A formula for the settling velocity of cohesive sediment flocs in water

Zhongfan Zhu

ABSTRACT

A simple formula is developed to relate the size and settling velocity of cohesive sediment flocs in both the viscous and inertial settling ranges. This formula maintains the same basic structure as the existing formula but is amended to incorporate the fact that the flocculated sediment has an internal fractal architecture and is composed of different-sized primary particles. The input parameters needed for calculating the settling velocity include the median size and size distribution of the primary particles, the fractal dimension of the floc, the density of the sediment, and two calibrated coefficients that incorporate the effects of floc shape, permeability, and flow separation on drag. The proposed formula is compared with four data sets of settling velocity–floc size collected from the published literature, and a good agreement between the model and these data can be found.

Key words | cohesive sediment, flocs, formula, sediment transport, settling velocity

Zhongfan Zhu
College of Water Sciences, Beijing Key Laboratory
of Urban Hydrological Cycle and Sponge City
Technology,
Beijing Normal University,
Beijing 100875,
China
E-mail: *zhuzhongfan1985@bnu.edu.cn*

INTRODUCTION

Different from non-cohesive sediments such as sand and gravel, cohesive sediment is a mixture of water and fine-grained sediments (such as silt and clay) and organic matter of diverse nature (Winterwerp *et al.* 2006; Son & Hsu 2008). Fine-grained sediment has obvious cohesive characteristics and can undergo flocculation effects due to evident electrochemical and biological–chemical attractions on the surface of particles (Stone & Krishnappan 2003; Mietta *et al.* 2009; Son & Hsu 2009). When these fine-grained sediments move into rivers, reservoirs, lakes and estuarine and coastal areas, where eddy motions in a turbulent flow can induce particles to collide, they can flocculate into flocs of different sizes via the binding of primary particles (Winterwerp 1998; Shen & Maa 2015). However, the turbulent shear motion may also cause the breakup of some porous and fragile flocs, leading to some small-sized flocs and/or primary particles (Dyer 1989; Winterwerp 1998). The transport of cohesive sediment plays an important role in some geophysical processes, such as

morphodynamic change and ecosystem function variation related to the water quality in rivers, reservoirs, lakes and estuarine and coastal waters (Kumar *et al.* 2010; Maggi 2013).

Cohesive sediment (or suspended solid matter) transport mainly occurs via horizontal advection and vertical gravitational sedimentation. Many works have focused on the advective motion of cohesive sediment (or suspended solid matter) (e.g. van Leussen 1991; Xu *et al.* 2008). However, the settling of cohesive sediment plays a crucial role in determining the vertical flux of sediment transport (Dyer 1989; Camenen 2007; Song *et al.* 2008; Maggi 2013). Many formulae for predicting the settling velocity of sands, gravels and cohesive sediment flocs in a quiescent or low-turbulence water column have been proposed (e.g. Watson 1969; Hallermeier 1981; Cheng 1997; Winterwerp 1998; Ferguson & Church 2004; Khelifa & Hill 2006; Strom & Keyvani 2011; Vahedi & Gorczyca 2012; Maggi 2013). The governing terms needed in the settling velocity calculation for cohesive sediment flocs include the submerged gravity

of the floc, $F_g = (\rho_f - \rho_w)Vg$, and the resistant drag exerted on the floc in water, $F_r = \frac{1}{2}C_d A \rho_w w_s^2$, where ρ_f and ρ_w are the densities of the floc and the water, respectively; V and A are the volume and the area projected normal to the direction of motion of the floc, respectively; C_d is the drag coefficient; g is the gravitational acceleration; and w_s is the terminal settling velocity of the floc. By considering the force balance between the submerged gravity and resistant drag terms (the equilibrium settling condition) and assuming that the floc can be treated as a sphere with modifications for floc shape and permeability, the expression for w_s can be written as:

$$w_s = \left(\frac{4}{3}\frac{\Delta\rho g d_f}{\alpha C_d \rho_w}\right)^{\frac{1}{2}} \tag{1}$$

Here, d_f is the floc diameter, $\Delta\rho = \rho_f - \rho_w$ is the effective density (excess density) of the floc (kg/m^3), and α is a non-dimensional factor that accounts for the deviation in the relationship among the diameter, projected area, volume and settling velocity of a floc from that of a smooth solid sphere due to floc shape and floc permeability. For a smooth solid sphere, $\alpha = 1$.

