Discharge capacity evaluation and hydraulic design of a piano key weir

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ABSTRACT

A piano key weir (PKW) is a new type of labyrinth weir that increases the unit discharge at the unregulated spillway inlet of the weir. It is considered to be an important structure in water supply and drainage systems. However, its complex geometry makes it difficult to achieve an optimal hydraulic design, and only a few design equations and criteria are available. This study investigates the discharge capacity of a PKW and evaluates the characteristics of its discharge using various sources of experimental data from a database. First, previously proposed discharge capacity formulas are summarized and analyzed. Then, a new formula that integrates the existing test data with the results of dimensional analysis and multiparameter optimization is proposed. The weir characteristics are evaluated using the proposed formula and a mathematical model. The results show that both the proposed formula and numerical model are promising approaches to evaluate the discharge capacity of an A-type PKW and can guide its design.

Key words | discharge capacity, physical and numerical models, piano key weir, spillway, upgrading and rehabilitation

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INTRODUCTION

A spillway is the most commonly used structure for discharging flood water beyond the designed capacity of a reservoir and avoiding overtopping. A new arrangement called a piano key weir (PKW) has been developed in recent years to rehabilitate spillways for the safety of reservoirs (Leite Ribeiro *et al.* 2012a, 2012b). It is a relevant solution to many problems such as water level control and emergency spillway operations. The PKW is a further development of the labyrinth weir, which was developed by Lempérière & Ouamane (2003). As in all labyrinth weirs, the weir axis of the PKW extends longitudinally, lengthening the front edge available for overflow by five times compared to that of a straight weir. A PKW looks like a set of piano keys because of its wide front edge, narrow bottom, ramps, and rectangular profile. For the same weir head, the discharge capacity of a PKW is substantially greater than that of a traditional crest weir because of this additional front-edge length. At present, there are more than 15 institutions working on PKWs (Laugier 2007; Erpicum *et al.* 2010; Anderson & Tullis 2012; Ho Ta Khanh *et al.* 2011; Leite Ribeiro *et al.* 2012a). Moreover, 15 years of research and development have enabled detailed investigations of the hydraulic behavior of a PKW. More than 25 PKWs are in operation or under construction globally (Erpicum *et al.* 2017). The discharge capacities of these dams vary from tens to thousands of cubic meters per second.

When designing a PKW, the first research target is its discharge capacity, which is directly related to the highest reservoir level or capacity. Although several PKW projects have been completed and many studies have been published on PKW discharge efficiency, their discharge features and

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optimization design were usually obtained based on model test studies conducted before construction. Fortunately, a PKW database has been created from different sources, and it is now easier to access experimental or field data for analysis (Boillat *et al.* 2011). A PKW has many geometric parameters that affect head discharge performance. Hence, to comprehensively investigate the discharge capacity of a PKW, this study reviews previous PKW discharge capacity formulas and then evaluates their characteristics using various sources of experimental data. A new formula is proposed that integrates the existing test data and the results of dimensional analysis and multi-parameter optimization. Additionally, the new design is compared with existing equations and a mathematical model to demonstrate its behavior as well as its advantages and disadvantages.

ANALYTICAL FORMULATIONS OF PKW DISCHARGE CAPACITY

The structure and characteristic parameters of a standard PKW are shown in Pralong *et al.* (2011). Typically, parameters W, B, P, and T_s represent the width, length, height, and sidewall thickness of the PKW, respectively, and R represents the height of the parapet wall. Subscripts i and o indicate the inlet and outlet keys, respectively. The developed length along the overflowing crest axis L is defined as $L = N(W_i + W_o + 2T_s + 2B)$ or as $L = W + 2NB$, where N is the number of cycles of the PKW. The slopes of the bottoms of the inlet and outlet keys, S_i and S_o, respectively, can be expressed as functions of P, R, and B.

From the perspective of the flow regime, the discharge capacity is primarily affected by the design parameters of the weir, which include the length, horizontal width of the weir, vertical height, widths of the inlet and outlet keys, crest length, the weir head H (including the velocity head of the approaching flow), flow rate V, acceleration of gravity g, kinematic viscosity v, and density ρ. Therefore, the discharge capacity can be expressed as follows:

$$Q = f(L, B, P, W, W_i, W_o, H, T_s, R, \rho, g, v) \quad (1)$$

In this study, ρ, g, and v are not considered because this investigation is only based on geometrical parameters. The slope of the keys can be omitted in the analysis because it is a function of the existing parameters. Using the π theorem and formula for flow, we obtain

$$\begin{cases} Q = C_d W \sqrt{2g} H^{1.5} \\ C_d = f\left(\frac{L}{W}, \frac{B}{P}, \frac{W_i}{W_o}, \frac{T_s}{H}, \frac{R}{P}, \frac{H}{P}\right) \end{cases} \quad (2)$$

where C_d is a discharge coefficient that can be obtained only by performing physical tests.

