Introducing modified version of penguins search optimization algorithm (PeSOA) and its application in optimal operation of reservoir systems
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ABSTRACT
In the current study, modified version of the penguins search optimization algorithm (PeSOA) was introduced, and its usage was assessed in the water resources field. In the modified version (MPeSOA), the Gaussian exploration was added to the algorithm. The MPeSOA performance was evaluated in optimal operation of a hypothetical four-reservoir system and Karun-4 reservoir as a real world problem. Also, genetic algorithm (GA) was used as a criterion for evaluating the performance of PeSOA and MPeSOA. The results revealed that in a four-reservoir system problem, the PeSOA performance was much weaker than the GA; but on the other hand, the MPeSOA had better performance than the GA. In the mentioned problem, PeSOA, GA, and MPeSOA reached 78.43, 97.46, and 98.30% of the global optimum, respectively. In the operation of Karun-4 reservoir, although PeSOA performance had less difference with the two other algorithms than four-reservoir problem, its performance was not acceptable. The average values of objective function in this case were equal to 26.49, 23.84, and 21.48 for PeSOA, GA, and MPeSOA, respectively. According to the results obtained in the operation of Karun-4 reservoir, the algorithms including MPeSOA, GA, and PeSOA were situated in ranks one to three in terms of efficiency, respectively.

Key words | Karun-4 reservoir, modified penguins search optimization algorithm, operation of reservoir, optimization

NOTATION

A(t,n) surface area of the nth reservoir at the beginning of period t
COA cuckoo optimization algorithm
DE differential evolution
DP dynamic programming during period t
e efficiency of power plant
EAs evolutionary algorithms
g acceleration of gravity
g1 penalty function related to carry over
g2 penalty function related to minimum storage
g3 penalty function related to maximum storage
GA genetic algorithm
H average of water level behind the dam at the beginning and end of the period t
ICA imperialist competitive algorithm
K1 penalty constant related to g1
K2 penalty constant related to g2
K3 penalty constant related to g3
M a N×N connection matrix among reservoirs
MPeSOA modified penguins search optimization algorithm
Mult 106 times the number of seconds in period t
MW megawatts
N total number of the reservoirs

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INTRODUCTION

Iran and other countries located in the Middle East encounter serious water shortage problems. Moreover, the population of the mentioned countries is growing, and there are no other alternative water resources for the population, and as a result, food security faces serious danger. Iran’s population growth shows an increasing rate, and actually, demand for water has improved; consequently, renewable water resources show decreasing rate. In addition, the results of climate change studies in the majority of Iran’s zones indicated the decreasing precipitation trend and increasing temperature trend (Tabari & Talaee 2011a, 2011b). In such a condition, it is necessary to use appropriate policies for optimal management of water resources. Among the policies, developing systems for drought monitoring and forecasting, management of reservoirs operation and, finally, increasing public awareness regarding water crisis issue can be taken into consideration.

Given that exploitation of surface water resources in Iran, similar to many areas of the world, is mostly performed by operation of dam reservoirs, extracting the optimal policies related to reservoir operation can play a significant role in the operation management (Moravej & Hosseini-Moghari 2016). For this reason, optimizing the operation of reservoir has long been one of the favorite research areas for researchers. Reservoirs operation has been carried out by means of different optimization methods, the following methods can be mentioned among them: linear programming (Yeh 1985), nonlinear programming (Arunkumar & Jothiprakash 2012), dynamic programming (DP) (Yakowitz 1982; Hall et al. 1968), and stochastic dynamic programming (SDP) (Stedinger et al. 1984; Karamouz & Vasiladiadis 1992). Each of the mentioned methods has its own limitations that reduce the popularity of their usage. Linear programming is merely used for linear problems, therefore, it cannot be used in a wide range of real-world problems. DP and SDP have difficulty with the curse of dimensionality, such that, as the problem dimensions increase, the computations will increase exponentially. Moreover, in order to use the DP and SDP, the considered problem must be discretized (Rani & Moreira 2013). Nonlinear programming has superiority over the mentioned methods, although it is possible to be dysfunctional regarding complex problems in local optimum; in other words, it is possible that nonlinear programming does not achieve the global optimum (Bozorg-Haddad et al. 2008).

