Application of Philip infiltration model to film hole irrigation

Yanwei Fan, Jiaguo Gong, Ying Wang, Xiaoxia Shao and Tong Zhao

ABSTRACT

Numerical simulations were carried out with HYDRUS-2D to investigate the influence of soil texture, initial water content, film hole diameter and water depth on cumulative infiltration from a film hole. Soil texture, film hole diameter and water depth are the dominant influencing factors. Philip infiltration model was used to fit the simulated results of the dominant influencing factors. For the same soil, the sorptivity (s) is a power function relationship with film hole diameter, while the steady infiltration rate (a) is a power function relationship with film hole diameter and water depth. On that basis, the calculation formulas for predicting s and a were established, and a simplified film hole infiltration model including the film hole diameter and water depth was proposed. The effectiveness of the model was verified by laboratory experiments and literature data. The predicted values of the model were in good agreement with the experimental observations. The model parameters can be determined only by a set of film hole infiltration experiment, which simplifies the experimental design and can be used as a tool for irrigation engineers or farmers to estimate cumulative infiltration.

Key words | cumulative infiltration, film hole infiltration, Philip model, sufficient water supply

INTRODUCTION

Plastic film mulching (PFM) is an important agricultural practice in arid and semi-arid areas of northwest China (Li et al. 2004a; Zhou et al. 2009). Film hole irrigation (FHI) has been also developed on the basis of PFM. FHI is a relatively new irrigation method that involves completely covering a bordered field with plastic film with uniform diameter holes (Saeed & Mahmood 2013; Li et al. 2017). When irrigating, water flows over the plastic film on the soil surface. There is a thin layer of water depth above the film hole, and water infiltrates into the soil through these holes on plastic film. Additionally, these infiltration holes are also seedling holes after germination of the crop. In this regard, FHI belongs to surface irrigation technology. Compared with traditional surface irrigation methods, FHI can effectively save water resources, improve irrigation uniformity and extend the length of surface irrigation (Wu et al. 2001; Li et al. 2005).

For crops with wide row and plant spacing, such as watermelon and melon, the wetting patterns of adjacent film holes do not overlap each other, which is macroscopically a single-point source infiltration. Several field and laboratory experiments have been carried out to study the influence of irrigation parameters (e.g. soil texture, initial water content, film hole diameter and water depth) on the
single-point source infiltration under FHI (Jiao & Wang 1999; Wu et al. 2001). Wang et al. (2001) and Wu et al. (2003) used experimental data to establish an infiltration model based on Philip and kostiakov model, respectively. However, these empirical models contained multiple undetermined coefficients, and the experimental design was more complicated.

Numerical simulation is a fast and inexpensive approach to studying optimal irrigation management practices. The numerical model HYDRUS-2D developed by Šimůnek et al. (2016) has been widely used in soil water movement for various irrigation methods. Several investigators have used this model to evaluate either field or laboratory experiments, or other mathematical models (e.g. Skaggs et al. 2004; Provenzano 2007; Naglić et al. 2014; El-Nesr et al. 2014; Karandish & Šimůnek 2017). The HYDRUS model enables its users to trace the movement of water and solutes and the wetting patterns in both simple and complex geometries for homogeneous or heterogeneous soils and for different combinations of initial and boundary conditions. Ma et al. (2015) showed that the HYDRUS-2D model can be used to simulate soil water movement under film hole infiltration with good accuracy.

In this study, the main objectives are to simulate and analyze the influence of soil texture, initial water content, film hole diameter, and water depth on cumulative infiltration and identify the dominant influence factors, and to develop an empirical model for predicting the cumulative infiltration of FHI.

**MATERIALS AND METHODS**

**Laboratory experiments**

Figure 1 illustrates the equipment for the laboratory experiments. The soil container was made of 10 mm thick Plexiglas sheets. The bottom of the container had many holes with a diameter of 2 mm for ventilation. Plastic film covered the soil surface and left a ¼ film hole at one corner of container. Water on the plastic film can only penetrated into the soil through the film hole. Holes with a diameter of 2 cm and a spacing of 5 cm were located on the side of soil container near the film hole in order to extract soil and measure its water content after irrigation. Mariotte bottle was fixed in the height adjustable stand and provides a constant head for the film hole through a rubber hose.

