–10</td>
<td>&gt;10</td>
</tr>
<tr>
<td>TP (mg/L)</td>
<td>&lt;0.01</td>
<td>0.01–0.025</td>
<td>0.025–0.05</td>
<td>0.05–0.1</td>
<td>0.1–0.2</td>
<td>&gt;0.2</td>
</tr>
<tr>
<td>TN (mg/L)</td>
<td>&lt;0.15</td>
<td>0.15–0.5</td>
<td>0.5–1.0</td>
<td>1.0–1.5</td>
<td>1.5–2.0</td>
<td>&gt;2.0</td>
</tr>
<tr>
<td>EC (μS/cm)</td>
<td>&lt;400</td>
<td>400–1,000</td>
<td>1,000–1,500</td>
<td>1,500–2,000</td>
<td>2,000–2,900</td>
<td>&gt;2,900</td>
</tr>
<tr>
<td>Temp (°C)</td>
<td>–2 ≤ 2 ≤ human-caused change in water temperature ≤1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TSS (mg/L)</td>
<td>&lt;20</td>
<td>20–25</td>
<td>25–30</td>
<td>30–60</td>
<td>60–150</td>
<td>&gt;150</td>
</tr>
<tr>
<td>Tur (NTU)</td>
<td>&lt;5</td>
<td>5–10</td>
<td>10–20</td>
<td>20–50</td>
<td>50–80</td>
<td>&gt;80</td>
</tr>
<tr>
<td>Mor (×10² CFU)</td>
<td>&lt;2</td>
<td>2–20</td>
<td>20–100</td>
<td>100–200</td>
<td>200–400</td>
<td>&gt;400</td>
</tr>
<tr>
<td>Phy (×10³ cells/L)</td>
<td>&lt;1.0</td>
<td>1.0–2.0</td>
<td>2.0–3.3</td>
<td>3.3–4.6</td>
<td>4.6–6.0</td>
<td>&gt;6.0</td>
</tr>
<tr>
<td>Hrt (days)</td>
<td>40–80</td>
<td>80–120</td>
<td>120–200</td>
<td>200–360</td>
<td>360–540</td>
<td>&gt;540</td>
</tr>
</tbody>
</table>

Note: The data derived from the National Environmental Quality Standards for Surface Water of China.
where $i$, $X_{i\text{std}}$ and $X_i$ denote the $i$th index, the normalized and monitoring value of the index $i$, respectively, $X_{i\text{--sup}}$ denotes the upper limit value of index $i$ for the fifth category.

When the fuzzy random variables in Formula (9) were replaced with crisp variables, using the index values shown in Table 2, a new integrated evaluation set was derived based on the calculation structure in Figure 4. The interval values for the new evaluation set are shown in Table 4.

The Lake Chaohu watershed has a northern subtropical monsoon climate. The annual average surface temperature is 16°C, the minimum and maximum temperatures are −1°C and 33°C and the annual average sunlight duration is 2,170 h. Therefore, the temperature and weekly sunlight duration parameters were taken as $(-1, \text{Temp}, 53)$, $\text{Temp} \sim N(16, 2)$ and $(11, \overline{SD}, 69)$, $\overline{SD} \sim N(40, 7)$. Following the steps below, the results were calculated using the model.

Step 1. Input the index monitoring data from Table 3, as well as the temperature and weekly sunlight duration parameter values.

Step 2. Standardize the input values using Formula (10).

Step 3. Using Formula (9), normalize the results of Step 2.

Step 4. According to the two principles, integrate the results from Step 3.

Step 5. Based on the calculation structure in Figure 4, calculate the model results.

Algorithms for calculating the fuzzy random variables in Steps 2–5 can be found in Kwakernaak (1978, 1979), Xu & Zhou (2011). All model calculation results are shown in Table 5.

The results in Table 5 are fuzzy random numbers, which cannot be directly compared with the interval numbers in Table 4. A chance measure (Xu & Zhou 2011) can be employed to deal with this situation. The chance measure is mathematically written as

$$Ch(a \leq \overline{x} \leq b)\{\theta_1, \theta_2\} \geq \{\theta_1, \theta_2\} \iff \begin{cases} Ch(a \leq \overline{x})\{\theta_1\} \geq \theta_1 \\ Ch(\overline{x} \leq b)\{\theta_2\} \geq \theta_2 \end{cases}$$

(11)

where $Ch(a \leq \overline{x})(\theta) \sim \text{Pos}[Pr(a \leq \overline{x}) \geq \theta]$, $Ch(\cdot)$, $\text{Pos}(\cdot)$ and $Pr(\cdot)$ denote event ‘∗’ chance, possibility and
probability measure respectively; $a$ and $b$ are crisp numbers, $\tilde{x}$ is a fuzzy random number; $\vartheta$, $\vartheta_1$ and $\vartheta_2$ are probability levels, $\theta$, $\theta_1$ and $\theta_2$ are possibility levels. Using Formula (11), the fuzzy random numbers in Table 5 and the interval numbers in Table 4 can be compared under different possibility and probability levels. The comparison results are listed in Table 6.