The above methods are classified into the category of classical optimization methods. There is another category of optimization methods, namely evolutionary algorithms (EAs). Appearing EAs are considered a revolution in the realm of optimization issues. Given that these algorithms had no limitations that classical optimization methods do, they were highly regarded for researchers. Some of the fields related to water sciences in which EAs have been
used include determining optimal crop pattern (Otieno & Adeyemo 2010; Noory et al. 2011; Lalehzari et al. 2015), optimizing the water distribution network (Soltanjalili et al. 2013; Jabbary et al. 2016; Gupta et al. 2017), project management (Zhang et al. 2005; Elbeltagi et al. 2007; Abdallah et al. 2009), and, finally, the optimal operation of the reservoir (He et al. 2014; Zhang et al. 2014; Akbari-Alashbtii et al. 2015; Nikoo et al. 2016).

Based on the theory of no free lunch, none of the EAs can be the best algorithm for all problems, unless the algorithms that are specifically developed for a specific problem (Wolpert & Macready 1997). This fact has caused the performance of new algorithms to be examined by researchers in different areas. Concerning the reservoir operation, as an example, Manatwy et al. (2005) used simulated annealing (SA) algorithm for optimal operation of the reservoir; eventually, the results indicated SA efficiency in optimization. Karaboga et al. (2008) extracted the optimal performance of the reservoir overflow discharge using tabu search (TS) algorithm and applying fuzzy logic. Eum et al. (2012) developed a reservoir integrated management system using differential evolution (DE) optimization algorithm and by extracting optimal rules of reservoirs operation under climate change condition. Hosseini-Moghari et al. (2015) examined two algorithms such as imperialist competitive algorithm (ICA) and cuckoo optimization algorithm (COA). The last mentioned researchers compared their results with genetic algorithm (GA) and stated that both methods had better performance than GA, and generally, COA had the best performance. They asserted that COA, ICA, and GA reached 99.88, 99.50, and 98.09% of the global optimum, respectively. Bozorg-Haddad et al. (2014b) examined the performance of water cycle algorithm (WCA) in operating the reservoir; the results of the study revealed the advantage of WCA over GA, in addition, WCA has approached close to the global optimum with high accuracy. Ehteram et al. (2017) compared the performance of Shark algorithm in operating the reservoir with particle swarm optimization (PSO) and GA, and solved several reservoir operation problems; finally, they declared that Shark algorithm had the best performance in general.

The majority of researchers believe that water resources for human consumption will be confronted with serious constraints in the future. In such a condition, better and more effective policies on water resources utilization can ease the shortages. In the current study, we attempt to evaluate the abilities of a new evolutionary algorithm, namely penguins search optimization algorithm (PeSOA) for the first time in the water resources management field. The objective of similar studies on different algorithms performed by other researchers is also to find the best algorithm to apply in water resources issues. The mentioned issue is vital because even a small percent of improvements in the optimization of operation process associated with large water resources systems can play a significant role in reducing the vulnerabilities. PeSOA, for the first time, was introduced by Gheraibia & Moussaoui (2013), and has been merely assessed in some benchmark problems and, naturally, its performance has not been evaluated in the real world issues. In the version introduced by Gheraibia & Moussaoui (2013), the exploration part was not defined for the algorithm, so this fact leads to decrease in algorithm performance in complex problems. Therefore, in the present study, in addition to examining the initial version of PeSOA, by adding exploration operator to the algorithm, its modified version (MPeSOA) has also been introduced. Since more comprehensive evaluation of the algorithm requires a real example, the performance of this algorithm in a real reservoir operation problem was examined in addition to benchmark functions. Subsequently, process of study has been described.