**Numerical modeling**

Soil water movement under FHI is an axisymmetric two-dimensional infiltration process. We simulated water infiltration using HYDRUS-2D (Šimůnek et al. 2006). Assuming a homogeneous and isotropic soil, the governing equation for water flow is the 2D Richards equation, which can be written in axisymmetric coordinates as follows:

\[
\frac{\partial \theta}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r K(\phi) \frac{\partial \phi}{\partial r} \right] + \frac{\partial}{\partial z} \left[ K(\phi) \frac{\partial \phi}{\partial z} \right] - \frac{\partial K(\phi)}{\partial z}
\]

where \( \theta \) is volumetric water content (cm\(^3\)/cm\(^3\)), \( t \) is time (min), \( r \) is radial coordinate (cm), \( K(\phi) \) is unsaturated hydraulic conductivity (cm/min), \( \phi \) is soil water pressure head (cm), and \( z \) is vertical coordinate that is positive downward (cm).
The soil hydraulic properties were modeled using the van Genuchten-Mualem (Mualem 1976; van Genuchten 1980) constitutive relationships, 

\[ \theta(\phi) = \theta_r + \left(\theta_s - \theta_r\right) \left[1 + \left|\alpha\phi\right|^{n}\right]^{m} \]  

(2)

\[ K(\phi) = K_s S_e^{0.5} \left[1 - \left(S_e^{1/m}\right)^{m}\right]^{2} \]  

(3)

where

\[ S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r}, \quad m = 1 - 1/n \]

and where \( \theta_s \) is saturated water contents (cm\(^3/cm^3\)), \( \theta_r \) is residual water contents (cm\(^3/cm^3\)), \( K_s \) is saturated hydraulic conductivity (cm/min), \( S_e \) is the effective degree of saturation, \( \alpha \) is an empirical parameter (1/cm), and \( m \) and \( n \) are empirical constants (-).

The initial conditions are as follows:

\[ \theta = \theta_0, \quad 0 \leq r \leq 50 \text{cm}, \quad 0 \leq z \leq 100 \text{cm}, \quad t = 0 \]  

(4)

where \( \theta_0 \) is the initial water potential before irrigation (cm).

The boundary conditions are as follows:

\[ \begin{align*}
\phi &= h, \quad 0 \leq r \leq \frac{d}{2}, \quad z > 0, \quad t > 0 \\
-K(\phi) \left(\frac{\partial \phi}{\partial z} + 1\right) &= 0, \quad 0 \leq r \leq 50 \text{cm}, \quad z = 0, \quad t > 0 \\
-K(\phi) \frac{\partial \phi}{\partial r} &= 0, \quad r = 0 \text{ or } r = 50 \text{cm}, \quad 0 \leq z \leq 100 \text{cm}, \quad t > 0 \\
-K(\phi) \left(\frac{\partial \phi}{\partial z} + 1\right) &= 0, \quad 0 \leq r \leq 50 \text{cm}, \quad z = 100 \text{cm}, \quad t > 0 
\end{align*} \]  

(5)

**Modeled scenarios**

Given the realities that constrain film hole irrigation in the field, the film hole diameters is typically 3–8 cm, the water depth on film mulch is kept constant within a range of 2–10 cm, and the crops need to be irrigated when soil water content is about 50% available water (AW) (Fan et al. 2008).

To address our main objectives, we have chosen to evaluate the following alternative scenarios:

1. Twelve soil textures. Soil hydraulic parameters for twelve textural classes were taken from the soil catalog provided by the HYDRUS software (Carsel & Parrish 1988), presented in Table 1.

2. Three soil initial water contents (30%, 50%, and 70% AW). The influence of initial water content was investigated by varying the %AW. AW corresponds to the amount of water between field capacity (FC) and permanent wilting point (PWP). FC corresponds to water content at a soil matric potential of −10 kPa, and \( \theta_r \) was used for the PWP (Li et al. 2004b; Naglič et al. 2014).

