Optimal estimation of unconfined aquifer parameters in uncertain environment based on fuzzy transformation method

Atefeh Delnaz, Gholamreza Rakhshandehroo and Mohammad Reza Nikoo

ABSTRACT

In this paper, a fuzzy simulation–optimization model coupled with the genetic algorithm based on Boulton’s equation is presented to estimate transmissibility ($T$), storage coefficient ($S$), specific yield ($S_y$) and leakage factor ($D_l$) of an unconfined aquifer. This model is capable of minimizing the deviation between observed and calculated drawdowns of pumping test data. To assess the applicability of the model, its results are compared with the graphically obtained solutions from Boulton’s equation. To this end, real pumping test data obtained from an unconfined aquifer in Dayton, Ohio, are considered as the case problem to evaluate the efficacy of the model. In the fuzzy approach, pumping rate is considered as an uncertain variable. For evaluation of the model, several statistical error indices are utilized. Results show better matches for the model as evidenced by much smaller errors. As an example, mean absolute relative error for the proposed model and graphical Boulton’s solution is 2.52% and 4.98%, respectively. It is concluded that the model is accurate and may replace the graphical Boulton’s solution.

Key words | fuzzy transformation, genetic algorithm (GA), groundwater, optimal estimation, pumping test, unconfined aquifer parameters

ABBREVIATIONS/SYMBOLS

- $a_i^{(0)}$: Lower limit of fuzzy membership level $\mu_i$ for the $i^{th}$ uncertain parameter
- $b_i^{(0)}$: Upper limit of fuzzy membership level $\mu_i$ for the $i^{th}$ uncertain parameter
- ACO: Ant colony optimization
- AIC: Akaike Information Criterion
- CMM: Cubic meter per minute
- $D_l$: Reciprocal of ‘delay index’ $[\text{m}]$
- $D_t$: Leakage factor $[\text{m}^-1]$
- FTM: Fuzzy transformation method
- GA: Genetic algorithm
- $J_0$: Bessel function of zero order
- $K_r$: Horizontal hydraulic conductivity $[\text{m/s}]$
- $K_z$: Vertical hydraulic conductivity $[\text{m/s}]$
- MARE: Mean absolute relative error ($\%$)
- $n$: Total number of observed drawdowns
- NLP: Non-linear programming
- $Q$: Pumping rate ($\text{m}^3/\text{min}$)
- $r$: Distance from pumping well to observation well ($\text{m}$)
- RMRE: Root Mean Relative Error
- RMSE: Root Mean Squared Error
- $R^2$: Correlation coefficient
- $s_o^t$: Observed drawdown at time $t$ ($\text{m}$)
\( s'_t \) Calculated drawdown at time \( t \) (m)

\( S \) Storage coefficient [–]

\( SI \) Scatter Index

SUMT Sequential unconstrained minimization technique

\( S_y \) Specific yield [–]

\( t \) Time (min)

\( T \) Transmissibility \((m^2/\text{min})\)

\( u_a \) Used for early-time in Boulton equation

\( u_y \) Used for late-time in Boulton equation

\( Z \) Deviation of the observed drawdown from calculated drawdown [–]