**METHODOLOGY AND CASE STUDY**

**Penguins search optimization algorithm**

This algorithm has inspired the collective hunting behavior of penguins to search for and find food. Penguins usually live in groups and go hunting in groups as well. As penguins eat fish, and in order to harvest the food, they need to swim, in addition, they do not have enough breath for long-term swimming. In the hunting process in penguins’ society, each penguin enters into a hole created for the ice for hunting fish, after searching the region, comes back to the ice surface; then, penguins share the observed information and whole group moves toward the area in which there is adequate fish for hunting. It is possible that the whole
group is divided into several categories, and each category moves toward the areas which are rich in terms of fish. If a group does not feel satisfied in terms of fish in a given area, the group will move to another area. Eventually, the best place for hunting is where the penguins are able to find adequate fish to the fullest. With inspiration from this natural process, Gheraibia & Moussaoui (2015) developed an optimization algorithm entitled penguins search optimization algorithm. The mentioned algorithm starts working via a set of the initial random solutions, which are the penguins’ population groups. In the next step, the amount of objective function related to each member of the population is calculated. Optimal value of the objective function actually indicates the location where sufficient food, i.e. considerable fish resources, exist; after recognizing this place, other solutions (penguins) will move toward the best other identified places. This movement is mathematically expressed as follows:

\[
Sol_{new}^{i} = Sol_{old}^{i} + (Sol_{best} - Sol_{old}^{i}) \times \text{rand}
\]  

(1)

where \(Sol_{new}^{i}\) is the \(i^{th}\) modified solution, \(Sol_{old}^{i}\) is the \(i^{th}\) solution before modifying, \(Sol_{best}\) is the best identified solution, and \(\text{rand}\) is a random number between zero and one that helps more places to be searched (creates responses diversity). The algorithm’s stages can be stated as follows:

1. Generating the random initial population.
2. Computing the objective function of each member from the population (solution).
3. Sorting the solutions based on the objective function.
4. Selecting the best response based on the best objective function.
5. Modifying other responses based on Equation (1).
6. Re-computing the objective function of new responses.
7. If stop conditions are not fulfilled, it will go back to step 3, otherwise it will go back to step 8.
8. Finishing the optimization process and reporting the best member of population as optimal solution of the problem.

**Modified penguins search algorithm**

As the initial version of the algorithm was explained, exploration operator has not been considered in this algorithm. This issue may be applied by the developers due to much more simplicity of the algorithm in addition to its suitable performance. Nevertheless, stochastic search of decision space in EAs has a great importance in the algorithm performance for complicated problems. Therefore, considering the real manner of penguin’s life in nature, it is possible that penguins find a better place for hunting when moving toward a given place, so suddenly and in a non-scheduled manner, they change their destination. The mentioned movement can be taken into account as an unplanned movement in the algorithm, and can be defined as exploration. In the current study, Gaussian exploration function was added to the algorithm based on Equation (2). In the next part, the results of initial version of the algorithm besides its modified version have been presented.

\[
Sol_{new}^{i} = Sol_{old}^{i} + \sigma \times \text{randn}
\]  

(2)

where \(Sol_{new}^{i}\) is the \(i^{th}\) modified decision variable related to \(i^{th}\) solution, \(Sol_{old}^{i}\) is the \(i^{th}\) decision variable before modification related to \(i^{th}\) solution, \(\sigma\) is Gaussian exploration parameter, and \(\text{randn}\) shows a number of normal distribution with mean equal to zero and standard deviation equal to one. Figure 1 shows a flowchart of the PeSOA and MPeSOA.

In Equation (2), change of some solutions is completely done at random. Consequently, in this equation, the best solution is not used like Equation (1). It should be mentioned that all algorithms were programmed in the MATLAB software.

**Benchmark functions**

In this part, in order to evaluate the performance of MPeSOA, two mathematical benchmark functions, which are defined as an index of assessing the ability of optimization techniques, were employed. These functions have a specific global optimum, and the purpose of their usage is to determine the ability of the algorithm in approaching the global optimum. Equations (3) and (4) show these functions.

\[
f(x) = \sum_{i=1}^{n} x_{i}^{2} - 5.12 \leq x_{i} \leq 5.12
\]  

(3)

\[
h(x, y) = 100(\sqrt{|y - 0.01x^{2}| + 0.01|x + 10|} - 15 \leq x \leq -5) - 3 \leq y \leq 3
\]  

(4)
where \( f() \) = called Sphere function, and \( h() \) = Bukin N.6 function; these two functions have unique features. Sphere function can be considered in high dimensions; but local optimum does not exist in this function. Nevertheless, Bukin N.6 function is a two-dimension function, which has plenty of local optimum (Bozorg-Haddad et al. 2018). In the present study, Sphere function is considered in 50 dimensions. The minimum value of the function is equal to zero, and is located in the point \( x_i = 0 \). The minimum amount of Bukin N.6 function is also zero that is situated in the point \((x,y) = (-10,1)\).

