Improved grey prediction method for optimal allocation of water resources: a case study in Beijing in China

Baohui Men, Zhijian Wu, Huanlong Liu, Zehua Hu and Yangsong Li

ABSTRACT

Water shortages and the deterioration of water quality in the natural environment have a negative effect on social development of many countries. Therefore, optimizing the allocation of water resources has become an important research topic in water resources planning and management. An essential step in improving the utilization efficiency of water resources is the prediction of water supply and demand. Because it has a great number of merits, the grey prediction method has been widely used in population prediction and temperature prediction. However, it also has limitations such as low prediction precision since original data seriously fluctuates. This paper aims to handle the sample values by an innovative method utilizing moving-average technique (MA) model and optimizing the background values to make them more typical. Results proved that the prediction accuracy of the traditional model was effectively improved by the proposed method. The proposed model was then applied in the multi-objective planning to establish an optimal water resources allocation model for Beijing in the short-term (2020) planning timeframe, including local water resources, transfer water volumes, and other water supplies. The results indicated that industrial and agricultural water use could be well met, while domestic and environmental water resources may face a shortage.

Key words | improved grey prediction method, multi-objective planning, optimal allocation, water resources

INTRODUCTION

In the past few decades, great progress has been made in water resources allocation due to the advances in computer technology and artificial intelligence. The study of water resources allocation has developed from simply allocating water quantity to a unified configuration that considers both water quantity and quality. In the real world, water resources systems are often complicated by social, economic, and environmental factors, leading to multi-objective problems with significant uncertainties in system parameters, objectives, and their interactions (Kanakoudis 2004; Han et al. 2011). For example, the growing water scarcity due to growing populations has imposed serious stress on irrigation systems (Xu et al. 2005).

Water resources managers and planners are challenged by the demand of maintaining sustainable development while facing the pressure of population growth, continuous development of the economy, and ever-changing weather. The water shortages are the major obstacles to regional sustainable development in water resources systems, particularly in many semiarid and arid regions. Therefore, optimal water allocation is an essential component to water resource management based on limited water supplies (Kanakoudis et al. 2016, 2017). An integrated nonlinear stochastic optimization model was developed for planning and evaluation of urban resources in order to optimize urban flows, and was successfully applied to a case study in Beijing (Huang et al. 2013). The multi-objective optimization model is a crucial tool for efficient water resources management and is widely used in water resources allocation. For example, Kilic & Anac (2010) developed a
multi-objective planning model and applied it to the Mene-
men Left Bank Irrigation System from the Lower Gediz Basin in Turkey, which was used to optimize the increase in
the total area irrigation and reduce water losses occurring
at the network level. Beh et al. (2017) put forward the use of
metamodels (surrogates for computationally expensive simulation models) to calculate robustness and other objectives,
which enables robustness to be considered explicitly as
an objective within a multi-objective optimization framework. Peng et al. (2015) developed a multi-objective
dynamic water resources allocation model by considering
the variation in overall users' satisfaction with time, and
following the principle that the variation in the satisfaction with the system within adjacent periods of time must be
minimal, which was applied to the Huai River for the pre-
sent situation (2010), short-term (2020) and long-term
(2030) planning timeframes. Guo et al. (2014) presented a
bi-level optimization model that optimized and allocated
water resources rationally among the social, economic, agri-
cultural, environment and groundwater preservation benefits, and prevented overexploitation as well. A model
comprised of four modules was applied to a case study of
optimization of water resources for the Pearl River Delta in China, which provided a useful tool for the operation of
reservoirs and freshwater allocation in a saltwater intrusion area (Liu et al. 2010). A water resources allocation model can be
optimized using genetic algorithms to derive near-optimal operating strategies for the water company’s multiple reser-
voir system for different projected rainfall scenarios, and also to test the robustness of drought contingency strategies
for taking the reservoirs down to a lower level under a severe drought condition (Rao et al. 2010). In addition, it
can be optimized by another genetic algorithm or particle swarm optimization algorithm (Vonk et al. 2016). Tang
(1995) proposed a method for solving complex multi-objective large-scale systems with nonlinear, multivariable and multi
constrained conditions to obtain the optimal allocation scheme for annual water resources in the Yellow River Basin.

