Analytical solutions for predicting the maximum pressure drop after pump failure in long-distance water supply project

Cheng-yu Fan, Jian Zhang and Xiao-dong Yu

ABSTRACT

The water hammer caused by pump failure in a long-distance pressurized pipe system generally poses a severe threat to the safety of the whole system. The maximum pressure drop at the pump end of the discharge line is significant for the safety assessment of the pipelines. In this study, the characteristics of the pump-stopping water hammer and its propagation in the pipelines are analyzed. The formula for predicting the maximum pressure drop is deduced based on the Method of Characteristics and the complete characteristics of the pumps. The application conditions of the formula and the solution procedures are presented as well. In addition, two engineering cases are introduced and the results calculated by the formula are compared with those resulting from the numerical simulation, and the agreement is satisfactory. The formula presented in this study is of simple form, practical and of high precision, and can provide a theoretical basis for the water hammer protection scheme of a long-distance water supply project.

Key words | direct pump-stopping water hammer, long-distance pressurized pipe system, maximum pressure drop, pump-stopping water hammer

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INTRODUCTION

There are many reasons for a pump failure accident in a pressurized project, among which operational problems are the most common ones. The water hammer caused by pump failure is a great threat to the safety of the system. Particularly, for those systems without any water hammer protection measures, once the pumps are power-off, the pressures in the discharge lines will then fall down sharply. The sharp pressure drops may lead to liquid column separation and rejoining, which may lead to leakage or even destroy the pipes as well as the pumps (Kanakoudis 2004). Hence, many scientific research results have come out on the reliability of pressurized systems in hydraulic transients (Duan *et al.* 2010; Wu *et al.* 2010; Liang *et al.* 2012), the influence factors of pump-stopping water hammer (Bergant *et al.* 2008; Halkijevic *et al.* 2013; Wu *et al.* 2015; Pozos-Estrada *et al.* 2016; Wan & Li 2016), water hammer protection measures (Boulos *et al.* 2005; Zhang *et al.* 2008; Kim 2010; Sun *et al.* 2011) and so on. In China, as the distribution of water resources is severely uneven, a considerable number of long-distance water supply projects are being constructed to alleviate regional water shortage. Only a few of them are gravity flow systems. The rest are either pressurized systems or systems that are partially pressurized. Therefore, it is essential to calculate the water hammer pressure in a long-distance pressurized pipe system, not only for the safety assessment of the pipelines, but also for the water hammer protection scheme.

In the past, some graphic methods were commonly used for the theoretical study of pump-stopping water hammer. As these methods are considered to be empirical methods, a large amount of work is needed on the interpolation and comparison among many graphs, which usually leads to high

doi: 10.2166/ws.2018.014

calculation errors. As a result, they gradually become inapplicable (Ghidaoui *et al.* 2005; Ametani 2007). Nowadays, the main method to calculate water hammer pressure is the numerical simulation method, which is a computer-aided calculating method based on the Method of Characteristics (MOC) (Wylie *et al.* 1993; Chaudhry 2014). The method is of clear physical concept, of high simulation accuracy and suitable for complex systems. However, as professionals are indispensable in operating the calculation programs and the related software, it cannot be used conveniently, especially to compare alternatives in the feasibility study stage of a project. So, it is meaningful to deduce a theoretical formula for pump-stopping water hammer calculation instead of using computer simulation. The formula should be of high precision and what is more, of simple form, so as to speed up the schedule and reduce the cost of the project.

PUMP-STOPPING WATER HAMMER

Figure 1 presents the different processes of pump-stopping water hammer in different long-distance water supply projects. As the time for a wave to complete a round trip in the discharge line, $2L/a$, in which L is the length of the discharge line and a is the wave speed, is usually defined as *one interval*, each process can be divided into several stages according to the intervals. For those systems with low head and large flow, as shown in Figure 1(a), the pumps will be of positive runaway speeds in the first interval, whereas for the systems with high head and small flow, as shown in Figure 1(b), the pumps will be of reverse runaway speeds in the first interval. In both cases, the pumps keep in their respective runaway conditions from the time their rotational speeds reach the runaway speeds to the end of the first interval, during which all the pump parameters keep almost constant. By comparing the head change in the first interval and those in the others, it can be seen that for each system the maximum head drop occurs in the first interval.

The characteristics of the pump-stopping water hammer in Figure 1 are similar to those of the negative first-interval water hammer in the classical water hammer theory. Two main differences between them are that the latter is caused by valves and the water hammer pressure is directly related to the valve overflowing property at $2L/a$, while the former is caused by pumps and the water hammer pressure is directly influenced by the characteristics of pump speed at the end of the first interval.