**Reservoir operation**

The main dominant equation on reservoir operation problem is continuity equation (Equation (5)). The equation determines the available water in the next time step based on the available volume of water in the reservoir, the amount of inputs and outputs. This equation is defined as follows (Garousi-Nejad et al. 2016):

\[
S_{(t+1,n)} = S_{(t,n)} + Q_{(t,n)} + M_{N \times N} \times R_{(t,n)} + M \times SP_{(t,n)} \quad \text{for} \quad t = 1, \ldots, T
\]

In this equation, \( S_{(t,n)} \), \( S_{(t+1,n)} \), \( Q_{(t,n)} \), \( R_{(t,n)} \), and \( SP_{(t,n)} \) = storage of the \( n^{th} \) reservoir (million m\(^3\)) at the beginning of period \( t \), and storage of the \( n^{th} \) reservoir (million m\(^3\)) at the end of period \( t \), input inflow into the \( n^{th} \) reservoir (million m\(^3\)) during period \( t \), release from the \( n^{th} \) reservoir (million m\(^3\)) during period \( t \), and overflow volume (million m\(^3\)) from the \( n^{th} \) reservoir during period \( t \), respectively. \( A_{(t,n)} = \) the surface area of the \( n^{th} \) reservoir (km\(^2\)) at the beginning of period \( t \), \( NEV_{(t,n)} = \) net evaporation from the \( n^{th} \) reservoir (mm) during period \( t \), \( M + a N^a N \) connection matrix (\( N \) is the total number of reservoirs) related to the reservoir system (Bozorg-Haddad et al. 2011), \( t = \) operation time step, and \( T = \) operation period length. The amount of overflow from the reservoir is calculated by the following equation (Bozorg-Haddad et al. 2017):

\[
\begin{align*}
&\text{if } S_{(t+1)} > S_{\text{max}} \rightarrow SP_{(t)} = S_{(t+1)} - S_{\text{max}} \quad t = 1, \ldots, T \\
&\text{if } S_{(t+1)} \leq S_{\text{max}} \rightarrow SP_{(t)} = 0 \quad t = 1, \ldots, T
\end{align*}
\]

where in this study, \( S_{\text{max}} = \) the maximum reservoir storage. There are constant constraints in the reservoir operation problem, regardless of the objective of operation, which determines the limits of release from reservoir and reservoir storage. The constraints are defined as follows (Solgi et al. 2017):

\[
R_{(0)}^{\text{min}} \leq R_{(t)} \leq R_{(0)}^{\text{max}} \quad \text{for} \quad t = 1, \ldots, T
\]
where \( R_{(t)}^{\min} \) and \( R_{(t)}^{\max} \) is the minimum and maximum permissible release from the reservoir during period \( t \), respectively. In addition, \( S_{(t)}^{\min} \) and \( S_{(t)}^{\max} \) is the minimum and maximum reservoir storage during period \( t \), respectively.