At low Reynolds numbers (Re $= w_s d_f / \nu$, where ν is the kinematic viscosity of the water (m^2/s)), Re < 1, the force balance between the creeping-flow resistant drag ($F_r = 3\pi\alpha\rho\nu d_f w_s$, which is an analytical solution to the governing Navier–Stokes equation solved by Stokes (1850)) and submerged gravity of a floc leads to the following expression of the floc settling velocity, i.e., the well-known Stokes' law:

$$w_s = \frac{1}{18}\frac{\Delta\rho g}{\alpha\nu\rho_w}d_f^2 \tag{2}$$

where the drag coefficient becomes $C_d = 24/\text{Re}$. The applicability of Stokes' law is strongly constrained to the assumption Re < 1, thus making it urgent to construct a more general settling velocity formula for sediment under moderate or high Reynolds number conditions, which are very common in geophysical systems (Winterwerp 1998; Khelifa & Hill 2006; Song et al. 2008).

Equation (1) can be used to calculate w_s directly when all of the quantities, including C_d, are known. Although there have been many works regarding C_d, an exact description of C_d does not exist at present, and only some empirical expressions relating C_d and Re are available for various ranges of Re and for various particle characteristics (Khelifa & Hill 2006; Song et al. 2008; Maggi 2013). Among these, the following expression for the drag coefficient given by Schiller and Naumann has commonly been presented in some works to match most empirical data (i.e., the modified Stokes' law) (Kelbaliyer 2011):

$$C_d = \frac{24}{\text{Re}}(1 + 0.15\text{Re}^{0.687}) \tag{3}$$

When Re ≤ 200, Equation (3) gives a good approximation of the real drag coefficient on spherical particles. However, it should be noted that the settling velocity formula obtained by substituting Equation (3) into Equation (1) may be not applicable for geophysical systems with Re higher than 200, and an iterative numerical technique is needed to solve the function of w_s since the Reynolds number also contains the settling velocity term.

Furthermore, to the best of our knowledge, at present, all of the developed models for the settling velocity of cohesive sediment flocs have been based on an assumption that the sediment floc is composed of primary particles with a uniform size. With this assumption, sediment flocs could be further assumed to be treated as a fractal object, and fractal geometry could be conveniently adopted to characterize their physical properties, as has been widely acknowledged in the cohesive sediment research field. However, as Vahedi & Gorczyca (2011, 2012) showed, this may not be true because flocs should be a mixture of a variety of primary particle sizes in actual flocculation systems. Neglecting the effect of the multi-size property of primary particles might lead to unexpected error in predicting the setting velocity of sediment floc in a quiescent water column.

Regarding the weakness of some proposed settling velocity formulae for flocs in that there is a constraint to the Reynolds number and an iterative numerical technique is needed to solve the function of w_s, some researchers have attempted to eliminate the need to use C_d in models for the settling velocity calculation by non-dimensionalizing both terms of the settling velocity and particle size and constructing a direct relationship between such non-dimensional terms (Hallermeier 1981; Dietrich 1982; Cheng 1997; Ferguson

& Church 2004). For example, Dietrich (1982) introduced two such non-dimensional parameters, $w_{s*} = w_s^3 \rho_w / (\Delta \rho g v)$ and $d_{f*} = (\Delta \rho g d_f^3)/(\rho_w v^2)$. By virtue of a similar non-dimensionalization method, Ferguson & Church (2004) constructed a simple and explicit relationship between the settling velocity and particle size in viscous, transitional and inertial regimes, and the proposed model had good agreement with measured experimental data of natural sands.

The work of Ferguson & Church (2004) motivates us to explore whether there exists a more general and applicable formula for estimating the settling velocity of sediment flocs. This study attempts to construct such a settling velocity formula. The structure of this paper is arranged as follows. All of the details of the model formulation are introduced in the section 'Model formulation'. A simple comparison of the developed model and collected settling velocity–floc size data from *in situ* field measurements and laboratory settling-column data are presented in the section 'Comparison with data', and finally, the last section presents concluding remarks.