When studying the optimal allocation of water resources, the grey forecasting model has often been used to predict the
supply and demand of water. The model was put forward by
Chinese scholars in the 1980s and has been improved con-
tinuously in the last 20 years. Wang & Liu (2016) used the
combination of multiple regression and a grey GM (1, 1)
model to predict the market demand for fresh agricultural products, which greatly improved the prediction accuracy
of the model. The method of automatic optimization to deter-
mine weight and the least-squares method were used to
increase the accuracy of grey prediction. The prediction example proved that the improved method was more effective
and perfect (Yang et al. 2011).

In addition to the improvement of the water supply and
demand forecasting method, the solution of multi-objective
problems is becoming more and more perfect. Zeng et al.
(2014) developed a two-stage inexact credibility-constrained
programming (TICP) method to identify water trading effi-
ciency with multiple uncertainties, and then applied the
developed TICP method to water resources allocation man-
agement and planning in the Kaidu-Kongque River Basin
to examine different water resource allocation policies.
Somayeh et al. (2014) developed a non-linear programming
optimization model based on the genetic algorithm (GA),
with an integrated soil/water balance to determine optimal
reservoir release policies and cropping patterns around Dor-
doudzan Dam in southwest Iran. Nouiri (2014) examined the
development of a multi-objective tool called ‘All Water’ that
can optimize water resource management by taking into
account water salinity requirements and hydraulic
constraints.

This study aims to develop a multi-objective planning optimal allocation model for water resources planning and
management based on an improved grey prediction method. The proposed method was applied to water allo-
cation in a case study in Beijing under the conditions of a
multi-objective water supply to obtain an optimal allocation
scheme for Beijing’s water resources in the planning year
2020. This study is important to sustainable planning and
management of the city’s water resources.

METHODS

Study area

Beijing, the capital of China, is located in the Haihe river basin with a semi-arid and sub-humid monsoon climate
(Figure 1). It has been facing challenges of serious water shortages where annual water resources per capita are less
than 200 m\(^3\). With population growth and economic development, the conflict between supply and demand of water resources has become more serious in recent years. Figure 1 shows that the water resources per capita in Beijing are less than 500 m\(^3\), indicating that Beijing is seriously short of water resources.

**Water supply and demand predictions**

Since the grey method (GM) has several advantages, such as the requirement for less sample data, it has been successfully applied in many research areas such as economics, management, meteorology, and medicine, and will be used in this study to predict city water supply and demand.

The GM (1, 1) model represents a typical model variation of the grey prediction method. Although it has been commonly used for prediction, the few limitations of this method hinder its wide application. For instance, as the discrete degree of the data increases, the prediction accuracy of the GM will deteriorate. In addition, the GM (1, 1) model assumes that the original data sequence approximates the exponential rule, while in fact many data series do not obey the index law (Deng 1989; Li & Wang 1998; Deng 2008).

**Improvement of the GM (1, 1) model**

The traditional GM (1, 1) model is formed by a first order differential equation containing a single variable, which is the basis of grey prediction.

Assume the existing data series is:

\[
X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))
\]  

The series generated through primary accumulation is:

\[
X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n))
\]

where

\[
x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \quad k = 1, 2, \ldots, n
\]

Define \(\varepsilon^{(1)}\) as the series generated from the mean value of consecutive neighbors of \(x^{(1)}\):

\[
\varepsilon^{(1)}(k) = \frac{1}{2} (x^{(1)}(k) + x^{(1)}(k - 1)), \quad k = 2, 3, \ldots, n
\]

The following grey differential equation can then be established as:

\[
x^{(0)}(k) + a\varepsilon^{(1)}(k) = b
\]