### Operation of four-reservoir system (a benchmark problem of reservoir operation)

Operation of four-reservoir system is a hypothetical operation problem, which was defined by Chow & Cortes-Rivera (1974). The mentioned problem is a complex linear operation problem with numerous constraints, which is used as a benchmark in the realm of optimal operation of reservoir (Bozorg-Haddad et al. 2011). Matrix \( M \) (Equation (5)) displays the method of reservoir connection (Bozorg-Haddad et al. 2011).

\[
M = \begin{bmatrix}
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 1 & -1 & 0 \\
1 & 0 & 1 & -1
\end{bmatrix}
\]

Operation of these reservoirs is based on a 12-month period. Therefore, the problem has 48 decision variables (each reservoir has 12 variables), which are reservoir outputs. The objective function related to this problem is defined as follows:

\[
\text{Max. } Z = \sum_{n=1}^{N} \sum_{t=1}^{T} \text{Benefit}_{(t)}^{n} + \text{R}_{(t)}^{n}
\]

where \( Z \) is the objective function (total benefit value), \( \text{Benefit}_{(t)}^{n} \) is the benefit related to the \( n \)th reservoir in period \( t \), \( N \) is the number of reservoirs, and \( \text{R}_{(t)}^{n} \) is the release of \( n \)th reservoir in the period \( t \). Furthermore, the following penalty functions is considered for the mentioned operation problem.

\[
\begin{align*}
\text{if } S_{(t+1)}^{n} < S_{T \text{arg.et}}^{n} \rightarrow g_1 &= \sum_{n=1}^{N} K_1 \left[ S_{(t+1)}^{n} - S_{T \text{arg.et}}^{n} \right]^2 \\
\text{otherwise } g_1 &= 0
\end{align*}
\]

In these equations, \( S_{(t+1)}^{n} \) is the storage of \( n \)th reservoir at the end of operation period, \( S_{T \text{arg.et}}^{n} \) is objective storage for the \( n \)th reservoir at the end of operation period, \( g_1 \) are, respectively, penalty function related to carry over, minimum storage, maximum storage while \( K_1 \) are, respectively, penalty constants, which are considered to be equal to 60, 40, and 40 (Bozorg-Haddad et al. 2015). Thus, final objective function will be as follows:

\[
\text{Max. } Z = \sum_{n=1}^{N} \sum_{t=1}^{T} \text{Benefit}_{(t)}^{n} + \text{R}_{(t)}^{n} - \sum_{i=1}^{3} g_i
\]

Required data for optimization of operation for this system such as inflow discharge to the reservoir, minimum and maximum storage of the reservoir, and relevant benefit amount are available in the study carried out by Chow & Cortes-Rivera (1974).

### A real world problem: optimal operation of Karun-4 reservoir

In this section, a specific case study was explained. The case study considered is Karun-4 reservoir, which has been constructed over Karun River as the biggest river of Iran, and with the aim of hydropower energy production. The mentioned dam is located in the southwest of Iran, at coordinates of 31° 35’ latitude and 50° 24’ longitude. Karun-4 dam is known as the greatest concrete dam of Iran and its maximum storage is equal to 2,190 million m³; moreove, installed plant power capacity (PPC) is 1,000 megawatts (MW). Operation of this reservoir was performed for a 15-year period during the years 2000 to 2015. Figure 2
The amount of power generated by a hydropower dam is calculated by the following equation (Bozorg-Haddad et al. 2015):

\[ P(t) = g.e \cdot \frac{R_p(t)}{PF} \cdot M_u(t) \cdot \frac{(H(t) - T_w(t))}{1000} \]

where \( P(t) \) = power produced (MW) at period \( t \), \( g \) = acceleration of gravity (m/s²), \( e \) = efficiency of power plant, \( R_p(t) \) = the amount of water entering the turbine (million m³) during period \( t \), \( PF \) = plant functional coefficient, \( M_u(t) \) = 10⁶ times the number of seconds in period \( t \), \( H(t) \) = the average of water level behind the dam at the beginning and end of the period \( t \) (m), \( T_w(t) \) = reservoir tailwater level during period \( t \) (m). It should be mentioned that \( 0 < P(t) < PPC \), therefore, when the amount of water which enters the turbine is more than the maximum amount, a part of this will spill over the turbine. Consequently, in Equation (14), \( R_p(t) \) was used instead of \( R(t) \) with less than or equal to \( R(t) \) value.