\( \mu \) Level of membership

### INTRODUCTION

Groundwater is an important vital resource for agricultural, industrial, and domestic water demands. Storage and transmissibility of the aquifers are controlled by their hydrogeological parameters \((T, S, S_y, \text{leakage factor})\). Therefore, the optimal estimation of these parameters is required for optimal groundwater management (Balkhair 2002; Jha et al. 2004, 2006). Since the fast decline of groundwater level has caused serious concerns, knowing the aquifer parameters has become more important, because groundwater recovery requires use of methods such as artificial recharge to minimize the adverse effects of over-pumping (Essl et al. 2014; Sakthivel et al. 2015; Choi et al. 2018; Farid et al. 2018). A popular conventional method for determination of aquifer parameters is the utilization of an inverse approach, such as a pumping test. On the other hand, estimation of aquifer parameters by graphical methods based on analytical solutions is subjective and often inaccurate. In particular, the estimation of aquifer parameters might not be reliable when observed time-drawdown data do not fit type curves with an acceptable precision in a curve-matching procedure. Hence, optimization algorithms such as the genetic algorithm (GA) may be utilized to minimize the deviation between observed and calculated drawdowns, and obtain a better estimation of aquifer parameters. Although evolutionary algorithms, and particularly GA, have been shown to be efficient in finding near-optimal input parameters in the context of aquifer and groundwater except for a few studies (Samuel & Jha 2003; Jha et al. 2004; Abdel-Gawad & El-Hadi 2009; Rajesh et al. 2010; Lu et al. 2011; Bateni et al. 2015), little effort has been undertaken to propose a GA-based optimization algorithm in estimation of aquifer parameters. To accommodate the field heterogeneity of aquifers, a recently developed technique called hydraulic tomography has been proposed (Yeh & Liu 2000; Zha et al. 2017a, 2017b). This technique is based on multiple pumping test data, as explained in the supplementary material (available with the online version of this paper). In this study, a GA algorithm is utilized to get an optimal drawdown by determining corresponding aquifer parameters while assuming homogeneity and isotropy for the aquifer, and full penetration for the well.

Uncertainties in pumping rate \( (Q) \) are very important because they will significantly affect prediction of decision variables. A spectrum of methods has been developed for quantitative analysis of uncertainty which vary in complexity, but all have the capability to implement uncertainty at each stage of the analysis and show how it propagates throughout the analytical chain. These methods include: sensitivity analysis, Taylor series approximation (mathematical approximation), Monte Carlo sampling (simulation approach) and Bayesian statistical modeling. Quantitative analyses suffer a number of disadvantages, however. In particular, not all sources of uncertainty may be quantifiable with any degree of reliability, especially those related to value-based judgments. Quantitative measures may therefore bias the description of uncertainty towards the more computational components of an assessment. For example, in the Monte Carlo method, which is considered a probabilistic method, one needs a hard-to-find probabilistic distribution function for the pumping rate, and the run time may become too high.

Since no known probabilistic distribution function is available for the pumping rates, fuzzy set theory is an appropriate approach for the uncertainty analysis. Fuzzy set theory, pioneered by Zadeh (1965), is shown to efficiently handle uncertainties when inexact input parameters exist in a model. Fuzzy set theory is an extension of the classical set theory. A fuzzy set includes members whose membership varies between 0 and 1. In order to obtain acceptable solutions for real numerical problems, exact values of the parameters should be available. However, in real practice, exact values might not be readily available, causing the
models to be unavoidably accompanied by uncertainties. These uncertainties might originate from the lack of particular information, measurement errors, and/or problem simplifications. In most cases, such uncertainties lead to non-deterministic or uncertain boundary and initial conditions. It is known that conventional fuzzy arithmetic in nonlinear optimization models deforms the output fuzzy membership function, and is not applicable in practice (Hanss 2002). Advanced fuzzy arithmetic methods such as the fuzzy transformation method (FTM), however, consider all uncertainties and avoid inherent problems that exist in conventional fuzzy arithmetic (Hanss & Willner 2000; Hanss 2002). FTM has been frequently incorporated in the context of water resources including water and waste-load allocation models (Aramaki & Matsuo 1998; Sadegh et al. 2010; Sadegh & Kerachian 2011; Nikoo et al. 2013), management of groundwater resources, water conflict resolution using fuzzy-based negotiation techniques (Kerachian et al. 2010), and river water quality management (Nasiri et al. 2007; Singh et al. 2007). Apparently, fuzzy logic has not been used for aquifer parameter estimation purposes, yet.

In this paper, a new simulation–optimization model using GA and FTM is developed based on Boulton’s equation. The model estimates unconfined aquifer parameters, when the pumping rate is considered as an uncertain fuzzy parameter. The proposed model is tested by real pumping test data in an unconfined aquifer in Dayton, Ohio. Results are compared with Boulton’s graphical solutions and shown to estimate aquifer parameters more accurately.