In Table 6, ‘*’ and ‘‘’ in the number pair (*, *) denote the probability and possibility levels for the water quality belonging to the corresponding category. For example, the number pair (0.80, 0.17) on row 2 column 2 shows that under a probability level of 0.8, the possibility level of the water quality in 2009 belonging to category III was 0.17. That is, under a probability level of 0.8 and a possibility level of 0.17, the integrated evaluation value of the water quality in Lake Chaohu was in the interval $[0.35, 0.60)$. Similarly, the number pair (0.80, 0.93) on row 3 column 2 indicates that the probability and possibility levels of the water quality in Lake Chaohu belonged to category IV (i.e. the integrated evaluation value belongs to interval $[0.60, 0.80]$) were 0.8 and 0.93, respectively. All number pairs in column 2 show that under a probability level of 0.8, the possibility levels of the water quality in 2009 belonging to category III, IV, V, and V bad were 0.17, 0.93, 0.21, 0.11 and under a probability level of 0.9, the possibility levels were respectively 0.09, 0.87, 0.13 and 0.05. This can be seen directly in Figure 5. For the number pairs in columns 3–6, it is similar. Therefore, using the model proposed in this paper to evaluate the water quality in Lake Chaohu, the integrated evaluation results of the Lake Chaohu water quality from 2009 to 2013 were as listed in Table 6.

### Analysis and discussion

From Table 6, it can be seen that from 2009 to 2013 the water quality in Lake Chaohu was a category IV according to the Chinese classification standards. That is, Lake Chaohu water was only suitable for industrial use or non-contact recreational activities. The worst water quality was in 2009 and, surprisingly, the best water quality was in 2010, but after that time, the water quality degraded again.

### Table 4 | The integrated evaluation set

<table>
<thead>
<tr>
<th>Categories</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>V bad</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interval values</td>
<td>[0.00, 0.15)</td>
<td>[0.15, 0.35)</td>
<td>[0.35, 0.60)</td>
<td>[0.60, 0.80)</td>
<td>[0.80, 1.00)</td>
<td>$\geq 1$</td>
</tr>
</tbody>
</table>

### Table 5 | Calculated results of IWQEMU method

<table>
<thead>
<tr>
<th>Years</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>Results</td>
<td>$(0.615, \bar{x}, 0.910)$</td>
<td>$(0.531, \bar{x}, 0.813)$</td>
<td>$(0.543, \bar{x}, 0.825)$</td>
<td>$(0.557, \bar{x}, 0.813)$</td>
<td>$(0.582, \bar{x}, 0.835)$</td>
</tr>
<tr>
<td>$\bar{x} \sim N(0.763, 0.115)$</td>
<td>$\bar{x} \sim N(0.672, 0.095)$</td>
<td>$\bar{x} \sim N(0.684, 0.103)$</td>
<td>$\bar{x} \sim N(0.685, 0.087)$</td>
<td>$\bar{x} \sim N(0.709, 0.107)$</td>
<td></td>
</tr>
</tbody>
</table>

### Table 6 | The integrated evaluation results of the water quality of Lake Chaohu from 2009 to 2013

<table>
<thead>
<tr>
<th>Categories</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>III</td>
<td>(0.80, 0.17)</td>
<td>(0.80, 0.29)</td>
<td>(0.80, 0.22)</td>
<td>(0.80, 0.21)</td>
<td>(0.80, 0.19)</td>
</tr>
<tr>
<td>IV</td>
<td>(0.80, 0.09)</td>
<td>(0.80, 0.23)</td>
<td>(0.80, 0.17)</td>
<td>(0.90, 0.16)</td>
<td>(0.90, 0.11)</td>
</tr>
<tr>
<td>V</td>
<td>(0.80, 0.93)</td>
<td>(0.80, 0.81)</td>
<td>(0.80, 0.85)</td>
<td>(0.80, 0.86)</td>
<td>(0.80, 0.89)</td>
</tr>
<tr>
<td>V bad</td>
<td>(0.90, 0.87)</td>
<td>(0.90, 0.79)</td>
<td>(0.90, 0.83)</td>
<td>(0.90, 0.83)</td>
<td>(0.90, 0.85)</td>
</tr>
<tr>
<td></td>
<td>(0.80, 0.21)</td>
<td>(0.80, 0.16)</td>
<td>(0.80, 0.19)</td>
<td>(0.80, 0.19)</td>
<td>(0.80, 0.20)</td>
</tr>
<tr>
<td></td>
<td>(0.90, 0.13)</td>
<td>(0.90, 0.11)</td>
<td>(0.90, 0.14)</td>
<td>(0.90, 0.15)</td>
<td>(0.90, 0.12)</td>
</tr>
<tr>
<td></td>
<td>(0.80, 0.11)</td>
<td>(0.80, 0.02)</td>
<td>(0.80, 0.05)</td>
<td>(0.80, 0.06)</td>
<td>(0.80, 0.09)</td>
</tr>
<tr>
<td></td>
<td>(0.90, 0.05)</td>
<td>(0.90, 0.00)</td>
<td>(0.90, 0.00)</td>
<td>(0.90, 0.01)</td>
<td>(0.90, 0.05)</td>
</tr>
</tbody>
</table>
the water quality slowly deteriorated from 2011 to 2013. The water quality improvement in 2010 could have been because of the strict water environment regulations implemented by the local government that year. Based on the data in Table 6, the observed change can be seen in Figure 6.