Four published rating curve equations for the discharge capacity of PKW are reviewed (Geng 2004; Kabiri-Samani & Javaheri 2012; Leite Ribeiro *et al.* 2012a, 2012b; Machiels 2012, Machiels *et al.* 2014). Table 1 lists the analytical formulas and their range of application. These empirical formulas were all obtained by performing a large number of tests and analyses on A-type PKWs. Although each parameter is defined within a fairly large range, these formulas still have certain limitations. Note that in Geng (2004), the effects of vertical height and crest length on discharge were not sufficiently considered ($B = 0.4$ m and $P = 0.15$ m) because the effect of B/P should include at least one secondary term. Because small discrepancies may result in significant overestimations of the discharge, the limitations of these formulas should be respected.

PROPOSED FORMULA

The above empirical formulas indicate that the discharge coefficient of a PKW primarily depends on L/W, B/P, W_i/W_o, and H/P and is hardly affected by the weir thickness or height of the parapet. Equation (2) can be rewritten as follows if the influence of the last two factors are ignored:

$$C_d = f\left(\frac{L}{W}, \frac{B}{P}, \frac{W_i}{W_o}, \frac{H}{P}\right) = f\left(\frac{L}{W}, \frac{B}{P}, \frac{W_i}{W_o}\right)\left(\frac{H}{P}\right)^a \quad (8)$$

where a is a coefficient to be determined. Note that the values of the above parameters are not determined randomly. By analyzing the existing experimental data of the PKW, a reasonable range for each parameter is selected as follows:

Table 1 | Published formulations for the discharge capacity coefficient

Geng (2004)	Expression	$C_d = \frac{f(H/P)}{f(L/W)\cdot f(W/P)}$, where $f(H/P) = -0.655\ln\left(\frac{H}{P}\right) + 0.711$, $f(L/W) = -1.158\left(\frac{L}{W}\right)^{1.66H/P-2} + 1.2\left(\frac{L}{W}\right)^{0.83H/P-1} + 0.771$ $f(W/P) = -9.023\left(\frac{W}{P}\right)^{2.4H/P-3} + 27.416\left(\frac{W}{P}\right)^{1.6H/P-2} - 27.39\left(\frac{W}{P}\right)^{0.8H/P-1} + 10.075$ (3)
	Features and limits	$0.21\ \text{m} \le W \le 0.30\ \text{m}$, $0.053\ \text{m} \le W_i \le 0.15\ \text{m}$, $0.043\ \text{m} \le W_o \le 0.15\ \text{m}$, $0.10\ \text{m} \le B_i \le 0.20\ \text{m}$, $0.00\ \text{m} \le B_o \le 0.1\ \text{m}$, $0.00\ \text{m} \le P_d \le 0.45\ \text{m}$, and $0.02\ \text{m} \le H \le 0.2\ \text{m}$.
Kabiri-Samani & Javaheri (2012)	Expression	$C_d = \frac{2}{3}\left\{0.606 + \left[0.202\left(\frac{H}{P}\right)^{-0.675}\left(\frac{L}{W}\right)^{0.377}\left(\frac{W_i}{W_o}\right)^{0.426}\left(\frac{B}{P}\right)^{0.306}\exp\left(1.504\frac{B_o}{B} + 0.093\frac{B_i}{B}\right)\right]\right\}$ (4)
	Features and limits	$0.1 \le H/P \le 0.6$, $2.5 \le L/W \le 7.0$, $1.0 \le B/P \le 2.5$, $0.33 \le W_i/W_o \le 1.22$, $0.0 \le B_i/B \le 0.26$
Leite Ribeiro et al. (2012a, 2012b)	Expression	$C_d = 0.42\left[1 + 0.24\left(\frac{(L-W)P_i}{WH}\right)^{0.9}\cdot wpba\right]$ (5) $C_d = 0.42\left[0.8 + 0.34\left(\frac{L}{W}\right)^{0.7}\left(\frac{W_i}{W_o}\right)^{0.08}\left(\frac{P_o}{P_i}\right)^{0.25}\left(\frac{P_i}{H}\right)^{0.82}\right]$ (6) where $w = \left(\frac{W_i}{W_o}\right)^{0.05}$, $p = \left(\frac{P_o}{P_i}\right)^{0.25}$, $b = \left(0.3 + \frac{B_i + B_o}{B}\right)^{-0.5}$, $a = 1 + \left(\frac{R_o}{P_o}\right)^2$
	Features and limits	$0.1 \le H/P \le 2.8$, $3.0 \le L/W \le 7.0$, $1.5 \le B/P \le 4.6$, $0.50 \le W_i/W_o \le 2.0$, $0.2 \le B_i/B \le 0.40$
Machiels (2012)	Expression	$C_d = C_{qu}\frac{W_o}{W_u} + C_{qd}\frac{W_i}{W_u} + C_{qs}\frac{2B}{W_u}K_{wo}$, where $W_u = W_i + W_o + 2T$, $C_{qu} = 0.374\left(1 + \frac{1}{1000H + 1.6}\right)\left(1 + 0.5\left(\frac{H}{H + P_T}\right)^2\right)$, (7) $C_{qd} = 0.445\left(1 + \frac{1}{1000H + 1.6}\right)\left(1 + 0.5\left(\frac{H}{H + P}\right)^2\right)$, $C_{qs} = 0.41\left(1 + \frac{1}{833H + 1.6}\right)\left(1 + 0.5\left(\frac{0.833H}{0.833H + P_e}\right)^2\right)\left(\frac{P_e^{\alpha} + \beta}{(0.833H + P_e)^{\alpha} + \beta}\right)K_{wi}$
	Features and limits	$0.1 \le H/P \le 5.0$, $4.2 \le L/W \le 5.0$, $1.0 \le B/P \le 6.0$, $0.50 \le W_i/W_o \le 2.0$, $0.29 \le B_i/B \le 0.33$