**METHODOLOGY**

The flowchart for the proposed fuzzy simulation–optimization model based on FTM is shown in Figure 1. In the developed FTM-based model, the GA optimization model is coupled with Boulton’s analytical solution, which requires numerical integration for predicting unconfined aquifer hydrogeological parameters. The flowchart is further explained in supplementary material (available with the online version of this paper).

**Formulation of the simulation–optimization model**

With the main objective being optimal estimation of aquifer parameters in an uncertain environment using GA, the

![Figure 1](https://iiwaponline.com/ws/article-pdf/19/2/444/592312/ws019020444.pdf)

**Figure 1** | Flowchart for the proposed model to develop a fuzzy method to estimate aquifer parameters by GA coupled with FTM.
objective function and constraints may be written as (Walton 1970):

$$\text{Min } Z = \left( \sum_{t=1}^{n} \frac{|s_t' - s_c'|/s_0'}{n} \right)$$  \hspace{1cm} (1)

$$s_c = \frac{Q}{4\pi T} \int_{0}^{\infty} 2J_0 \left( \frac{r}{D_t} x \right) \left[ 1 - \left( \frac{1}{x^2+1} \right) \exp \left( -\frac{Ditx^2}{x^2+1} \right) - \frac{x^2}{x^2+1} \exp \left( -D_t N t (x^2 + 1) \right) \right] dx$$  \hspace{1cm} (2)

$$u_a = \frac{r^2 S}{4T}$$  \hspace{1cm} (3)

$$u_y = \frac{r^2 S_y}{4T}$$  \hspace{1cm} (4)

$$D_i = \frac{(r/D_t)^2}{u_y}$$  \hspace{1cm} (5)

$$D_t = \sqrt{\frac{T}{D_t S_y}}$$  \hspace{1cm} (6)

$$N = \frac{u_y}{u_a} + 1$$  \hspace{1cm} (7)

Subject to:

$$T_{\text{min}} \leq T \leq T_{\text{max}}$$  \hspace{1cm} (8)

$$S_{\text{min}} \leq S \leq S_{\text{max}}$$  \hspace{1cm} (9)

$$S_{y\text{-min}} \leq S_y \leq S_{y\text{-max}}$$  \hspace{1cm} (10)

$$\left( \frac{r}{D_i} \right)_{\text{min}} \leq \left( \frac{r}{D_t} \right) \leq \left( \frac{r}{D_t} \right)_{\text{max}}$$  \hspace{1cm} (11)

where $Z$ represents deviation of the observed drawdown, $s_t'$, at time $t$ (min) from $s_c'$ as calculated according to Boulton's equation. $n$ is the total number of observed drawdowns, $Q$ is the pumping rate (m$^3$/min), $T$ is the transmissibility (m$^2$/min), $J_0$ is the Bessel function of zero order, $r$ is the distance from pumping well to observation well (m), $S$ is the storage coefficient (Ss-h), $S_y$ is the specific yield, $D_t$ is the leakage factor, $D_i$ is reciprocal of the ‘delay index’, $u_a$ is for the early- and $u_y$ is for the late-times. $T$, $S$, $S_y$ and $r/D_t$ are decision variables of the model. Subscripts $\text{min}$ and $\text{max}$ for $T$, $S$, $S_y$ and $r/D_t$ are their corresponding lower and upper bounds, respectively.

Transmissibility of an unconfined aquifer is defined as:

$$T = K \times h$$  \hspace{1cm} (12)

where $h$ is the saturated thickness of the aquifer (Freeze & Cherry 1979), and the storage coefficient is defined here as:

$$S = S_s \times h$$  \hspace{1cm} (13)

where $S_s$ is the specific storage equal to $\gamma(\alpha + \beta)$.