The following are two key reasons why the pump-stopping water hammer in the long-distance water supply project has the characteristics mentioned in the previous paragraphs. For one thing, the conduits are long enough. The longer the conduits are, the longer the first interval is, which means more time is needed for the positive pressure wave reflected by the outlet sump to return back to the pump end of the discharge line. Thus, one interval could be longer than the time for the pump to change its rotational speed to the runaway speed. So the pump is able to reach the runaway condition in $2L/a$. For another, the rotational inertia of the pump is small enough. The rotational inertia of a pump is mainly composed of three parts: the inertia of the fluid in the pump, the inertia of the pump impeller and the rotational inertia of the motor, in which the rotational

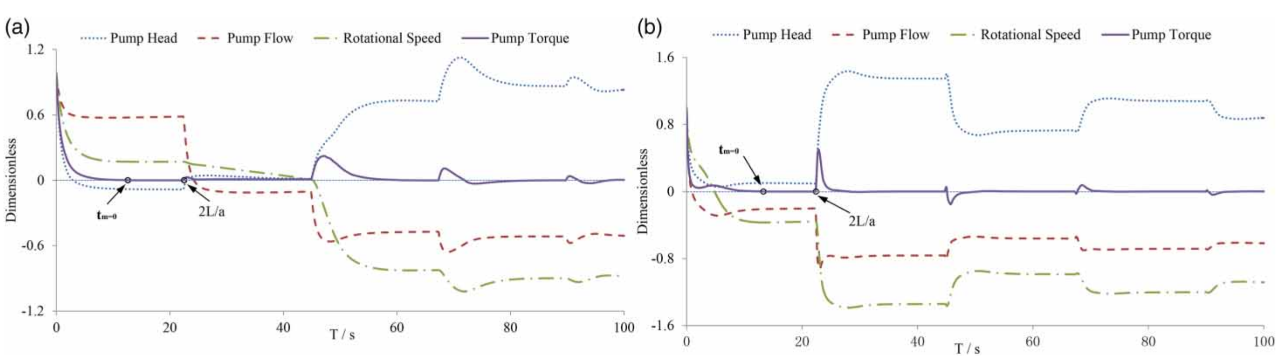

Figure 1 | Different processes of pump-stopping water hammer in different long-distance water supply projects: (a) low head and large flow, (b) high head and small flow.

inertia of the motor accounts for the largest proportion (more than 80%). However, with the development of the manufacturing technology, on the one hand, the number of magnetic poles is decreased gradually owing to the increase of the rated speed, which directly leads to a reduction of windings. On the other, as copper is gradually substituted by aluminum as the material for the rotor windings, the weight of a single winding is reduced (Olivares-Galván et al. 2010). Therefore, the rotational inertia of the motor is significantly reduced, which is the main cause of the reduction of the pump inertia. Although the cost of the pump is hence saved, in hydraulic transients the change rate of the rotational speed is increased as well. The increased change rate of the rotational speed leads to a sharp pressure drop in the discharge line, which is a great threat to the safety of the system. The time needed for the pump to change its rotational speed to the runaway speed is then shortened, and so the pump can reach the runaway condition in the first interval. In summary, for the majority of long-distance water supply projects, as the rotational inertia of the pump tends to be smaller and smaller, when a power failure accident happens, the pumps can be in the runaway condition before the reflected wave returns back to the pump end. The pump-stopping water hammer that happens in this condition can be named as the *direct pump-stopping water hammer*.

In this study, according to the characteristics of the direct pump-stopping water hammer and its propagation in the pipelines, the formula and its application conditions are deduced based on the MOC and the complete characteristics of the pumps for predicting the maximum pressure drop at the pump end of the discharge line. The formula can provide a theoretical basis for the water hammer protection scheme of a long-distance water supply project.

FORMULA DERIVATION

For the direct pump-stopping water hammer in Figure 1, the pump heads at the end of the first interval both approximate to the minimum. All the pump parameters keep almost constant from the time the rotational speeds reach the runaway speeds to the end of the first interval. So, it can be assumed that the pumps keep in the runaway conditions and the pressure drops keep the maximum values unchangeable during that time interval. That is, at the end of the first interval the pumps are in the runaway conditions and at the same time the pressure drops are maximum, which are the targets for solving in the following derivation process.

The layout of the water supply project is shown in Figure 2, in which B and P are separately referred to as the locations of the outlet sump and the pump end of the discharge line. The compatibility equation of the MOC can be integrated along the negative characteristic line from B to P as (Zhang et al. 2015):

$$H_P - H_B - \frac{a}{gA}(Q_{PT} - Q_B) + \frac{f}{2gDA^2}\int_B^P |Q|Q\mathrm{d}x = 0 \qquad (1)$$

in which H_P is the piezometric head at P at the end of the first interval, m, H_B is the water level of the outlet sump,