$H/P > 0.1$, $2.5 < L/W < 8.5$, $0 < W_i/W_o < 2.45$, and $1 < B/P < 6$. The geometric parameters of the completed prototype weirs also fall in these ranges. Here, the height of the parapet and crest shape are secondary factors and therefore not considered when $H/P > 0.1$.

Because the main problem in PKW hydraulic design is the number of geometric parameters, in this study, data from more than 18 specific PKW geometries were used to set up the equation (as listed in Figure 1). The data include results from experimental studies carried out by the Indian Institute of Technology (IIT), Ho Chi Minh City University of Technology (HCMUT), EDF, and LHC (Leite Ribeiro et al. 2012a, 2012b), which are plotted in Figure 1. To facilitate the analysis, all data were converted to C_d as a function of H/P. It can be easily inferred that C_d is close to a negative power-exponential function that can be obtained through data fitting. The PKW is efficient for relatively lower heads, and its efficiency decreases rapidly as the head increases. The discretization of various data sources is large when $H/P < 0.4$. However, because C_d is still related to L/W, B/P, and W_i/W_o and, moreover, several coefficients need to be determined, overfitting can occur when an optimization algorithm is used. In an optimization algorithm, a cluster of curves are generated, and multiple optimal coefficients are obtained. To describe this cluster of curves, the representative data points were analyzed and compared with each other. Several techniques have been employed to solve optimization problems (Azamathulla et al. 2016; Zheng et al. 2016, 2017). Here, all the data points were sorted in ascending order, and then the simple method of least squares was used to fit these points, where H/P is taken as a variable. The value of a was found to be -0.465 given $R^2 = 0.89$. The fitted formula is expressed as follows:

$$C_d = 0.1 + b\left(\frac{L}{W}\right)^c\left(\frac{B}{P}\right)^d\left(\frac{W_i}{W_o}\right)^e\left(\frac{H}{P}\right)^{-0.465} \quad (9)$$

where b, c, d, and e are coefficients to be determined. The thick solid line in Figure 1 represents all the fitted test values, where $b(L/W)^c(B/P)^d(W_i/W_o)^e = 0.625$. The functions of the data points obtained from the various tests oscillate around the fitted curve because of their different values for L/W, B/P, and W_i/W_o. Therefore, the problem of determining these parameters can be converted into a problem of optimizing multiple parameters in an objective function. For this optimization, a genetic algorithm is an appropriate tool. It was utilized to form an objective function with four parametric variables. The convergence threshold was set to 0.005. The optimal solution