Fuzzy transformation method

Fuzzy transformation is a suitable and applicable method for simulation of models that have nondeterministic parameters. This method has two forms, a general and a reduced one, used for simulating and analyzing non-monotonic and monotonic models, respectively. More details on FTM and its formulation are explained in the supplementary material.

RESULTS AND DISCUSSION

The proposed simulation-optimization model based on Boulton’s numerical solution and GA is developed in MATLAB R2010a®. To evaluate the model, time-drawdown data of a pumping test in an unconfined aquifer in Dayton, Ohio, conducted by Walton and presented in Todd & Mays (1980), are used. Then, the model is run, and calculated drawdowns are compared against both the corresponding observed drawdowns and Boulton’s graphical solution. To assess the model's accuracy, statistical error indices such as Mean Absolute Relative Error (MARE), Root Mean Squared Error (RMSE), bias and Scatter Index (SI), the correlation
coefficient \((R^2)\) and the Akaike Information Criterion are calculated (Table 1). Table 1 demonstrates that the proposed model outperforms Boulton’s graphical method as evidenced by 15% to 50% lower statistical error indices, MARE, RMSE, Bias, and SI, as well as 24% increase in Akaike index. Correlation coefficients are almost 1 (0.99982–0.99979). This means there are no differences between calculated drawdown and observed drawdown.

Once applicability and accuracy of the model are assessed, the nondeterministic parameter of the problem is investigated. In this paper, pumping rate is assumed to be an uncertain parameter; an assumption that is deemed reasonable considering its inherent imprecise field measurement. Based on field experiences, \(\pm 10\%\) change in the pumping rate was considered in this study. To investigate the monotonicity of the model, the GA-based model is run for different values of the pumping rate, and the corresponding objective function is determined. The objective function variation for different values of the pumping rate based on the obtained results and its interpretation is explained in the supplementary material (available with the online version of this paper).

For each \(\alpha\)-cut, FTM considers all possible combinations of the points (lower bound, upper bound, or extra values between them). Then, several scenarios are computed based on every possible pairwise combination of the points on each \(\alpha\)-cut for uncertain parameters for further evaluation. Once the fuzzy model based on FTM is developed and run, the fuzzy membership function is retransformed in different \(\alpha\)-cuts (Figure 2). As shown, the objective function varies from 2.5 to 3.5 at zero fuzzy degree, and as the fuzzy degree increases (i.e. uncertainty bound decreases), the range of objective function variation reduces. This feature allows the model to assume different levels of uncertainty when considering the pumping rate.

After retransformation of the output fuzzy intervals, the range of decision variables in different \(\alpha\)-cuts is determined (Table 2). As shown, the maximum variation in estimating \(T\), \(S_y\) and \(r/D_t\) for the unconfined aquifer is when the uncertainty level is 0.25. At this level, when the pumping rate \((Q)\) varies between \([3.78–4.39] \text{ m}^3/\text{min}\), \(T\), \(S_y\) and \(r/D_t\) vary between \([0.5595–0.7116]\) \text{ (m}^2/\text{min)}, \([0.0890–0.1517]\) and \([0.3877–0.4918]\), respectively.

Table 1 | Aquifer parameters computed by the graphical Boulton’s solution and the proposed model, and statistical error indices for drawdown calculations compared with observed ones

<table>
<thead>
<tr>
<th>Method</th>
<th>Aquifer parameters and statistical error indices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graphical Boulton’s solution</td>
<td>Proposed model</td>
</tr>
<tr>
<td>(T [m^2/\text{min}])</td>
<td>2.45</td>
</tr>
<tr>
<td>(S [Ss \cdot h])</td>
<td>0.0027</td>
</tr>
<tr>
<td>(S_y)</td>
<td>0.088</td>
</tr>
<tr>
<td>(r/D_t)</td>
<td>0.4</td>
</tr>
<tr>
<td>MARE (%)</td>
<td>4.98</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.0583</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.99982</td>
</tr>
<tr>
<td>Bias</td>
<td>0.0456</td>
</tr>
<tr>
<td>SI</td>
<td>0.0264</td>
</tr>
<tr>
<td>Akaike</td>
<td>-380</td>
</tr>
</tbody>
</table>