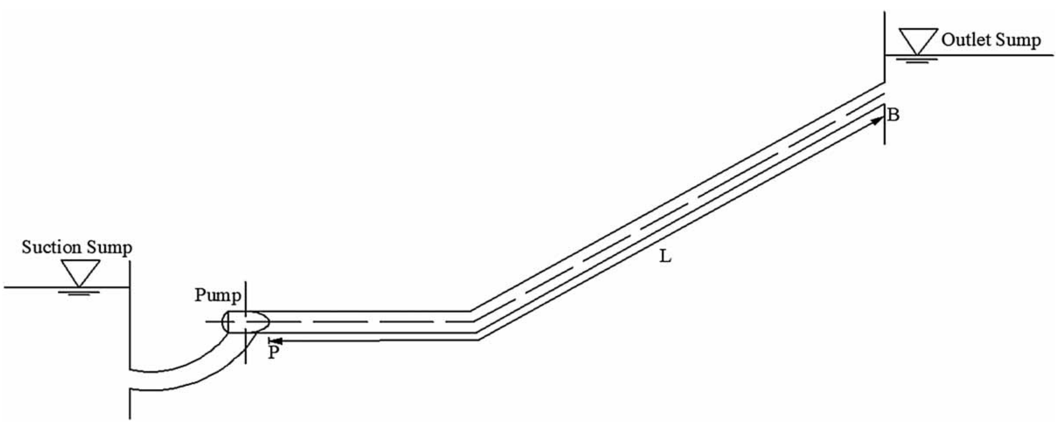

Figure 2 | Layout of the water supply system.

m, Q_{PT} is the total flow at P at the end of the first interval, m^3/s, Q_B is the total flow at B at L/a, m^3/s, L is the length of the discharge line, m, a is the wave speed, m/s, g is the acceleration due to gravity, m/s^2, A is the cross-sectional area of the pipe, m^2, f is the Darcy–Weisibach resistance coefficient, and D is the diameter of the pipe, m.

The following four items can easily be derived:

1. $\int_B^P |Q|Q\mathrm{d}x \approx -|Q_B|Q_{PT}L$ can be derived by expanding $\int_B^P |Q|Q\mathrm{d}x$ in Equation (1) according to the Taylor formula and then keeping the linear items.
2. If the number of the same parallel pumps in the system is i, then $Q_{PT} = iQ_P$.
3. When the system is in the initial steady state, then $H_{P_0} = H_B + \frac{fLQ_0^2}{2gDA^2}$.
4. At L/a, $Q_B = Q_0$.

By substituting $\int_B^P |Q|Q\mathrm{d}x \approx -|Q_B|Q_{PT}L$, $Q_{PT} = iQ_P$, $H_{P_0} = H_B + \frac{fLQ_0^2}{2gDA^2}$ and $Q_B = Q_0$ into Equation (1), the resulting equation becomes:

$$\Delta H_P = H_P - H_{P_0} = \left(\frac{a}{gA} + \frac{fLQ_0}{2gDA^2}\right)iQ_P - \left(\frac{aQ_0}{gA} + \frac{fLQ_0^2}{2gDA^2}\right) \tag{2}$$

in which ΔH_P is the pressure drop at P at the end of the first interval, m, H_{P_0} is the piezometric head at P in the initial steady state, m, i is the number of the same parallel pumps, Q_P is the flow of a single pump at the end of the first interval, m^3/s, and Q_0 is the total flow in the discharge line in the initial steady state, m^3/s.

As ΔH_P and Q_P are the only two unknown parameters in Equation (2), one more equation is needed to solve out ΔH_P, which is the target for solving according to the foregoing analysis. So the pump characteristics should be taken into consideration. The Suter characteristic curves of a centrifugal pump with specific speed of 89 are shown in Figure 3, in which $WH(x)$ and $WB(x)$ are separately the head and torque characteristics of the pump under different flow and rotational speed conditions (Yang et al. 2010):

$$WH(x) = \frac{h}{q^2 + n^2} \tag{3}$$

$$WB(x) = \frac{m}{q^2 + n^2} \tag{4}$$

$$x = \pi + \tan^{-1}\left(\frac{q}{n}\right) \tag{5}$$

in which the dimensionless head $h = \frac{H}{H_r}$, H is the pump head, m, H_r is the rated pump head, m, the dimensionless flow $q = \frac{Q_{pump}}{Q_{Pr}}$, Q_{pump} is the pump flow, m^3/s, Q_{Pr} is the rated pump flow, m^3/s, the dimensionless rotational speed

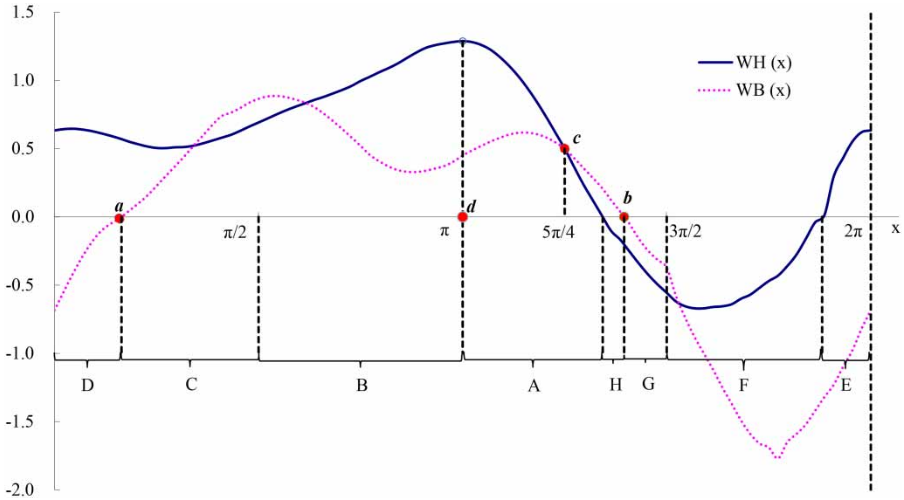

Figure 3 | Suter characteristic curves of a centrifugal pump with specific speed of 89.