3. Three irrigation water depths (2, 4 and 6 cm).

4. Three film hole diameters (3, 5 and 7 cm).

**Analytic method**

The cumulative infiltration of HYDRUS was determined by the change in water content of each node. The relationship
between cumulative infiltration and time were described using the Philip infiltration model (Philip 1957) and expressed as Equation (6).

\[ I = st^{0.5} + at \]  

where \( I \) is cumulative infiltration (mL), \( s \) is sorptivity (mL/min\(^{0.5}\)), and \( a \) is steady infiltration rate (mL/min).

### Statistical analysis

The root mean square error (RMSE) and modelling efficiency (EF) were adopted for further analysis to examine the performance of the simplified infiltration model against observed values. The expressions are:

\[
RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (M_i - S_i)^2}
\]

\[
EF = \frac{\sum_{i=1}^{n} (M_i - M_{mean})^2 - \sum_{i=1}^{n} (M_i - S_i)^2}{\sum_{i=1}^{n} (M_i - M_{mean})^2}
\]

where \( M_i \) and \( S_i \) are the observed and predicted values, respectively. \( M_{mean} \) is the mean of observed data, and \( n \) is the total number of observations.

### RESULTS AND DISCUSSION

Different factors affecting cumulative infiltration of FHI

The influence of soil texture, initial water content \( (\theta_0) \), film hole diameter \( (d) \) and water depth \( (h) \) on cumulative infiltrations is shown in Figure 3. Soil texture has a very significant effect on the cumulative infiltration. For a given time, the cumulative infiltration is smaller for fine-textured soil and larger for coarse-textured soil. Film hole diameter and water depth have a great influence on the cumulative infiltration. As the film hole diameter and water depth increase, the infiltration increases. The initial water content has little effect on the cumulative infiltration, and the cumulative infiltration decreases with the increase of the initial water content. Based on the above analysis, we can conclude that the soil texture, film hole diameter, and water depth are dominant influencing factors.

### Table 1 | The van Genuchten–Mualem model parameters for simulated soils

<table>
<thead>
<tr>
<th>Soil texture</th>
<th>( \theta_r ) (cm(^3)·cm(^{-3}))</th>
<th>( \theta_s ) (cm(^3)·cm(^{-3}))</th>
<th>( \alpha ) (cm(^{-1}))</th>
<th>( n ) (-)</th>
<th>( K_s ) (cm·min(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silty clay</td>
<td>0.070</td>
<td>0.360</td>
<td>0.005</td>
<td>1.09</td>
<td>0.0003</td>
</tr>
<tr>
<td>Silty clay loam</td>
<td>0.089</td>
<td>0.430</td>
<td>0.010</td>
<td>1.23</td>
<td>0.0012</td>
</tr>
<tr>
<td>Sandy clay</td>
<td>0.100</td>
<td>0.380</td>
<td>0.027</td>
<td>1.23</td>
<td>0.0020</td>
</tr>
<tr>
<td>Clay</td>
<td>0.068</td>
<td>0.380</td>
<td>0.008</td>
<td>1.09</td>
<td>0.0033</td>
</tr>
<tr>
<td>Clay loam</td>
<td>0.095</td>
<td>0.410</td>
<td>0.019</td>
<td>1.31</td>
<td>0.0043</td>
</tr>
<tr>
<td>Silt</td>
<td>0.034</td>
<td>0.460</td>
<td>0.016</td>
<td>1.37</td>
<td>0.0042</td>
</tr>
<tr>
<td>Silt loam</td>
<td>0.067</td>
<td>0.450</td>
<td>0.020</td>
<td>1.41</td>
<td>0.0075</td>
</tr>
<tr>
<td>Loam</td>
<td>0.078</td>
<td>0.430</td>
<td>0.036</td>
<td>1.56</td>
<td>0.0173</td>
</tr>
<tr>
<td>Sandy clay loam</td>
<td>0.100</td>
<td>0.390</td>
<td>0.059</td>
<td>1.48</td>
<td>0.0218</td>
</tr>
<tr>
<td>Sandy loam</td>
<td>0.065</td>
<td>0.410</td>
<td>0.075</td>
<td>1.89</td>
<td>0.0757</td>
</tr>
<tr>
<td>Loamy sand</td>
<td>0.057</td>
<td>0.410</td>
<td>0.124</td>
<td>2.28</td>
<td>0.2432</td>
</tr>
<tr>
<td>Sand</td>
<td>0.045</td>
<td>0.430</td>
<td>0.145</td>
<td>2.68</td>
<td>0.4950</td>
</tr>
</tbody>
</table>