\( A_0(t) \) in Equation (5) and \( H_0(t) \) in Equation (14) were calculated using area-storage equation \( A_0(t) = 2.08 \times 10^{-9}S_3(t) - 9.79 \times 10^{-6}S_2(t) - 2.42 \times 10^{-2}S_1(t) + 1.82 \) and Elevation-Storage equation \( H_0(t) = 1.74 \times 10^{-8}S_3(t) - 8.60 \times 10^{-5}S_2(t) + 17.71 \times 10^{-2}S_1(t) + 869.55 \) of Karun-4, respectively (Bozorg-Haddad et al. 2016). Objective function related to this problem is considered as minimizing the square of difference between the amount of energy produced and the maximum amount of energy generated, i.e. PPC by the following equation:

\[ \text{Min. } F = \sum_{t=1}^{T} \left( 1 - \frac{P}{PPC} \right)^2 + \sum_{i=1}^{2} g_i \]

where \( F \) = objective function. In this problem, it should be noted that since overflow from the reservoir has been considered, the reservoir storage never exceeds the maximum storage. Hence, the amount of \( g_3 \) will not be entered in this case. \( K_1 \) and \( K_2 \) constants for this case are 100 and 50, respectively.

It should be emphasized that considering the proper performance of GA in the realm of reservoir operation (see e.g. Cheng et al. 2008) in this study, GA was selected as the basis of comparison. Moreover, with respect to GA suitable performance in optimizing the reservoir operation of Karun-4 (see e.g. Ahmadi et al. 2014), for the problem, GA was merely used; in other words, global optimum amount has not been extracted for this problem.

### RESULTS

#### Results of benchmark functions

In this section, the results related to running of the PeSOA and MPeSOA for Sphere and Bukin6 functions were presented. In order to use a criterion for assessing the performance of the two algorithms, GA algorithm was also employed. The number of objective function evaluations for Sphere function and for all three algorithms was considered to be equal to 10,000, and equal to 9,000 for Bukin6. After determining optimal parameters of each algorithm, due to stochastic nature of the algorithms, only one run was not performed, and instead, five runs were performed. The results of five runs are presented in Table 1. According to the obtained results, all three algorithms demonstrated appropriate performance, but PeSOA and MPeSOA had somewhat better performance. Regarding Sphere function, no difference between ordinary version of PeSOA and its modified version has been observed, but in Bukin6 function, there has been more difference. This can be attributed to local optimum of Bukin6 function; given that PeSOA algorithm does not use exploration operator, it is possible that PeSOA would get stuck in the local optimum. In the best runs for each algorithm regarding Sphere function, GA, PeSOA, and MPeSOA algorithms achieved 0.006, 0.000, and 0.000, respectively, and were 0.023, 0.014, and 0.005, respectively, for Bukin6 function; this
issue indicates that performance of PeSOA and MPeSOA have had proper performance. Figure 3 shows algorithm convergence for the average performance of each algorithm in five runs; in this figure, with respect to small amount of objective function, vertical axis is considered logarithmically. Based on the figure, GA algorithm averagely showed better performance in five runs, but it was weaker than MPeSOA.

The results related to four-reservoir system

Operation of this system was performed in order to maximize the gained benefit from a four-reservoir system (Table 2); this problem was expressed and solved for the first time by Chow & Cortes-Rivera (1974). They solved the problem using LP method, and optimal solution equal to 308.26 was determined by them. Murray & Yakowitz (1979) also solved the mentioned problem applying differential dynamic programming (DDP) method; finally, they reported the number 308.23 as problem optimal solution. Bozorg-Haddad et al. (2015) considered nearly 50,000 objective function evaluations (the number of population for GA was equal to 200 and reiteration number was equal to 2,500)
for this problem, the best condition of 10 GA runs reached 500.47, and in the average of 10 runs, it reached 299.70. In the current study, the problem was solved using PeSOA and MPeSOA. To compare the obtained results with the GA results, the number of evaluations was considered to be equal to 50,000. It should be noted that $\sigma$ parameter related to the exploration in Equation (2) was considered as 0.03 of differences between upper and lower limits of decision variables. In addition, exploration rate was selected to be equal to 0.08.