MODEL FORMULATION

In the work of Ferguson & Church (2004), $\sqrt{\Delta \rho g d_f / \rho_w}$ was chosen as the velocity scale. With this, the non-dimensional forms of the settling velocity, w_s, and the floc diameter, d_f, can be written as $w_s' = \dfrac{w_s}{\sqrt{\Delta \rho g d_f / \rho_w}}$ and $d_f' = \dfrac{\sqrt{\Delta \rho g d_f / \rho_w} d_f}{v}$. Introducing such expressions into Equation (1) yields the following expression: $w_s' = \left(\dfrac{4}{3 \alpha C_d} \right)^{\frac{1}{2}}$.

Under low Reynolds number conditions ($Re = w_s d_f / v < 1$), Stokes' law (Equation (2)) can be expressed in the following non-dimensional form: $w_s' = \dfrac{d_f'}{18 \alpha}$. However, for $Re > 1,000$, some studies have shown that C_d is a constant and that its value depends on particle shape and roundness (Dietrich 1982; Cheng 1997; Camenen 2007). Introducing this constant as C can produce the following form of the settling velocity: $w_s' = \left(\dfrac{4}{3 \alpha C} \right)^{\frac{1}{2}}$.

Any proposed formulae for predicting the settling velocity of sediment flocs must satisfy the abovementioned particular relation under very low Reynolds number

conditions and another abovementioned relation under very high Reynolds number conditions (Maggi 2013). Ferguson & Church (2004) suggested the following settling-velocity relation to satisfy the above characteristics: $\dfrac{1}{w_s'} = \dfrac{c_1}{d_f'} + c_2$. For $Re = w_s' d_f' / v < 1$, the first term of the right-hand side of this equation dominates, which yields $c_1 = 18 \alpha$, whereas for large Re conditions, the second term of the right-hand side of the equation should be dominant, leading to $c_2 = \sqrt{3 \alpha C / 4}$. Recovering the dimensional form of this equation leads to the settling velocity formula presented in Ferguson & Church (2004):

$$w_s = \frac{\Delta \rho g d_f^2}{c_1 v \rho_w + c_2 \sqrt{\Delta \rho \rho_w g d_f^3}} \tag{4}$$

In the work of Ferguson & Church (2004), Equation (4) was tested against settling column experimental data, and good agreement was presented for a range of particle shapes in the viscous, transitional, and inertial regimes.

When considering that cohesive sediment floc is a fractal object and that it is formed by different-sized primary particles, which is more realistic in natural water bodies, the effective density of the floc can be expressed as

$$\Delta \rho = \rho_f - \rho_w = (\rho_s - \rho_w) \frac{\sum_{i=1}^{k} d_{pi}^3}{d_f^3} \tag{5}$$

where ρ_s is the density of primary particles, d_{pi} represents the diameter of the i-th primary particle forming the floc, and k is the number of primary particles forming the floc. Regarding the floc diameter d_f, if a floc of diameter d_f is assumed to be composed of k primary mono-sized particles of diameter d_p, then the floc diameter d_f can be expressed as a function of the number of primary particles, k, and the fractal dimension of the flocs, D_f, as follows (Jiang & Logan 1991): $d_f = d_p k^{\frac{1}{D_f}}$. In the case that the floc is composed of k primary multi-sized particles, we could extend this relation to the following expression, as presented in some additional studies (Khelifa & Hill 2006): $d_f = \left(\sum_{i=1}^{k} d_{pi}^{D_f} \right)^{1/D_f}$. Substituting this mathematical relation into Equation (5) could yield

$\Delta\rho = \rho_f - \rho_w = (\rho_s - \rho_w)k^{(D_f-3)/D_f}\phi$,   where $\phi = m_3/m_F^{3/D_f}$, $m_3 = \sum_{i=1}^{k} d_{pi}^3/k$ and $m_F = \sum_{i=1}^{k} d_{pi}^{D_f}/k$; the term ϕ represents the effect of the size distribution of primary particles forming natural flocs, whereas the term $k^{(D_f-3)/D_f}$ denotes the effect of the fractal dimension and floc size. An explanation of the internal fractal architecture of the flocs and the fractal dimension of the floc D_f is as follows. Fractal theory describes the geometry of many natural structures that show a rough or fragmented geometric shape that can be split into parts, each of which is a reduced-size copy of the whole. Many studies have shown that flocs could be considered as self-similar fractal entities. According to fractal theory, the structure of a fractal entity is considered to follow a power-law behaviour. The fractal dimension is the value of the power to indicate the structure of a fractal entity and the number of primary particles in a fractal entity as follows: $k = \left(\dfrac{d_f}{d_p}\right)^{D_f}$, as mentioned above. For the flocs, D_f is defined to be a measure of how the primary particles fill the floc, and it can range from 1 to 3, where larger values indicate a highly compact object ($D_f = 3$ implies the floc is a highly compact sphere).