Denote \(\hat{a} = \begin{pmatrix} a \\ b \end{pmatrix}\), then the least square estimation of parameters of the grey differential equation can be expressed as:

\[
\hat{a} = (B^T B)^{-1} B^T Y
\]
where

\[
B = \begin{bmatrix}
-\frac{[x^{(1)}(1) + x^{(1)}(2)]}{2} & 1 \\
-\frac{[x^{(1)}(2) + x^{(1)}(3)]}{2} & 1 \\
\vdots & \vdots \\
-\frac{[x^{(1)}(n-1) + x^{(1)}(n)]}{2} & 1
\end{bmatrix}, \quad Y = \begin{bmatrix}
x^{(0)}(2) \\
x^{(0)}(3) \\
\vdots \\
x^{(0)}(n)
\end{bmatrix}
\]

(7)

The albino equation of the grey differential equation can be expressed as:

\[
\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b
\]

(8)

Then the time response series of the GM (1, 1) grey differential equation is:

\[
x^{(1)}(k) = ce^{-a(k-1)} + \frac{b}{a}, \quad k = 1, 2, \cdots, n
\]

(9)

One then obtains the prediction equation:

\[
x^{(0)}(k) = x^{(1)}(k + 1) - x^{(1)}(k)
\]

\[
= (1 - e^a)\left[x^{(1)}(1) - \frac{b}{a}\right]e^{-a(k-1)}, \quad k = 1, 2, \cdots, n
\]

(10)

From the introduction above, it can be seen that the GM (1, 1) model relies on the exponential fitting of the original data, which is essentially a biased exponential model. The prediction precision of the GM (1, 1) model depends on two factors: (1) the original sequence, (2) the values of the unknown parameters \(a\) and \(b\), which depend on the forming structure of the background value \(z^{(1)}\). Based on the two impact factors, the moving average process is applied to smooth the original data series.

Assume the original series are: \(X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))\), and the new series are \(X^{(0)}\). The two end points of the new series can be calculated with the following equations:

\[
x^{(0)}(1) = \frac{3x^{(0)}(1) + x^{(0)}(2)}{4}
\]

(11)

\[
x^{(0)}(n) = \frac{x^{(0)}(n-1) + 3x^{(0)}(n)}{4}
\]

(12)

For each intermediate data point, the following equation can be used for the calculation:

\[
x^{(0)}(k) = \frac{3x^{(0)}(k-1) + x^{(0)}(k)}{4} + 2x^{(0)}(k) + x^{(0)}(k + 1)
\]

(13)

Equations (10)–(12) can be used to pre-process the original series to generate a new sequence \(X^{(0)}\), and then make a cumulative generation sequence

\[
X^{(1)}(i) = (x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n))
\]

(14)

where

\[
x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \quad k = 1, 2, \cdots, n
\]

(15)

In the traditional GM (1,1) model, for computational convenience, the background value is assumed to be generated from the mean value of consecutive neighbors. In other words, the background value of the traditional GM (1,1) model is computed as: \(z^{(1)}(k) = \mu x^{(1)}(k) + (1-\mu)x^{(1)}(k-1), \quad k = 2,3,\ldots,n, \mu = 0.5\). However, it is stated that this cannot account for the highest prediction precision, when \(\mu = 0.5\). In this paper, automatic optimization methods were applied to select the most appropriate \(\mu\). According to the form of the general solution of the original equation (Equation (5)), the values of the unknown \(\mu\) can be obtained by applying Equations (6) and (7), respectively. Here, parameters \(a\) and \(b\) are changing with \(\mu\), and Equation (7) is revised as (16):

\[
B = \begin{bmatrix}
z^{(1)}(2) & 1 \\
z^{(1)}(3) & 1 \\
\vdots & \vdots \\
z^{(1)}(n) & 1
\end{bmatrix}, \quad Y = \begin{bmatrix}
x^{(0)}(2) \\
x^{(0)}(3) \\
\vdots \\
x^{(0)}(n)
\end{bmatrix}
\]

(16)

According to the least squares method, the square sum of the difference of the estimated value \(\hat{x}^{(1)}(k)\) and the analog value \(x^{(1)}(k)\) of the original data should represent a minimum. Thus, an additional constraint is specified as follows:

\[
S = \sum_{k=1}^{n} (\hat{x}^{(1)}(k) - x^{(1)}(k))^2
\]