$n = \frac{N}{N_r}$, N is the rotational speed of the pump, r/min, N_r is the rated rotational speed, r/min, the dimensionless torque $m = \frac{M}{M_r}$, M is the torque of the pump, kN·m, M_r is the rated torque, kN·m, and x is the abscissa of these curves.

The characteristics of some special operating points are shown in Table 1. Although all the summarized characteristics of these special operating points are of the centrifugal pump with specific speed of 89 in Figure 3, these characteristics are also generally applicable to any other pump. To any pump at point a on its characteristic curves, the pump is of reverse runaway speed and of reverse flow. As a little head loss is caused for the reverse flow going through the pump and overcoming frictional resistance, the pressure at the export of the pump should be a bit greater than at the inlet, which leads to the pump head being positive. Similarly, as the pump is of positive runaway speed and of positive flow at point b, the pressure at the export of the pump should be a bit smaller than at the inlet, which leads to the pump head being negative.

As according to the assumption, the pump is of runaway speed at the end of the first interval, according to Equation (4), the pump torque is zero at that time. So according to Table 1, the pump must be at point a or point b at the end of the first interval. Hence at the end of the first interval $x = x_0$ and $Q_{pump} = Q_P$. By substituting $h = \frac{H}{H_r}$, $H = H_0 + \Delta H$, $q = \frac{Q_{pump}}{Q_{Pr}}$, $Q_{pump} = Q_P$, Equation (5) and $x = x_0$ into Equation (3), we have:

$$WH(x_0) = \frac{(H_0 + \Delta H)/H_r}{[1 + (1/(\tan^2(x_0 - \pi)))](Q_P/Q_{Pr})^2} \tag{6}$$

in which H_0 is the pump head in the initial steady state, m, ΔH is the head change at the end of the first interval, m, and $x_0 = x_a$ or $x_0 = x_b$.

As the water supply system we considered is a simple system with a short suction line which can be neglected, it can be assumed that the piezometric head at the inlet of the pump keeps constant as H_U, which is the water level of the suction sump. Under this assumption, the head change is approximately equal to the pressure drop at P at the end of the first interval. That is, $\Delta H \approx \Delta H_P$. The flow of a single pump at the end of the first interval, Q_P, can then be solved out according to Equation (2) and Equation (6):

$$Q_P = \frac{iC_1 \pm \sqrt{(iC_1)^2 + 4C_2(H_0 - C_1Q_0)}}{2C_2} \tag{7}$$

in which $C_1 = \frac{fLQ_0}{2gDA^2} + \frac{a}{gA}$ and $C_2 = WH(x_0)\left[1 + \frac{1}{\tan^2(x_0 - \pi)}\right]\frac{H_r}{Q_{Pr}^2}$.

DISCUSSION

As mentioned above, the pump is in the runaway condition at the end of the first interval. So the pump flow at the end of the first interval, Q_P in Equation (7), should meet the pump characteristics at point a or point b:

1. If the pump is at point a at the end of the first interval, then $x_0 = x_a$. According to Table 1, $C_1 > 0$, $C_2 > 0$ and $Q_P < 0$. So if the solution of Equation (7) exists, $iC_1 - \sqrt{(iC_1)^2 + 4C_2(H_0 - C_1Q_0)} < 0$ and $H_0 - C_1Q_0 = H_B - H_U - \frac{a}{gA}Q_0 > 0$.
2. Similarly, if the pump is at point b at the end of the first interval, then $x_0 = x_b$. According to Table 1, $C_1 > 0$,

Table 1 | Characteristics of special operating points

Special operating points	x	WH(x)	WB(x)	h	m	q	n
a	$x = x_a = \pi + \tan^{-1}\left(\frac{q_a}{n_a}\right)$	+	0	+	0	–	–
b	$x = x_b = \pi + \tan^{-1}\left(\frac{q_b}{n_b}\right)$	–	0	–	0	+	+
c	$x = x_c = \pi + \tan^{-1}\left(\frac{q_c}{n_c}\right) = \frac{5\pi}{4}$	0.5	0.5	1	1	1	1
d	$x = x_\pi = \pi = \pi + \tan^{-1}\left(\frac{q_\pi}{n_\pi}\right)$	+	+	+	+	0	+

$C_2 < 0$ and $Q_P > 0$. If the solution of Equation (7) exists, $iC_1 - \sqrt{(iC_1)^2 + 4C_2(H_0 - C_1Q_0)} < 0$ as well and $H_0 - C_1Q_0 = H_B - H_U - \frac{a}{gA}Q_0 < 0$.