### Figure 3 | Simulated cumulative infiltration for different influencing factors. (a) \( \theta_0 = 50\%\text{AW}, h = 4\ cm, d = 5\ cm \). (b) Loam, \( h = 4\ cm, d = 5\ cm \). (c) Loam, \( \theta_0 = 50\%\text{AW}, h = 4\ cm \). (d) Loam, \( \theta_0 = 50\%\text{AW}, d = 5\ cm \).
Establishment of the simplified infiltration model

In this section, soil texture, water depth, and film hole diameter were simulated. The simulated cumulative infiltration data were fitted using Equation (6) to obtain infiltration parameters $s$ and $a$, as shown in Table 2. For different combinations of dominant influencing factors, the coefficient of determination ($R^2$) were all greater than 0.99, indicating that the relationship between cumulative infiltration and time can be expressed by Philip infiltration model.

For the same soil texture, $s$ is mainly affected by $d$, while $h$ has little effect on it, and $a$ is related to $d$ and $h$. Based on extensive verification, we found that the relationship between $s$ and $d$ satisfies the power function, while $a$ is a power function relationship with $d$ and $h$. Predictive model for $s$ and $a$ were proposed as below:

$$s = m_1 d^{n_1}$$
$$a = m_2 d^{n_2} h^{n_3}$$

where $m_1$, $m_2$, $n_1$, $n_2$, and $n_3$ are the fitting parameters.

The $m_1$ and $n_1$ were obtained from fitting $s$ using Equation (9) for each soil in Table 2. Similarly, the $m_2$, $n_2$, and $n_3$ were determined by fitting $a$ with Equation (10). We found that $n_1$ changed little (from 1.35 to 1.73), and $n_2$ and $n_3$ varied greatly for fine-textured soils (silty clay, silty clay loam, sandy clay, and clay) and little for coarse-textured soils ($n_2$ from 0.76 to 0.91, and $n_3$ from 0.25 to 0.35). It can be seen from Equation (6) that $f$ is affected by $s t^{0.5}$ and $a t$. For fine-textured soils, $s$ is much larger than $a$, and the effect of $s t^{0.5}$ on cumulative infiltration is much greater than $a t$ during irrigation time. Although the fitting results of $a$ were not good, it had little effect on cumulative infiltration. To simplify the parameters, $n_1$ can take the average value of twelve soils, and $n_2$ and $n_3$ can take the average value of coarse-textured soils, i.e. $n_1 = 1.6$, $n_2 = 0.8$, and $n_3 = 0.3$. Therefore, Equations (9) and (10) can be further varied to:

$$s = k_1 d^{0.6}$$
$$a = k_2 d^{0.8} h^{0.3}$$

where $k_1$ and $k_2$ are the fitting parameters.

Table 2 | Fitted infiltration parameter values

<table>
<thead>
<tr>
<th>Soil texture</th>
<th>Film hole diameter</th>
<th>Water depth</th>
<th>Sandy clay</th>
<th>Clay</th>
<th>Silt</th>
<th>Clay loam</th>
<th>Sandy clay loam</th>
<th>Silty clay loam</th>
<th>Silt loam</th>
<th>Loam</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>0.07</td>
<td>1.35</td>
<td>0.06</td>
<td>2.70</td>
<td>0.05</td>
<td>1.35</td>
<td>0.06</td>
<td>2.70</td>
<td>0.05</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>0.07</td>
<td>1.35</td>
<td>0.06</td>
<td>2.70</td>
<td>0.05</td>
<td>1.35</td>
<td>0.06</td>
<td>2.70</td>
<td>0.05</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>0.07</td>
<td>1.35</td>
<td>0.06</td>
<td>2.70</td>
<td>0.05</td>
<td>1.35</td>
<td>0.06</td>
<td>2.70</td>
<td>0.05</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>0.07</td>
<td>1.35</td>
<td>0.06</td>
<td>2.70</td>
<td>0.05</td>
<td>1.35</td>
<td>0.06</td>
<td>2.70</td>
<td>0.05</td>
</tr>
</tbody>
</table>