(17)
To find the minimum value of $S$, let $\frac{dS}{dc} = 0$, and one obtains:

$$c = \frac{\sum_{k=1}^{n} \left( x^{(1)}(k) - \frac{b}{a} \right) e^{-a(k-1)}}{\sum_{k=1}^{n} e^{-2a(k-1)}}$$

(18)

Substituting Equation (18) into Equation (9), $x^{(1)}(k)$ can then be formulated as:

$$x^{(1)}(k) = \frac{\sum_{k=1}^{n} \left( x^{(1)}(k) - \frac{b}{a} \right) e^{-a(k-1)}}{\sum_{k=1}^{n} e^{-2a(k-1)}} e^{-a(k-1)} + \frac{b}{a}$$

$k = 1, 2, \ldots, n$

(19)

Then $x^{(0)}(k)$ can be expressed as:

$$x^{(0)}(k) = x^{(1)}(k + 1) - \hat{x}^{(1)}(k)$$

(20)

In order to obtain the best $\mu$ value, a calculation process using Matlab programming is shown with the support of a flow chart in Figure 2.

In this case, the calculation process starts from the assumption $\mu = 0$, then uses Equations (6)–(8) to calculate $a$, $b$, and Equations (13)–(15) to obtain the parameters $c$ and $\hat{x}^{(1)}(k)$. Afterwards, Equation (13) is used to calculate the sum of squares of deviations $S$ under this weight $\mu$ and a small amount of $\Delta \mu$ of greater than zero is added on this basis (that is $\mu \rightarrow \mu + \Delta \mu$) to repeat the process until $\mu = 1$. In this process, a comparison of the sum of squares of deviations of predicted value and the actual value under different weights would be gained. The fittest $\mu$ will be selected as the best weight under the case of the sum of squares of deviations (namely $S$) to get to the minimum.

Data simulation analysis

To verify the improvement effect, the authors used a simulation series to test both the traditional model and the improved model, and then analyzed the simulation and prediction precision. Here, $x_i^{(0)}(k + 1) = e^{-mk}$ was used as an example for the simulation analysis, where $k = 0, 1, 2, 3, 4$ and 5. Thus, $x_i^{(0)} = \{x_i^{(0)}(1), x_i^{(0)}(2), x_i^{(0)}(3), x_i^{(0)}(4), x_i^{(0)}(5)\}$.

$x_i^{(0)}(6)$] with $-m$ equal to 0.5 and 1.0. The researchers defined the traditional and improved GM (1, 1) models as mod1 and mod2, respectively. Both models were then applied to make predictions and subsequently compared with corresponding sequence values of $x_i^{(0)}$ (Table 1, Figure 5).

Prediction results

In order to compare prediction effects, this paper established traditional and improved grey prediction models to
forecast environmental water use for data of 2016 based on the data on environmental water from 2000 to 2015. Table 2 presents the results.

Table 2 shows that the improved GM (1, 1) model has better prediction effects than the traditional GM (1, 1) model. These results show that improvements to the traditional GM (1, 1) model are effective. The team then applied the improved grey prediction GM (1, 1) model to predict Beijing’s water resources and water demand for 2020 (Table 3).

Multi-objective optimization allocation of water resources

Optimization model

The goal of water allocation is not to optimize a particular aspect or object, but to pursue the overall efficiency of the

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Optimization model

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Figure 3 | Simulation precision comparison of two models. Diagram (a) corresponds to the case of $-m = 0.5$ and diagram (b) corresponds to the case of $-m = 1$. The longitudinal axis of the left side corresponds to mod1 and the right one corresponds to mod2.