In summary, Equation (7) can be simplified as:

$$Q_P = \frac{iC_1 - \sqrt{(iC_1)^2 + 4C_2(H_0 - C_1Q_0)}}{2C_2} \tag{8}$$

in which if $H_0 - C_1Q_0 = H_B - H_U - (a/gA)Q_0 > 0$, the pump will be at point a at the end of the first interval, $x_0 = x_a$, while if $H_0 - C_1Q_0 = H_B - H_U - (a/gA)Q_0 < 0$, the pump will be at point b at the end of the first interval, $x_0 = x_b$.

By substituting Q_P that is calculated according to Equation (8) into Equation (2), we have:

$$\Delta H_P = iC_1Q_P - C_1Q_0 = C_1(iQ_P - Q_0) \tag{9}$$

The calculated pressure drop according to Equation (9), ΔH_P, is exactly the maximum pressure drop for the direct pump-stopping water hammer.

Generally, as for a long-distance water supply project, the wave speed is between 800 and 1,000 m/s and flow velocity is between 1 and 2 m/s. So according to Equation (8), only if the net head of the system $\Delta H_{UB} = H_B - H_U > 80 \sim 200$ m can the pump be at point a at the end of the first interval. As $H_0 = \Delta H_{UB} + (fLQ_0^2/2gDA^2)$, the pump head will be even larger. So for the direct pump-stopping water hammer, only in a few water supply systems with high head can the pumps be of reverse runaway speeds at the end of the first interval (at point a); in most cases the pumps will be of positive runaway speeds at the end of the first interval (at point b).

APPLICATION CONDITIONS

The pump-stopping water hammer that happens should be confirmed as the direct pump-stopping water hammer, otherwise the formula is not applicable. So Q_P and ΔH_P calculated by Equation (8) and Equation (9) should meet some application conditions. As the key characteristic of the direct pump-stopping water hammer is that the pumps can be of runaway speeds in the first interval, the time needed for the pumps turning to be in the runaway conditions should be no more than one interval, which can be determined as the application condition.

The rotation equation for a pump in the power-off accident is:

$$T_a(n_t - n_0) = -\overline{m} \cdot t \tag{10}$$

in which $T_a = (GD^2N_r^2/365P_r)$ is the inertia constant of the pump, s, GD^2 is the flywheel moment of the pump, $\mathrm{t \cdot m^2}$, P_r is the rated power of the motor, kW, n_t is the dimensionless rotational speed at t, n_0 is the dimensionless rotational speed in the initial steady state, $\overline{m}$ is the average dimensionless torque, $\overline{m} = (\int_0^t m\mathrm{d}t/t)$, and t is the time for the torque variation, s.

For the direct pump-stopping water hammer in Figure 1, the dimensionless pump torque in the first interval can be assumed to conform to the following equation:

$$m = \begin{cases} -\alpha\sqrt[\phi]{t} + m_0 & , 0 \le t \le t_{m=0} \\ 0 & , t_{m=0} \le t \le 2L/a \end{cases} \tag{11}$$

in which α is an undetermined coefficient, the index $\phi = (T_r/2T_a) + 1$, $T_r = (2L/a)$ is one interval, s, m_0 is the dimensionless torque in the initial steady state, and $t_{m=0}$ is the time needed for the pump changing its rotational speed to the runaway speed in the power-off accident, s.

According to Equation (11) the average dimensionless torque from the moment the pump is power-off to $t_{m=0}$ will be:

$$\overline{m} = \frac{\int_0^{t_{m=0}} (-\alpha\sqrt[\phi]{t} + m_0)\mathrm{d}t}{t_{m=0}} = \frac{\phi - 1}{\phi} m|_{t=t_{m=0}} + \frac{1}{\phi} m_0 \tag{12}$$

As the pump is in the runaway condition at $t_{m=0}$, then $m|_{t=t_{m=0}} = 0$. Equation (12) can be simplified as:

$$\overline{m} = \frac{1}{\phi} m_0 \tag{13}$$

As the pump is in the runaway condition at $t_{m=0}$, then $x = x_0$. So according to Equation (5), the dimensionless rotational speed at $t_{m=0}$ is:

$$n_{t_{m=0}} = \frac{q_{t_{m=0}}}{\tan(x_0 - \pi)} = \frac{Q_{t_{m=0}}}{Q_{Pr}\tan(x_0 - \pi)} \tag{14}$$

As the pump flow keeps constant from $t_{m=0}$ to the end of the first interval, $Q_{t_{m=0}} = Q_P$. By substituting $t = t_{m=0}$, $Q_{t_{m=0}} = Q_P$, Equation (13) and Equation (14) into Equation (10), we have:

$$t_{m=0} = \frac{T_a[n_0 - Q_P/Q_{Pr}\tan(x_0 - \pi)]}{(1/\phi)m_0} \quad (15)$$

in which Q_P can be calculated by Equation (8).