If we substitute this form of the effective density of the floc into the original model of Ferguson & Church (2004) (i.e., Equation (4)), we could obtain a new formulation for estimating the settling velocity of natural sediment flocs in quiescent water bodies as follows:

$$w_s = \frac{(\rho_s - \rho_w)gd_f^{D_f-1}\phi}{c_1\nu d_{50}^{D_f-3}\rho_w + c_2\sqrt{(\rho_s - \rho_w)\rho_w gd_f^{D_f}d_{50}^{D_f-3}\phi}} \quad (6)$$

where d_{50} is the median of the size distribution of primary particles for simplicity.

Equation (6) maintains the same basic structure as that in Ferguson & Church (2004), which should be applicable over the viscous, transitional, and inertial settling ranges but is amended to incorporate the fact that the flocculated sediment has an internal fractal architecture, characterized by D_f, and is composed of different-sized primary particles, characterized by ϕ. On the right-hand side of Equation (6), the first term of the denominator represents the effect of flow resistance over the surface of the falling sediment floc, whereas the second term represents the effect of flow separation around the falling floc. For the case of mono-sized particles and a solid Euclidian particle for the floc, $\phi = 1$ and $D_f = 3$; thus, Equation (6) can be reduced to Stokes' law (Equation (2)) when $c_2 = 0$.

COMPARISON WITH DATA

To examine the ability of the proposed model (Equation (6)) to estimate the settling velocity of cohesive sediment flocs, this study attempts to compare the model with four existing data sets of the settling velocity collected from the literature: two are from *in situ* field observations, and the others are from laboratory settling column measurements, introduced as follows.

Table 1 summarizes the information for these four data sets. The second and third columns show sediment conditions and experimental conditions, respectively. The number of data points in each data set is given in the last column.

Table 1 | Summary of four floc settling-velocity data sets collected from the literature

References	Sediment condition	Experimental condition	Data points
Medema *et al.* (1998)	Oocysts and cysts, fresh water	Laboratory settling column	10
Smith & Friedrichs (2011)	Suspended sediment flocs, San Francisco Bay, US west coast, USA	Field observation, settling column, camera and image analysis system	19
Ahn (2012)	Suspended sediments with maximum entropy classification group 1, Southern California, USA	Field observation, settling tube, LISST-ST	12
Zhao *et al.* (2018)	Sediment, Guan Qiao Lake, Wuhan, China	Laboratory experiment, sediment settling velocity measurement system	64

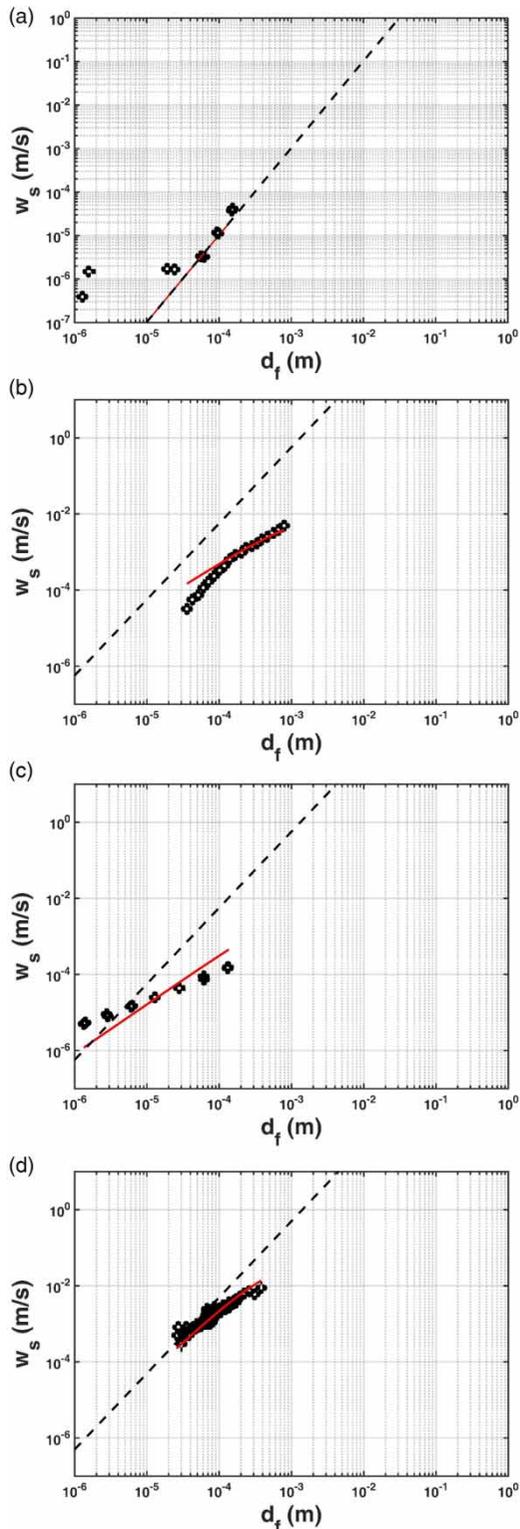LISST: laser in situ scattering and transmissometry.