Table 2 | Range of estimated decision variables at different \(\alpha\)-cuts (fuzzy degrees) | Aquifer parameters

<table>
<thead>
<tr>
<th>Fuzzy degree</th>
<th>(T [\text{m}^2/\text{min}])</th>
<th>(S [Ss \cdot h])</th>
<th>(S_y)</th>
<th>(r/D_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Min</td>
<td>0.6043</td>
<td>0.0027</td>
<td>0.1194</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>0.6097</td>
<td>0.0028</td>
<td>0.1516</td>
</tr>
<tr>
<td>0.25</td>
<td>Min</td>
<td>0.5595</td>
<td>0.0027</td>
<td>0.0890</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>0.7116</td>
<td>0.0028</td>
<td>0.1517</td>
</tr>
<tr>
<td>0.5</td>
<td>Min</td>
<td>0.6371</td>
<td>0.0027</td>
<td>0.0925</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>0.7535</td>
<td>0.0029</td>
<td>0.1283</td>
</tr>
<tr>
<td>0.75</td>
<td>Min</td>
<td>0.7541</td>
<td>0.0029</td>
<td>0.0896</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>0.7832</td>
<td>0.0030</td>
<td>0.0911</td>
</tr>
<tr>
<td>1</td>
<td>Min</td>
<td>0.8059</td>
<td>0.0027</td>
<td>0.0880</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>0.8059</td>
<td>0.0027</td>
<td>0.0880</td>
</tr>
</tbody>
</table>
Showing the percentage variation in each fuzzy degree, Figure 3 also depicts a clear confirmation of how aquifer parameters vary. In other words, at 0.25 fuzzy degree level, the percentage variations of $T$, $S_y$ and $r/D_t$ are 27.2%, 70.5% and 26.8%, respectively. Results of this optimization, which consider uncertainties in pumping rate measurements, show that for $\pm 10\%$ change in the pumping rate, parameter estimation may vary significantly.

To check the fuzzy model with other pumping test data, it was applied to Analooche and Batlagh wells, located near Esfahan, Iran, and the range of variations in decision variables for different $\alpha$-cuts (levels of fuzzy degree) was obtained. Results are presented in the supplementary material, and show a similar trend as Figure 3 (Figures S3 and S4, available online).

**SUMMARY AND CONCLUSIONS**

In this study, the efficiency of a fuzzy simulation–optimization model coupled with the GA based on Boulton’s equation and FTM is examined in predicting hydrogeological parameters of an unconfined aquifer from its real time-drawdown pumping test data. An objective function is formulated based on the difference between drawdown observations from the pumping test and corresponding drawdown estimates from Boulton’s equation. The proposed simulation–optimization model is utilized to estimate the hydrogeological parameters of the aquifer by minimizing the objective function. Performance of the proposed method is compared with the traditional graphical method. Results show that the proposed model outperforms the graphical method, and thus is a more reliable method for predicting hydrogeological parameters. For example, MARE indices for the proposed model and graphical Boulton’s solution are 2.52% and 4.98%, respectively. Also, Boulton’s graphical method errors in estimating $T$ and $S_y$ are more than its error in estimating other hydrogeological parameters of the aquifer. When the applicability of the proposed model was assessed, the pumping rate was determined as the nondeterministic parameter. Observing a non-monotonic behavior for the proposed model, the pumping rate was decomposed based on a general FTM, and the decision-variable bounds were determined. The maximum variation in estimating $T$, $S_y$ and $r/D_t$ for the unconfined aquifer was observed when the uncertainty level was 0.25.

**CONFLICT OF INTEREST**

Authors of this research study clearly declare that they have no conflict of interest.

**REFERENCES**


First received 2 December 2017; accepted in revised form 26 April 2018. Available online 9 May 2018