The results gained from 10 runs of PeSOA and MPeSOA as well as the results of study carried out by Bozorg-Haddad et al. (2015) on the GA have been presented. According to the obtained results, PeSOA reflected much weaker performance than GA, and in the best condition, it reached 241.80 which has a major difference with the best performance of GA, which is 300.47. However, MPeSOA has reflected a more acceptable performance than GA, and has reached 303.04. The mentioned amount shows 0.85% improvement compared to GA. Bozorg-Haddad et al. (2015) reported 308.29 as global optimum for the problem and, hence, based on the best performance of each algorithm in this problem, GA, PeSOA, and MPeSOA have reached 97.46, 78.43, and 98.30% of the global optimum, respectively. Therefore, it can be concluded that the initial version of PeSOA do not reflect a proper performance, although the modified version has an acceptable performance. Another criterion used in the evaluation of EAs is coefficient of variation related to algorithm responses in different runs (without changing the parameters). Based on the mentioned criterion, the GA with coefficient of variation equal to 0.71 has shown better performance in comparison to PeSOA and MPeSOA. In this regard, PeSOA with coefficient of variation equal to 9.68 has demonstrated poor performance and it represents significant impacts of stochastic processes on the algorithm performance. On the other hand, there is no big difference between the coefficient of variations related to two algorithms including MPeSOA and GA; however, based on the coefficient of variation, MPeSOA has been worse than GA. A convergence diagram of PeSOA and MPeSOA for the best obtained results is shown in Figure 4.

This issue should be taken into account that Bozorg-Haddad et al. (2014a), Bozorg-Haddad et al. (2014b), Bozorg-Haddad et al. (2015), Asgari et al. (2015), Garousi-Nejad et al. (2016), Solgi et al. (2017), Bozorg-Haddad et al. (2016), and Bozorg-Haddad et al. (2017) also solved this problem using different algorithms such as bat algorithm, weighted clustering algorithm (WCA), biogeography-based optimization algorithm, weed optimization algorithm, modified firefly algorithm, the enhanced honey-bee mating optimization algorithm, state of matter search and symbiotic organisms search, and they reached the best condition objective function equal to 308.20, 306.92, 308.12, 308.15, 308.25, 308.24, 308.26, and 306.50, respectively. The mentioned fact indicates the better performance of aforementioned algorithms compared to MPeSOA for solving the problem. However, the much simpler structure of MPeSOA than the above-mentioned algorithms can turn this algorithm into a rival for other algorithms.

**Results of the real world problem: Karun-4 reservoir operation**

Reservoir operation was done in a 15-year period with monthly time steps. In the real world problem, three algorithm including GA, PeSOA, and MPeSOA were also applied. For all the three algorithms, the number of objective function evaluations was considered as 150,000. In the GA algorithm, crossover rate was set to 0.7 with two-point crossover function, and the exploration rate was considered as 0.05 with Gaussian function. In addition, tournament function was employed as selection operator. In the MPeSOA, the amount of $\sigma$ value is considered to be equal to 0.03 of upper and lower bound difference of variable according to

![Convergence diagram related to PeSOA and MPeSOA algorithm for four-reservoir system.](image-url)
trial and error; in this algorithm, exploration rate was also considered to be equal to 0.05. Then, according to optimal parameters, each algorithm was run 10 times; Table 3 shows the results of these runs. Based on the results presented, PeSOA algorithm at the best condition converged on 25.36 which when compared to the best condition of GA, which converged on 22.93, its performance was about 10% inferior to GA. The modified algorithm of PeSOA, i.e. MPeSOA has reached 19.48 at the best performance, which has been better than either the initial version of PeSOA or the GA. Also, in this section, based on standard deviation, the GA algorithm has been better than the other algorithms, and the worst algorithm has been associated with MPeSOA. However, the range of changes in the objective function of MPeSOA in most cases (except for the third and fourth runs) was better than the best performance of GA.

Figure 5 shows convergence diagram regarding the best performance of studied three algorithms in the 10 runs. According to the figure, MPeSOA has shown a sharp increasing trend at the beginning, which is located under PeSOA and GA diagrams and, subsequently, before evaluation number 10,000 was converged on the solution 19.48. Although PeSOA at first acted like GA, after the evaluation number 2,500 has nearly reached a premature convergence, its graph is located above the GA.