Figure 1 | Comparison of the proposed model (Equation (6)) with four collected data sets: (a) Medema *et al.* (1998), (b) Smith & Friedrichs (2011), (c) Ahn (2012) and (d) Zhao *et al.* (2018), respectively. Here, the dashed line denotes Stokes' law, the circles represent the settling velocity–floc size data, and the red line represents the proposed model using the fitting parameter values shown in Table 2.

Figure 1(a)–1(d) show the comparison of these collected data sets with the proposed model (Equation (6)), using a constant fractal dimension as suggested by Strom & Keyvani (2011). In this figure, the dashed line denotes Stokes' law, and the red solid line represents the fitted model. The fitting parameters of the proposed model for each case are summarized in the fourth through seventh columns of Table 2. Here, simple values of $c_1 = 20$ and $c_2 = 1.258$ are adopted for flocs, as suggested by Camenen (2007), and $\phi = 0.7$ is adopted (Khelifa & Hill 2006). The fitting correlation coefficients, R^2, of the proposed model for each case are presented in the eighth column of Table 2. It can be found that all of the examined real cases agree with the proposed model with a high correlation coefficient. It can be simply concluded that the proposed model is valid for the collected settling-velocity data sets. However, it can be seen that either the proposed model or Stokes' law does not agree well with the data when the floc size is small, especially in Figure 1(a)–1(c), possibly due to data scattering. The data scattering in both laboratory and *in situ* settling-velocity data could originate from two sources. The first source is that sediment flocs were formed in different water turbulent flow and salinity conditions, and cohesive sediment is a mixture of water, fine-grained sediments (such as silt, clay), and inorganic and organic matter of diverse nature. They greatly affect the structure of the formed flocs, as well as settling velocity. Furthermore, even for the same floc size, the flocs exhibit many different structures and different fractal dimensions due to the variety of flocculation mechanisms, floc breakup, floc restructuring and different unit masses of primary particles (Vahedi & Gorczyca 2011, 2012). Another source for data scattering is possible experimental error for measuring the settling velocity of the sediment flocs. It could be much more difficult to measure the floc size and the floc settling velocity due to the fragility of the flocs and their slow settling velocities in the water column (Kumar *et al.* 2010; Vahedi & Gorczyca 2012; Priya *et al.* 2015). It could also be difficult to control the water flow condition in the laboratory to be zero, and for *in situ* observation, a completely stagnant water environment for individual sedimentation of the floc is rare (Kumar *et al.* 2010). The camera video system and attached imaging technique used to measure the floc size properties and the floc settling velocity could have its

Table 2 | Summary of parameters used for the lines (that is, Equation (6)) plotted in Figure 1 and fitting coefficients

Reference	Primary particle size d_p (micron)	$\rho_s//\rho_w$ (kg/m³)	c_1	c_2	Φ	D_f	R^2	Experimental condition
Medema *et al.* (1998)	No	1,008//1,005	20	1.258	0.7	3	0.98	Laboratory settling column
Smith & Friedrichs (2011)	[5]	2,650//1,000	20	1.258	0.7	2.2	0.99	Field observation
Ahn (2012)	1.7	2,650//1,000	20	1.258	0.7	2.3	0.97	Field observation
Zhao *et al.* (2018)	6.11	2,455//1,000	20	1.258	0.7	2.75	0.93	Laboratory settling column

The symbol '[ ]' means that the value was predicted from the original reference.

inherent difficulties (Strom & Keyvani 2011; Vahedi & Gorczyca 2011, 2012).