Table 1 | Simulation precision comparison of two models

<table>
<thead>
<tr>
<th>$-m$</th>
<th>mod1</th>
<th></th>
<th></th>
<th>mod2</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sim. val</td>
<td>Rel. err</td>
<td>Sim. val</td>
<td>Rel. err</td>
<td>Sim. val</td>
<td>Rel. err</td>
</tr>
<tr>
<td>$m$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>1.6063</td>
<td>2.5689</td>
<td>1.6506</td>
<td>0.1139</td>
<td>2.4045</td>
<td>11.5345</td>
</tr>
<tr>
<td>1.0</td>
<td>2.6216</td>
<td>3.5559</td>
<td>2.7203</td>
<td>0.0747</td>
<td>6.0592</td>
<td>17.9966</td>
</tr>
<tr>
<td></td>
<td>4.2787</td>
<td>4.5306</td>
<td>4.4834</td>
<td>0.0378</td>
<td>15.2689</td>
<td>23.9805</td>
</tr>
<tr>
<td></td>
<td>6.9830</td>
<td>5.4949</td>
<td>7.3891</td>
<td>0.0016</td>
<td>38.4767</td>
<td>29.5276</td>
</tr>
<tr>
<td></td>
<td>11.3966</td>
<td>6.4511</td>
<td>12.1781</td>
<td>0.0365</td>
<td>96.9588</td>
<td>34.6697</td>
</tr>
<tr>
<td></td>
<td>Average error % 4.5203 0.0529</td>
<td>Average error % 23.5436 0.3450</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2 | Prediction effect comparison between the traditional model and the improved model (environmental water, 10^3 m³)

<table>
<thead>
<tr>
<th>Year</th>
<th>Actual value</th>
<th>Predicted value of traditional model</th>
<th>Relative error %</th>
<th>Predicted value of improved model</th>
<th>Relative error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>2016</td>
<td>11.1</td>
<td>11.32</td>
<td>1.98</td>
<td>11.20</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table 3 | Predicted Beijing water supply and demand in 2020 (10^3 m³)

<table>
<thead>
<tr>
<th>Year</th>
<th>Surface water</th>
<th>Ground-water</th>
<th>Reclaimed water</th>
<th>Transferred water</th>
</tr>
</thead>
<tbody>
<tr>
<td>2020</td>
<td>3.4</td>
<td>16.1</td>
<td>14.7</td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>Domestic water</th>
<th>Agricultural water</th>
<th>Industrial water</th>
<th>Environmental water</th>
</tr>
</thead>
<tbody>
<tr>
<td>2020</td>
<td>19.8</td>
<td>5.7</td>
<td>3.4</td>
<td>16.0</td>
</tr>
</tbody>
</table>
system. Therefore, an effective water allocation model must consider multiple objectives in an optimization setting. Multi-objective optimization can be defined as the problem of identifying a set of decision-relevant variables, which satisfy constraints and optimize a vector function whose elements represent objective functions (Xevi & Khan 2005; Chen et al. 2007). A fundamental multi-objective model for allocating water resources optimally is expressed as:

\[ Z = \text{opt}\{f_1(x), f_2(x), \ldots, f_p(x)\} \]

\[ s.t. \begin{cases} g_i(x) \leq b_i, & i = 1, 2, \ldots, m \\ x_i \geq 0, & j = 1, 2, \ldots, n \end{cases} \]

where \( x \) represents the decision variable, \( f_p(x) \) represents the objective function of the independent p-th vector, \( g_i(x) \) represents a set of constraints, and \( b_i \) represents a set of constant vectors.

Based on maximizing the overall efficiency of economic, social, and environmental benefits, three goals were chosen as the target of the optimization model.

**Objective functions**

(i) Economic benefit: Suppose that the maximum net benefit of the regional water supply is expressed as:

\[ f_1(x) = \max \sum_{i=1}^{l} \sum_{j=1}^{l} (b_{ij} - c_{ij})x_{ij}\alpha_i\beta_j \]

where \( x_{ij} \) represents decision-relevant variables that stand for water volume distributed from independent water source \( i \) (unit: \( 10^6 \) m\(^3\)) to user \( j \), \( b_{ij} \) represents a unit water supply’s benefit coefficients to user \( j \) from an independent water source \( i \), \( c_{ij} \) represents the water supply’s cost coefficient to user \( j \) from an independent water source \( i \), \( \alpha_i \) represents a water supply’s order coefficients that are relevant to the user type, and \( \beta_j \) represents the fair coefficients of the water supply, which reflect a user’s priority in relation to other users according to the degree of importance of water use. The total number of water supplies \( l = 4 \), and the number of users \( j = 4 \).