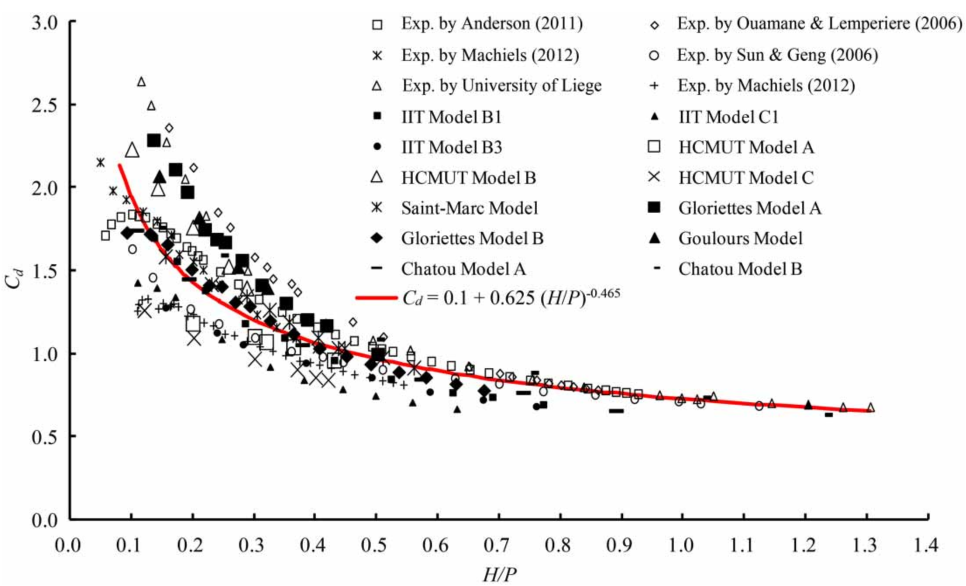

Figure 1 | Relationships of PKW discharge coefficients obtained from various tests and H/P values.

for the coefficients was obtained through hundreds of generations of optimization and is as follows:

$$C_d = 0.1 + 0.285\left(\frac{L}{W}\right)^{0.45}\left(\frac{B}{P}\right)^{0.1}\left(\frac{W_i}{W_o}\right)^{0.05}\left(\frac{H}{P}\right)^{-0.465} \quad (10)$$

COMPARISON AND DISCUSSION

The test parameters listed in Table 2 were used to compare the formulas of Kabiri-Samani & Javaheri (2012; Equation (4)), Leite Ribeiro *et al.* (2012a, 2012b; Equations (5) and (6)), and Machiels (2012; Equation (7)). The data selected here include the typical and representative model test data from Anderson (2011), Ouamane & Lempérière (2006), and Sun & Geng (2006) as well as the test data for the Escouloubre PKW in France (Machiels 2012). In addition to the analytical formulas, a numerical approach using WOLF-1D software was also implemented to improve the understanding of PKW flow behaviour and optimal design. Erpicum *et al.* (2010) presented such a model for preliminary PKW design with an accuracy of ±10%, and it was used for comparison. Because of limited space, only two sources were used to show the comparison results. Note that, in Anderson's formula for calculating PKW flow (Anderson 2011), the total width L was multiplied by a factor of 2/3.

Table 2 | Characteristics of the reference models

Crest-shape	Utah Anderson (2011) Half rounded	Biskra Ouamane & Lempérière (2006) Sharp crested	IWHR Sun & Geng (2006) Sharp crested	Liège Erpicum *et al.* (2010) Sharp crested	Escouloubre Machiels (2012) Flat topped
W	0.933	0.169	0.5	0.6	5.11
L	4.845	1.514	2.1	2.49	21.91
P	0.197	0.155	0.15	0.525	1.77
P_d	–	0	0.45	0.2	4.53
W_u	0.233	0.169	0.25	0.4	2.8
W_i	0.116	0.089	0.125	0.18	1.3
W_o	0.092	0.074	0.125	0.18	0.9
B	0.489	0.412	0.4	0.63	5.1
B_i	0.121	0.103	0.1	0.184	1.2
B_o	0.121	0.103	0.1	0.184	1.2
T_s	0.013	0.002	0	0.02	0.3
R	0.013	0	0	0	0

As shown in Figures 2 and 3, when $H/P < 0.15$, the values predicted by Equation (10) are larger than the ones obtained using Machiels' formula and the measured data. This implies that the efficiency is overestimated. However, the flow rate difference is small in this situation because of the lower head. Considering the applicable range of each formula and the influence of water surface tension in the models, H/P must be greater than 0.1.