As the time needed for the pump turning to be in the runaway condition should be no more than one interval, according to Equation (15), the application condition for the direct pump-stopping water hammer should be:

$$t_{m=0} = \frac{T_a[n_0 - Q_P/Q_{Pr}\tan(x_0 - \pi)]}{(1/\phi)m_0} \le T_r \quad (16)$$

Equation (16) can be transformed as:

$$T_a \le \frac{T_r \times (1/\phi)m_0}{n_0 - Q_P/Q_{Pr}\tan(x_0 - \pi)} = \left[\frac{m_0}{\phi}\frac{1}{n_0 - (Q_P/Q_{Pr}\tan(x_0 - \pi))}\right]T_r = CT_r \quad (17)$$

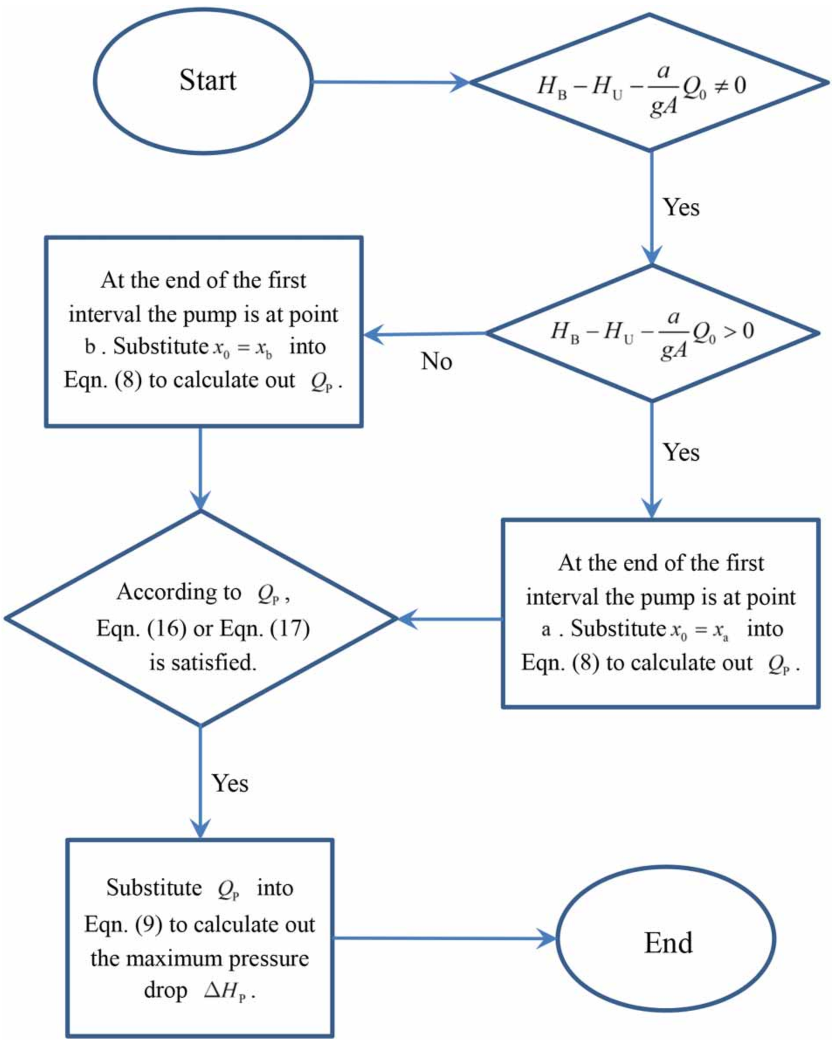

Figure 4 | Flow chart for the formula calculation.

In summary, Equation (16) and Equation (17) are the application conditions for the direct pump-stopping water hammer. The unique unknown parameter in them is Q_P. However, it can be calculated according to Equation (8). That is, the Q_P calculated by Equation (8) should meet Equation (16) or Equation (17).

SOLUTION PROCEDURES

1. The procedures to calculate the maximum pressure drop at the pump end of the discharge line with the formula for the direct pump-stopping water hammer are summarized, as shown in Figure 4. According to the assumptions, the solving target is the pressure drop for the pump with runaway speed at the end of the first interval. If $H_0 - C_1Q_0 = H_B - H_U - (a/gA)Q_0 > 0$, the pump will be at point a at the end of the first interval, $x_0 = x_a$, while if $H_0 - C_1Q_0 = H_B - H_U - (a/gA)Q_0 < 0$, the pump will be at point b at the end of the first interval, $x_0 = x_b$. By substituting the corresponding x_0 into Equation (8), Q_P can be calculated. As the pump-stopping water hammer that happens should be confirmed as the direct pump-stopping water hammer, Q_P should meet Equation (16) or Equation (17), which are the application conditions. At last, according to Q_P and Equation (9), the maximum pressure drop can then be derived.
2. As $(0, m_0)$ and $(t_{m=0}, 0)$ obviously are two solutions for Equation (11), then the undetermined coefficient $\alpha = \frac{m_0}{\sqrt[\theta]{t_{m=0}}}$. As $t_{m=0}$ can be calculated by Equation (15), α actually can then be derived.
3. If $H_B - H_U - (a/gA)Q_0 \to 0$, according to Equation (8) $Q_P \to 0$. As the pump is at a or b at the end of the first interval, $m = 0$, and as a result, according to Equation (14) $n \to 0$. What is more, according to Equation (3) $H \to 0$. In summary, when $H_B - H_U - (a/gA)Q_0 = 0$, the pump will be neither at point a nor at point b but 'at rest' from the time the pump reaches its runaway speed to the end of the first interval. During this time interval, all the pump parameters keep constant as zero. $\Delta H_P = -H_0$ and $Q_P = 0$ can directly be derived. In this condition, by substituting $Q_P = 0$ into Equation (16) or Equation (17), if these application conditions are satisfied, the pump-stopping water hammer that happens, as shown in Figure 5, can be regarded as a special kind of direct pump-stopping water hammer.