(ii) Social benefit: The social objective is to minimize regional water shortages, which reflect the principle of fairness in water resources allocation. The objective function is expressed as:

\[ f_2(x) = \min \sum_{j=1}^{l} \left[ D_j - \sum_{i=1}^{l} x_{ij} \right] \]  

where \( D_j \) represented the water demand volume of user \( j \) (yuan/m\(^3\)).

(iii) Environmental benefit: The environmental objective is to minimize pollutant discharge on the basis of guaranteed ecological water demand. The objective function is expressed as:

\[ f_3(x) = \min \sum_{j=1}^{l} \sum_{i=1}^{l} p_i x_{ij} \]

where \( p_i \) represents the waste water discharge coefficient of user \( j \).

**Constraints**

(i) Constraint of water supply capacity: In order to maintain the balance of supply and demand, the amount of available water from source \( i \) should be less than the water supply it could afford, as described in Equation (25).

\[ \sum_{j=1}^{l} x_{ij} \leq W_i \]  

where \( W_i \) represents the upper limit of the water supply from the independent water source \( i \).

(ii) Constraint of supply-demand range: The amount of water obtained from sources for \( j \) users should be between the users’ upper and lower water demands:

\[ L_j \leq \sum_{i=1}^{l} x_{ij} \leq H_j \]  

where \( L_j \) and \( H_j \) are the upper and lower limit for user \( j \)’s water demands, respectively. The upper and lower limits for water-supply requirements were taken as a forecast of domestic water demand according to the characteristics of domestic water use. The upper and lower environmental water needs were also used to forecast environmental water demand.
(iii) Non-negative constraints:

\[ x_{ij} \geq 0 \]  
(27)

Here every decision variable \( x_{ij} \) should be greater than zero.

A fundamental multi-objective model for allocating water resources optimally can be formed by combining the above objective functions and constraints.

**Model development**

Based on the above analysis, the number of water-supply sources \( I = 4 \) and the number of water-users \( J = 4 \), the optimal water resources allocation model for Beijing has 16 decision-related variables and 10 constraints in total. After the estimation of model parameters, the optimized multi-objective allocation model for Beijing's water resources can be expressed as follows.

**Decision variables:**

Based on the optimal allocation model and the actual situation of Beijing, the decision-related variables are shown (Table 4).

**Economic objectives:**

\[
\begin{align*}
mf(1) &= -(11984 \cdot x(1))/25 - (417 \cdot x(10))/50 \\
&-(139 \cdot x(11))/25 - (139 \cdot x(12))/50 \\
&-(2999 \cdot x(13))/25 - (8997 \cdot x(14))/100 \\
&-(2999 \cdot x(15))/50 - (2999 \cdot x(16))/100 \\
&-(8988 \cdot x(2))/25 - (5992 \cdot x(3))/25 \\
&-(2996 \cdot x(4))/25 - (11982 \cdot x(5))/25 \\
&-(17973 \cdot x(6))/50 - (5991 \cdot x(7))/25 \\
&-(5991 \cdot x(8))/50 - (278 \cdot x(9))/25 
\end{align*}
\]  
(28)

**Social objectives:**

\[
\begin{align*}
mf(2) &= 44.9 - \sum_{i=1}^{16} x_i 
\end{align*}
\]  
(29)

**Table 4 | Model decision variables**

<table>
<thead>
<tr>
<th>Water type</th>
<th>Domestic</th>
<th>Industry</th>
<th>Agriculture</th>
<th>Environment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Groundwater</td>
<td>( x_1 )</td>
<td>( x_2 )</td>
<td>( x_3 )</td>
<td>( x_4 )</td>
</tr>
<tr>
<td>Surface water</td>
<td>( x_5 )</td>
<td>( x_6 )</td>
<td>( x_7 )</td>
<td>( x_8 )</td>
</tr>
<tr>
<td>Reclaimed water</td>
<td>( x_9 )</td>
<td>( x_{10} )</td>
<td>( x_{11} )</td>
<td>( x_{12} )</td>
</tr>
<tr>
<td>Transferred water</td>
<td>( x_{13} )</td>
<td>( x_{14} )</td>
<td>( x_{15} )</td>
<td>( x_{16} )</td>
</tr>
</tbody>
</table>