When the weir head is small, i.e., $0.1 < H/P < 0.15$, the discharge coefficient obtained by the tests tends to increase. However, none of the empirical formulas predict this phenomenon, which could be attributed to the influence of the weir thickness and shape. In the tests by Anderson (2011), the PKW was made of Plexiglas with weir thickness $T_s = 1.27$ cm. In the case of a very small value of H/T_s, the PKW would have a flow pattern similar to that of a broad-crested weir and the discharge coefficient would be close to that of a broad-crested weir. However, the above empirical formulas do not consider the influence of thickness.

When $0.15 < H/P < 0.25$, the discharge coefficient predicted by the proposed formula is closer to the measured value and the average error is less than 8%. With further increases in the head ratio, the predicted values are very close to the measured values in all cases. The characteristic parameters $W_i = W_o$, $B_i = B_o$, and $P_i = P_o$ in Figure 2 represent a special weir type. The results calculated from both empirical formulas of Leite Ribeiro *et al.* are close to each other. For $H < 0.02$ m, the result obtained using Machiels' formula is too low. In this case, the water surface tension must be considered because of the small weir head. However, all the other results for different heads are large. For $H/P > 0.5$, the value estimated using either of Leite Ribeiro *et al.*'s formulas is the closest to the measured value and is slightly better than the value obtained using Machiels' formula.

The three empirical formulas for discharge coefficient calculation were obtained using a large number of tests, and their areas of focus are slightly different from each other. Each parameter in the test was restricted to a certain range. For example, in Kabiri-Samani & Javaheri's formula, the parameters are restricted as follows: $2.5 < L/W < 7.0$, $0.1 < H/P < 0.6$, $0.33 < W_i/W_o < 1.22$, $1.0 < B/P < 2.5$, and $0.0 < B_i/B < 0.26$. These restrictions cause the estimated

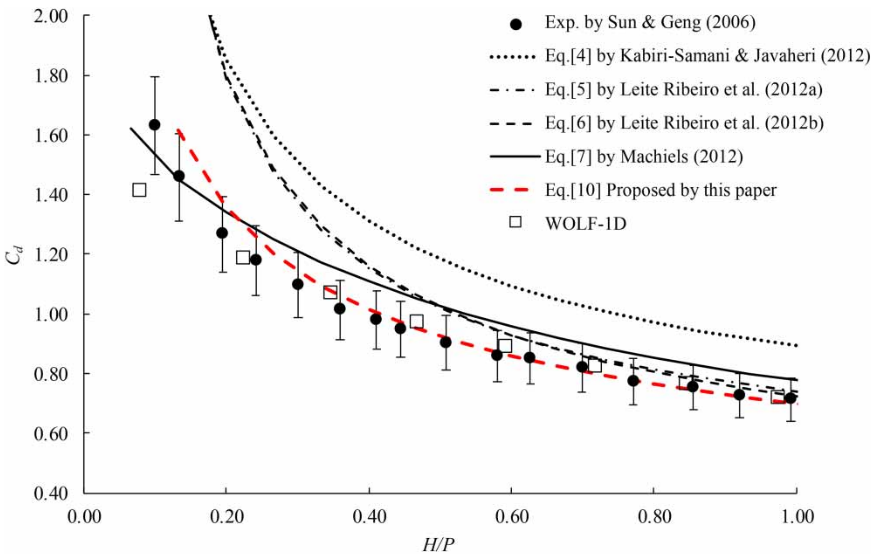

Figure 2 | Comparison of discharge coefficient for various formulas and the experimental results from Sun & Geng (2006).

values to vary by a substantial margin. Machiels *et al.* (2014) pointed out that significant differences in capacity estimations may result from the application of a formulation outside its parameter ranges. These results reveal that no data of the prototype characteristics are completely in conformity with the applicable scope of this formula, indicating that the application of this formula may be limited. Note that the different crest shapes should be considered when comparing different PKW datasets. For thick crested weirs, the shape and thickness may influence the discharge capacity, especially at low heads. The results of experiments conducted on labyrinth weirs with varied crest shapes show that a quarter-round crest provides discharges almost 9% higher than those of a sharp crest (Erpicum *et al.* 2016). The different crest shapes explain the differences among the formulations inside their common parameter ranges.