CASE STUDY AND VALIDATION

The parameters of the pressurized water supply systems are shown in Table 2. Separately, the maximum pressure drop at the pump end of the discharge line for each system is calculated, not only by the deduced formula, but also by the numerical simulation.

RESULTS

According to Figure 4, the formula calculation procedures and results are shown in Table 3.

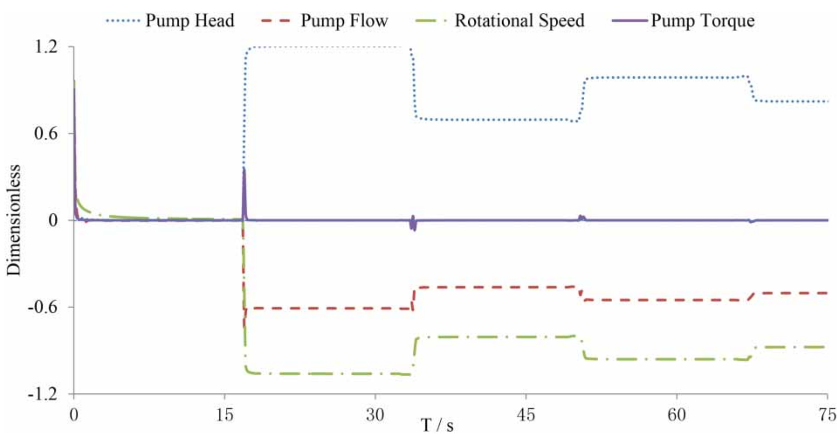

Figure 5 | Process of pump-stopping water hammer when $H_B - H_U - \frac{a}{gA}Q_0 = 0$.

Table 2 | Parameters of the systems

Case 1			
Water level of suction sump (m)	20	Quantity of pumps	2
Water level of outlet sump (m)	66	Rated head (m)	52
Pipe length (m)	8,380	Rated flow (m^3/s)	2.6
Pipe diameter (m)	2.2	Rated rotational speed (r/min)	600
Elevation of pipe center (m)	15	Rated motor power (kW)	1,800
Design flow (m^3/s)	5	Flywheel moment (kg·m^2)	2,600
Case 2			
Water level of suction sump (m)	20	Quantity of pumps	2
Water level of outlet sump (m)	175	Rated head (m)	160
Pipe length (m)	8,380	Rated flow (m^3/s)	1.04
Pipe diameter (m)	1.5	Rated rotational speed (r/min)	1,500
Elevation of pipe center (m)	15	Rated motor power (kW)	2,400
Design flow (m^3/s)	2	Flywheel moment (kg·m^2)	200

Based on the MOC, the numerical simulation program was written in FORTRAN language on our own. The calculation results are shown in Figure 6.

DISCUSSION

According to Tables 3 and 4 and Figure 6, the pump-stopping water hammer that happened in case 1 is similar to the direct pump-stopping water hammer shown in Figure 1(a), while the pump-stopping water hammer that happened in case 2 is similar to the direct pump-stopping water hammer shown in Figure 1(b). For both cases, the pressure drops at the end of the first interval are close to the pressure drops at $t_{m=0}$, both approximating to the maximum. So it is feasible to assume that the pumps keep in the runaway conditions and the pressure drops keep the maximum values unchangeable during that time interval.

By comparing the results calculated by the formula and those by the numerical simulation, we can see the following:

1. As a result of the assumptions and the simplifications in the formula derivation, for both cases the time of one interval in the numerical simulation calculation is a bit different from that in the formula calculation.
2. For case 1, the maximum pressure drop and the time needed for the pump to change its rotational speed to the runaway speed calculated by the formula are very

Table 3 | Formula calculation procedures and results

Procedures	Case 1	Case 2
1	$H_B - H_U - \frac{a}{gA}Q_0 = -88.08 < 0$ $x_0 = x_b$	$H_B - H_U - \frac{a}{gA}Q_0 = 39.63 > 0$ $x_0 = x_a$
2	$Q_P = 1.52m^3/s$	$Q_P = -0.21m^3/s$
3	$t_{m=0} = 8.33s < T_r = 16.76s$ Application conditions are satisfied	$t_{m=0} = 12.36s < T_r = 16.76s$ Application conditions are satisfied
4	$\Delta H_P = -54.23m$ $H = H_0 + \Delta H_P = -4.34m$ $N = \frac{Q_P}{Q_{Pr}\tan(x_b - \pi)}N_r = 102.54r/min$	$\Delta H_P = -145.42m$ $H = H_0 + \Delta H_P = 14.39m$ $N = \frac{Q_P}{Q_{Pr}\tan(x_a - \pi)}N_r = -532.7r/min$