According to China’s current water quality evaluation method, i.e. the max-index method (Table 1), the expected value of the monitoring data in Table 3 was taken as the annual average value of the pollutants. The upper limit of the corresponding pollutant concentration for category V water (Table 2) was taken as the benchmark and the annual average was standardized, from which the max-index value was obtained. The results are shown in Table 7.

The last row of Table 7 shows that the values of the max-index method in Lake Chaohu from 2009 to 2012 were all in the interval \([0.80, 1]\), and more than 1 in 2013. Referring to the integrated evaluation set listed in Table 4, the water quality in Lake Chaohu was category V from 2009 to 2012, and reached category V bad in 2013. The annual change in pollutant concentrations from 2009 to 2013 are shown in Figure 7.

The results in Table 7 and Figure 7 also reveal that the water pollution causes in Lake Chaohu were total suspended solids (TSS) in 2009 and 2012, Phy in 2010 and 2013, and Tur in 2011. However, since 2000, the actual cause of water pollution in Lake Chaohu has been mainly eutrophication.

When comparing the results listed in Tables 6 and 7, it can be seen that the results of the IWQEMU are lower than that of the max-index method. The comparison of the results between the proposed method and max-index method is intuitively drawn in Figure 8.

The principle of the max-index method is that the maximum value of the various pollutant concentrations is used as the evaluation criteria. Therefore, compared to other methods, the results of this evaluation method are always the highest.

However, there are two real-world phenomena that should not be ignored in water quality evaluations. First, because of large amounts of sediment, the total suspended solids level downstream in the Yellow River are unusually high (generally more than 1,000 mg/L, the highest up to 14,000 mg/L) from June to October every year, yet water quality incidents rarely occur downstream in the Yellow River in this period. Secondly, if the temperature, sunlight duration, algal community and other water environmental conditions are at a certain level, water quality events such
as algal bloom may occur, such as the cyanobacterial bloom in Lake Chaohu in June 2012. If using the max-index evaluation method to evaluate water quality, the Yellow River may be judged as being seriously polluted while the Lake Chaohu is relatively optimistic. However, in reality, opposite judgments should be made. The water quality evaluation model proposed in our research objectively evaluates the water quality according to its internal interaction mechanisms, thereby avoiding the emergence of the two extremes described above.

Five key elements can be seen in Figures 5–8. (1) If the probability level is under 0.8 or 0.9, the possibility of the water quality being a category IV are both higher than 0.8. It demonstrates the stability of the IWQEMU. (2) Employing fuzzy random variables to describe the water quality using both probability and possibility measures may be closer to the actual water quality situation because of the wide uncertainty range in the water system. (3) From 2009 to 2013, Lake Chaohu water quality was category IV. Although the water quality in Lake Chaohu improved marginally in 2010, it remained a category IV, and from 2011 began to slowly deteriorate again. These results highlighted that the local government pollution control policies implemented in 2010 were still insufficient. (4) For public health, the use of Lake Chaohu water for drinking, grazing, or swimming purposes should be prohibited. (5) When the probability and possibility level reach 1, the IWQEMU in this research degenerates into an exact evaluation model. It shows that exact model is a special case of IWQEMU.

CONCLUSIONS

By considering the previously overlooked interaction between pollutants and the water environment when evaluating water quality, the IWQEMU was established in this paper, in which catastrophe theory was used to treat the
ambiguous internal mechanism of the interaction between the pollutants and the water environment, and fuzzy random variables were employed to describe the uncertainty in the water system. As the parameters were fuzzy random numbers, a new algorithm was designed to solve the model. The model was applied to an actual case at Lake Chaohu, China and some interesting results were obtained from the analysis.

A detailed comparison for the IWQEMU and the max-index method was given and the relationship between the IWQEMU and the weighted sum model was also clarified. Because of the uncertainty existing in actual water bodies, the IWQEMU used probability and possibility levels to judge the quality of an actual water body, the results of which were closer to reality.

In the case study, an annual evaluation of Lake Chaohu’s water quality was conducted using the IWQEMU. However, timely water quality evaluation, either weekly or monthly, is needed for actual water quality management. Using the monitoring data from seven water quality monitoring stations in Lake Chaohu, in future research, we hope to conduct weekly and monthly water quality evaluations using the IWQEMU to study its performance in practice and evaluate its usefulness for water management so as to improve the model.

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