The flow ratio coefficient, i.e., the improvement in discharge in the case of Sun & Geng (2006), is in the range of 1.2–3.5. Smaller H/P values lead to better discharge capacities. As H/P increases, the overall length L of the

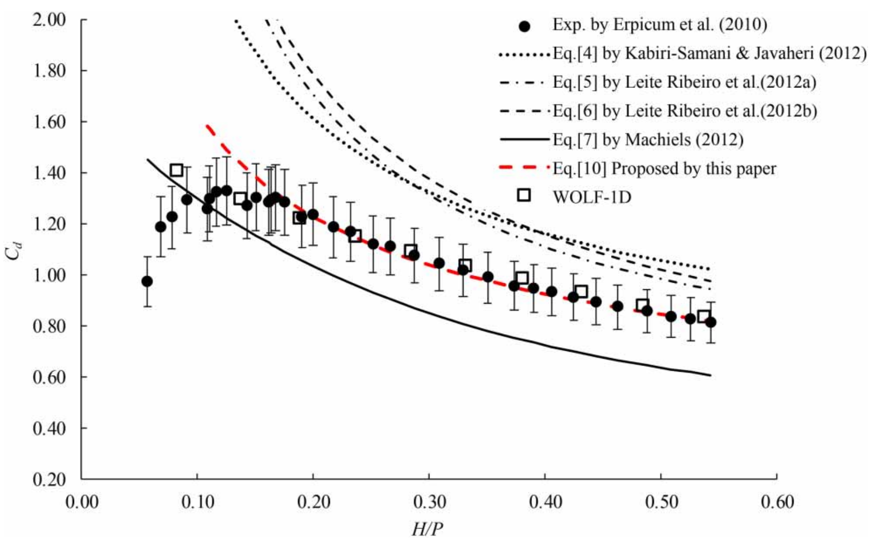

Figure 3 | Comparison of discharge coefficients for various formulas and the experimental results from Erpicum *et al.* (2010).

weir plays a dominant role in the discharge flow and the increase in discharge reduces. When $H/P > 1$, the discharge improvement ratio is less than 1.5. For example, the discharge coefficient calculated using the proposed formula is 0.65 and the discharge improvement ratio is approximately 1.3 when $H/P = 1.2$. This indicates that if the weir is designed as a PKW, the head required to realize the same flow is much lower than that required by a standard ogee-crested weir; this difference in the heads is referred to as the new head addition. Note that the designed head ratio of the existing prototype PKWs is limited within the range $0.13 < H/P < 0.66$, indicating that the prediction accuracy of the proposed formula is less than 8% and that it has higher practicability.

The numerical results of WOLF-1D are in satisfactory agreement with the results obtained using the three datasets. For large and small head ratios, the errors in the predicted values are less than 10%, and for moderate head ratios, the predicted values appear to be very promising. The values predicted using the formulas proposed with the data listed in Table 2 are also close to their corresponding measured values with an overall average error of up to 8% if a head ratio $H/P > 0.1$ is considered.

CONCLUSION

This study compared the PKW discharge capacity formulas developed in recent years and proposed a formula based on existing test data. Analysis results showed that the values predicted by the proposed formula are in good agreement with the published test data, with an average error in the range of 5–8% if the head ratio $H/P > 0.15$. The proposed formula is an easy and practical approach for predicting the release capacity of PKWs. Considering the scale effect of a model and the influence of water surface tension, the head ratio should be larger than 0.1 in practical applications. The numerical results of WOLF-1D are in satisfactory agreement with the experimental data, with a prediction error of less than 8%. Because no well-established and practical standard is currently available for PKW design and evaluation, both the proposed formula and WOLF-1D can be used as a reference for evaluating the discharge capacity of an A-type PKW and for guiding its design. In the case of a small upstream head (normally $H/P < 0.2$), the discharge coefficient of the PKW is fairly large and the discharge improvement ratio is higher than 3, which indicates an obvious increase in the discharge flow. However, the absolute flow is still not high because of the small head in this case. Considering that $H/P < 1$ for normal operations, a practical range for the discharge improvement ratio is 1.2–3.5. As the upstream head increases, the overall length L of the weir plays a dominant role in the discharge flow and the increase in the discharge flow reduces. When $H/P > 1.2$, the flow ratio coefficient is in the range of 1.2–1.3. Therefore, the discharge flow of the PKW in this sense cannot increase in an unrestricted manner, and instead, a reasonable range for the head should be defined.

It should be noted that the side wall thickness, height of the parapet on the weir, and parapet shape are secondary factors that may have insignificant influence on the discharge capacity and their specific effects can be further investigated from a quantitative point of view in future work.

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