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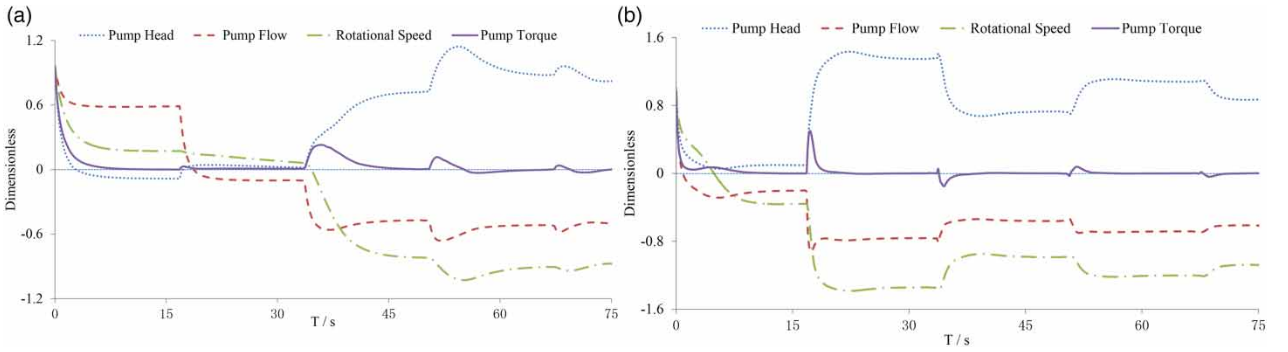

Figure 6 | Results calculated by numerical simulation: (a) case 1, (b) case 2.

Table 4 | Comparison of the calculation results

Cases	Compared parameters	Numerical simulation	Formula calculation	Errors (%)
Case 1	Maximum pressure drop (m)	54.44	54.23	−0.39
	Corresponding time (s)	16.65	8.33–16.76	/
	Pressure drop in runaway conditions (m)	54.41	54.23	−0.33
	$t_{m=0}$ (s)	8.40	8.33	−0.83
	Pressure drop at the end of the first interval (m)	54.44	54.23	−0.39
	One interval (s)	16.65	16.76	0.66
Case 2	Maximum pressure drop (m)	151.29	145.42	−3.88
	Corresponding time (s)	5.70	12.36–16.76	/
	Pressure drop in runaway conditions (m)	144.07	145.42	0.94
	$t_{m=0}$ (s)	12.60	12.36	−1.90
	Pressure drop at the end of the first interval (m)	144.54	145.42	0.61
	One interval (s)	16.65	16.76	0.66

close to those resulting from the numerical simulation. According to the formula calculation results, at the end of the first interval, the pump head is negative, the pump flow is positive, the rotational speed is positive and the pump is at point b, which are consistent with the numerical simulation results.

3. For case 2, the maximum pressure drop and the time needed for the pump to change its rotational speed to the runaway speed calculated by the formula are very close to those resulting from the numerical simulation. According to the formula calculation results, at the end of the first interval, the pump head is positive, the pump flow is negative, the rotational speed is negative and the pump is at point a, which are consistent with the numerical simulation results as well.

In summary, the assumptions and the simplifications in the formula derivation are reasonable. The formula is of high precision and of simple form, especially practical for the comparison among alternatives in the feasibility study stage of a project.

CONCLUSIONS

The theoretical formula and its application conditions for the direct pump-stopping water hammer are deduced based on the MOC and the complete characteristics of the pumps. The formula is of simple form. It can be used conveniently to derive the analytical solutions for predicting the maximum pressure drop at the pump end of the discharge line instead of using computer simulation. Although some assumptions and simplifications based on the characteristics of the direct pump-stopping water hammer and its propagation in the pipelines are put forward in the formula derivation, according to the case study, the maximum pressure drop and other results calculated by the formula are close to those resulting from the numerical simulation, which indicates that the formula is of high precision and the assumptions and the simplifications in the formula derivation are reasonable. Although the formula is of simple form and of high precision, it can only be used to calculate the direct pump-stopping water hammer in simple systems, so it is especially suitable for the comparison among alternatives in the feasibility study stage of a project so as to speed up the schedule and reduce the cost of the project. The formula can provide a theoretical basis to the water hammer protection scheme of a long-distance water supply project.

ACKNOWLEDGEMENT

This research is funded by the National Key R&D Program of China (2016YFC0401810), the Fundamental Research Funds for the Central Universities (No. 2016B10814; No. 2016B04914) and the Priority Academic Program Development of Jiangsu Higher Education Institutions.

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First received 22 March 2017; accepted in revised form 9 January 2018. Available online 24 January 2018