**Environmental objectives:**

\[
\begin{align*}
mf(3) &= (9 \cdot x(1))/10 + (4 \cdot x(10))/5 + (5 \cdot x(11))/10 \\
&+(9 \cdot x(12))/10 + (9 \cdot x(13))/10 + (4 \cdot x(14))/5 \\
&+(5 \cdot x(15))/10 + (9 \cdot x(16))/10 + (4 \cdot x(2))/5 \\
&+(5 \cdot x(5))/10 + (9 \cdot x(4))/10 + (9 \cdot x(5))/10 \\
&+(4 \cdot x(6))/5 + (5 \cdot x(7))/10 + (9 \cdot x(8))/10 \\
&+(9 \cdot x(9))/10 
\end{align*}
\]  
(30)

**Water supply constraints:**

\[
\begin{align*}
x_1 + x_2 + x_3 + x_4 &\leq 16.1 \\
x_5 + x_6 + x_7 + x_8 &\leq 3.4 \\
x_9 + x_{10} + x_{11} + x_{12} &\leq 14.7 \\
x_{13} + x_{14} + x_{15} + x_{16} &\leq 10 
\end{align*}
\]  
(31)

**Water demand constraints:**

\[
\begin{align*}
x_3 + x_4 + x_9 + x_{15} &\leq 19.8 \\
2.72 &\leq x_2 + x_5 + x_{10} + x_{14} \leq 3.4 \\
3.99 &\leq x_3 + x_7 + x_{11} + x_{15} \leq 5.7 \\
x_4 + x_9 + x_{12} + x_{16} &\leq 16.0 
\end{align*}
\]  
(32)

**Non-negative constraints:**

\[
x_i \geq 0, i = 1, 2, \ldots, 16 
\]  
(33)

**RESULTS AND DISCUSSION**

**Model results**

The planning year is 2020. The model parameters are estimated with the Matlab optimization toolbox `fgoalattain` function. This paper used different initial estimated values for optimizing water characteristics according to different regions of water users, water sources, and several other factors.

The model allocating Beijing's water resources optimally should yield the best integrated effects, which represent the greatest net benefits for Beijing, considering the total minimum amount of water demanded and the total minimum amount of waste water produced. This model establishes the lower and upper water demand to ensure minimum water demand while not exceeding
maximum water demand to allocate water resources to achieve maximum benefits. Given their different initial values $x_0$, the researchers determined the weight of the objective functions by weighting coefficients from the fgoalattain function. Table 5 and Figure 4 include the final configuration results.

**Discussion**

It can be seen that the traditional grey prediction model has its limitations when the data do not conform to the exponential distribution from the prediction results. There are some random errors in each original sequence. The traditional model does not preprocess the original sequence and directly accumulates the original sequence, which cannot effectively eliminate the random error. In addition, the traditional model cannot theoretically explain the best prediction accuracy when $\mu$ is 0.5. However, the improved model preprocessed the original sequence and adopted an automatic optimization method for $\mu$ to improve the accuracy of prediction. In fact, many sequences do not obey such an exponential law. If the traditional model is used, not only the simulation error will increase, but also the prediction effect will be affected to a great extent. The automatic optimization methods and the moving average process were used to improve the traditional model. The automatic optimization methods changed the assumption of $\mu = 0.5$ in the traditional model. It is of great significance to use the method of automatic optimization to determine the value of $\mu$ with the highest prediction accuracy to improve prediction accuracy in practice. The moving average process strengthened the general trend of the original series, avoided