$k_1$ and $k_2$ are the fitting parameters.
Combining Equations (6), (11) and (12), a simplified infiltration model for predicting the cumulative infiltration of single film hole was proposed:

\[ I = k_1 d^{1.6} t^{0.5} + k_2 d^{0.8} h^{0.3} t \]  

(13)

**Evaluation of the simplified infiltration model**

A soil tank experiment was designed to verify whether the simplified infiltration model is accurate. The soil sample was taken from Yangling area, China. Given the USDA soil taxonomy, the soil texture is silt loam and sandy loam. The cumulative infiltration data with \( d \) of 5 cm and \( h \) of 5 cm was used to fit and determine the parameters \( s \) and \( a \) with Equation (6). Then, using the Equations (11) and (12), we get the fitting parameters \( k_1 \) and \( k_2 \). The simplified prediction models were established.

For Yangling silt loam:

\[ I = 1.8d^{1.6} t^{0.5} + 1.1d^{0.8} h^{0.3} t \]  

(14)

For Yangling sandy loam:

\[ I = 1.4d^{1.6} t^{0.5} + 1.7d^{0.8} h^{0.3} t \]  

(15)

Additionally, other experiments were also carried out with three film hole diameters and two water depths to verify the accuracy of Equations (14) and (15). The comparison result between predicted and observed values was illustrated in Figure 4(a).

To further judge whether the proposed model is reliable, the published data were also compared for three different soils, i.e. Dingbian sandy loam from Wu et al. (2005), Xi’an silt from Li et al. (2001) and Yulin loam from Jiao & Wang (1999). Similarly, the \( s \) and \( a \) were determined from fitting the cumulative infiltration with \( d \) of 5 cm and \( h \) of 2 cm, the \( k_1 \) and \( k_2 \) were obtained by Equations (9) and (10). To this end, the cumulative infiltration prediction models for the other three soils were established and written in Equations (16), (17) and (18) below. The comparison result between predicted and observed values for three soils was presented in Figure 4(b).

For Yulin loam:

\[ I = 2.5d^{1.6} t^{0.5} + 1.0d^{0.8} h^{0.3} t \]  

(16)

For X’an silt:

\[ I = 1.9d^{1.6} t^{0.5} + 0.9d^{0.8} h^{0.3} t \]  

(17)
For Dingbian sandy loam:

\[ I = 3.9d^{1.6}h^{0.5} + 4.8d^{0.8}h^{0.3}t \quad (18) \]

The values of RMSE is close to 0 (RMSE = 0.06 L); meanwhile, the values of EF approach 1.0 (EF = 0.99). It was noteworthy that all the results are consistent, indicating that the simplified model can accurately describe the cumulative infiltration from a film hole.

CONCLUSIONS

HYDRUS-2D simulation showed that soil texture, film hole diameter and water depth are the dominant factors affecting the cumulative infiltration from a film hole. For the same soil texture, the sorptivity (s) increases with the film hole diameter, and the relationship between them is a power function, while the steady infiltration rate (a) increases with increases of film hole diameter and water depth; the a is a power function relationship with film hole diameter and water depth. The calculation formulas of s and a were created, and a simplified model of cumulative infiltration of single-hole, including film hole diameter and water depth, was proposed. The effectiveness of the proposed model was verified by infiltration experiments of film hole infiltration and literature data. The calculated values of the model were in good agreement with the measured values. The model has only two undetermined coefficients, which can be determined only by a set of film hole infiltration tests. The simplified model in this study can provide a reference for agricultural irrigation project designers to estimate cumulative infiltration.

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