Figure 6 illustrates the number of months related to operation period length in which generated power was equal to PPC, more than half of PPC, less than half of PPC, or zero. According to the figure, among 180 operation months, PeSOA, GA, and MPeSOA have reached the maximum generated power in 22, 14, and only 4 months, respectively. Nevertheless, the number of times power has been generated between PPC and half PPC was 164 months based on the MPeSOA algorithm and also, 145 and 124 months based on GA and PeSOA, respectively. Moreover, MPeSOA has had power produced for all months, while its generated power for the other two algorithms has been zero in 3 months. The mentioned fact shows that MPeSOA has been able to prevent great failures and vulnerabilities of the system. This outcome can play an important role in water resource systems because managers are always trying to reduce the magnitude of system failures and change them to several smaller failures. Therefore, considering the critical conditions of water in Iran, applying optimizing methods such as MPeSOA algorithm can serve as a solution to deal with these shortages and lead to an optimal usage of the existing water resources. Utilizing such methods in dams such as Karun-4, which provides only hydropower demand, can be strongly useful because in dams that provide agricultural demand, due to existence of social issues, practical use of optimization methods is more difficult than others.

Limitations and suggestions for future research

Although the obtained results of the conducted study show more appropriate performance of MOPeSA than its initial

<table>
<thead>
<tr>
<th>Number of runs</th>
<th>GA</th>
<th>PeSOA</th>
<th>MPeSOA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23.12</td>
<td>28.12</td>
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<tr>
<td>SD</td>
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version, as well as GA, the studied problems are not enough to make a comprehensive comment on the efficiency of the algorithm. Hence, it is recommended that in other researches, the efficiency of the MOPeSA algorithm would be examined; and in addition to GA, other algorithms be used for comparing the results. Moreover, given that the current research just introduced the modified version of the algorithm, the considered real problem was a simple example of the real world, thus applying this algorithm in multi-reservoir systems for various purposes can confirm or deny the performance and ability of the algorithm in the complex problems. Finally, it is recommended that multi-objective version of the algorithm would be developed and be used and assessed in multi-objective problems.

CONCLUSION

The main aim of this study was to evaluate the ability of PeSOA as a new optimization algorithm in the field of water resources as well as to introduce modified version of the algorithm called MPeSOA. The purpose of introducing a modified version was the fact that in the initial version, the exploration operator was not considered, such that this feature of the PeSOA algorithm may cause it to be irresponsible and get stuck in the local optimum. To this end, after modifying the PeSOA algorithm, at first, two algorithms (PeSOA and MPeSOA) were assessed in finding optimal points of two mathematical benchmark functions. Then, in order to have a basis for comparing the results, the GA was used besides two algorithms. The results of this section indicated that the performance of both algorithms in finding optimal points of specified mathematical functions were proper, and both were better than GA. The results of Gherabia & Moussaoui’s (2013) study were in line with the current study on evaluating the initial version of PeSOA for finding optimal value of some mathematical benchmark functions and its comparison with GA results. Subsequently, performance of algorithms in optimizing a four-reservoir system operation, was examined. In this section, the performance of PeSOA was not acceptable, and merely reached 78.43% of optimum solution, whereas GA and MPeSOA reached 97.46 and 98.30% of optimum solution, respectively. Finally, the algorithms were employed in a real world problem; thus, for this purpose, operation of Karun-4 hydropower reservoir was tested for a 15-year period. Based on the obtained results of the mentioned problem, MPeSOA performance was more proper than the two other algorithms. The results revealed that coefficient of variation related to objective function values in 10 runs of MPeSOA was greater than two other algorithms; and this issue is surely a negative point for this algorithm. However, in summary, MPeSOA performance was entirely satisfactory. Eventually, with respect to suitable performance of MPeSOA, it is recommended that in other optimization issues of water resources field, MPeSOA algorithm would be utilized and evaluated.

Figure 6 | The number of months in which the generated power was equal to PPC, between PPC and half of PPC, between half of PPC, and zero or equal to zero.
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