**Table 5** Optimized water resources allocation for Beijing (Unit: 10^8 m³)

<table>
<thead>
<tr>
<th>Item</th>
<th>Domestic</th>
<th>Agriculture</th>
<th>Industry</th>
<th>Environment</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface water</td>
<td>0.9911</td>
<td>0.7791</td>
<td>0.6387</td>
<td>0.9911</td>
<td>3.4</td>
</tr>
<tr>
<td>Groundwater</td>
<td>9.0236</td>
<td>1.0000</td>
<td>2.0002</td>
<td>4.0762</td>
<td>16.1</td>
</tr>
<tr>
<td>Reclaimed water</td>
<td>3.4932</td>
<td>3.0001</td>
<td>0.0023</td>
<td>8.2043</td>
<td>14.7</td>
</tr>
<tr>
<td>Transferred water</td>
<td>5.4940</td>
<td>0.9001</td>
<td>0.7008</td>
<td>2.6863</td>
<td>10</td>
</tr>
<tr>
<td>Total</td>
<td>19.0019</td>
<td>5.6793</td>
<td>3.3420</td>
<td>16.1768</td>
<td>44.2</td>
</tr>
</tbody>
</table>

**Figure 4** The proportion of different water-supply resource and water-use sectors in the optimal allocation results.
excessive fluctuations in the value and reduced the impact of extreme values. The improved grey forecasting model was used to forecast the supply and demand of water in Beijing, which effectively improved the accuracy of forecasting and increased the reliability and credibility of forecasting results, which laid the foundation for the next multi-objective water resources allocation.

Based on Table 5, the industrial water and agricultural water allocations are predicted to be $3.34 \times 10^8$ m$^3$ and $5.68 \times 10^8$ m$^3$, respectively. Because of recent adjustments in Beijing’s industrial structure, which has eliminated many water-consuming industrial enterprises and raised industrial water use efficiency, the industrial water allocation will satisfy the water demand of industry. The agricultural water allocation will basically achieve a balance in supply and demand because more than 30 acres of paddy fields in the south-eastern region of Beijing have been converted into upland fields. Moreover, the effective utilization coefficient of irrigation water in Beijing was 0.697 in 2016, while the national average was less than 0.5.

Efforts to improve industrial and agricultural systems in Beijing have sharply reduced water use in these sectors. Meanwhile, industrial prosperity has increased domestic water use in the city, leading to a significant domestic water shortage. The water shortage rate has risen to 9.2% since 1999, when Beijing entered a continuous dry season. A sharp decrease in natural runoff has contributed to the current shortage of water resources in Beijing, while, at the same time, the rapid population growth and urban development have intensified domestic water demand. Although a south-north water transfer project can play a significant role in addressing the water shortage in Beijing, it is not enough to solve the problem fundamentally. The environmental water demand has gradually increased in recent years, which is not as significant as the domestic water shortage due to the use of reclaimed water, lake water, and landscape water.

The use of reclaimed water for sanitation combined with the south-north water transfer project can help alleviate the environmental pressure on Beijing’s water resources. Measures must be taken to address the shortage of water resources in Beijing. Scientific planning can help to establish a reasonable policy and technical innovation is necessary to promote water-saving measures.

CONCLUSIONS

This study developed an optimal model for water resources allocation using the multi-objective planning approach based on the improved grey prediction method. An innovative method is proposed to address shortcomings of the traditional grey prediction model by using the moving-average technique and improving background values. Through simulation analysis, it has been shown that the model precision consistently exceeds 95%, which is better than that of the traditional grey prediction model.

The optimal allocation of water resources has attracted increased attention as an effective method to manage regional water demand. The multi-objective planning method is used to construct an optimal allocation model to produce an optimal water resources allocation scheme for Beijing for the planning year 2020. The planning scheme shows that industrial and agricultural water demands can be met, while domestic and environmental water resources will experience shortages. Thus, the rational allocation of water for domestic use and ecology must be strengthened to meet the water shortage challenge. This research takes the water allocation in the typical arid city of Beijing as a case study, but the proposed method can also be used for allocating water resources among other regions in an optimal way.

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CONFLICTS OF INTEREST

The authors declare no conflict of interest.

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