CONCLUDING REMARKS

A formula for calculating the settling velocity of cohesive sediment flocs is proposed in this study. This formula utilizes the basic structure of the formulation of Ferguson & Church (2004) proposed for natural sands but amends the previous model to incorporate the fact that the flocculated sediment has an internal fractal architecture and is composed of different-sized primary particles. The proposed formula is applicable for the settling of sediment flocs across the viscous, transitional and inertial settling regimes, and it overcomes the shortcoming of the traditional approaches in which an iterative numerical technique is needed to calculate the settling velocity.

A comparison of the proposed model with four collected groups of settling velocity data is performed in this study, and a good agreement is found.

ACKNOWLEDGEMENTS

REFERENCES

Ahn, J. H. 2012 Size distribution and settling velocities of suspended particles in a tidal embayment. *Water Res.* **46**, 3219–3228.

Camenen, B. 2007 Simple and general formula for the settling velocity of particles. *J. Hydraul. Eng.* **133** (2), 229–233.

Cheng, N. S. 1997 Simplified settling velocity formula for sediment particle. *J. Hydraul. Eng.* **123** (2), 149–152.

Dietrich, W. E. 1982 Settling velocity of natural particles. *Water Resour. Res.* **18**, 1615–1626.

Dyer, K. R. 1989 Sediment processes in estuaries: future research requirements. *J. Geophys. Res.* **94**, 14327–14339.

Ferguson, R. I. & Church, M. 2004 A simple universal equation for grain settling velocity. *J. Sediment Res.* **74**, 933–937.

Hallermeier, R. J. 1981 Terminal settling velocity of commonly occurring sand grains. *Sedimentology* **28**, 859–865.

Jiang, Q. & Logan, B. E. 1991 Fractal dimensions of aggregates determined from steady-state size distributions. *Environ. Sci. Technol.* **25**, 2031–2038.

Kelbaliyev, G. I. 2011 Drag coefficients of variously shaped solid particles, drops and bubbles. *Theor. Found. Chem. Eng.* **45** (3), 248–266.

Khelifa, A. & Hill, P. S. 2006 Models for effective density and settling velocity of flocs. *J. Hydraul. Res.* **44** (3), 390–401.

Kumar, R. G., Strom, K. B. & Keyvani, A. 2010 Floc properties and settling velocity of San Jacinto estuary mud under variable shear and salinity conditions. *Continent. Shelf Res.* **30**, 2067–2081.

Maggi, F. 2013 The settling velocity of mineral, biomineral, and biological particles and aggregates in water. *J. Geophys. Res. Ocean* **118**, 2118–2132.

Medema, G. J., Schets, F. M., Teunis, P. F. M. & Havelaar, A. H. 1998 Sedimentation of free and attached *Cryptosporidium* oocysts and *Giardia* cysts in water. *Appl. Environ. Microbiol.* **64** (11), 4460–4466.

Mietta, F., Chassagne, C. & Winterwerp, J. C. 2009 Shear-induced flocculation of a suspension of kaolinite as function of pH and salt concentration. *J. Colloid Interface Sci.* **336**, 134–141.

Priya, K. L., Jegathambal, P. & James, E. J. 2015 On the factors affecting the settling velocity of fine suspended sediments in a shallow estuary. *J. Oceanogr.* **71**, 163–175.

Shen, X. & Maa, J. P. Y. 2015 Modeling floc size distribution of suspended cohesive sediments using quadrature method of moments. *Mar. Geol.* **359**, 106–119.

Smith, S. J. & Friedrichs, C. T. 2011 Size and settling velocities of cohesive flocs and suspended sediment aggregates in a trailing suction hopper dredge plume. *Cont. Shelf Res.* **31**, S50–S63.

Son, M. & Hsu, T. J. 2008 Flocculation model of cohesive sediment using variable fractal dimension. *Environ. Fluid Mech.* **8**, 55–71.

Son, M. & Hsu, T. J. 2009 The effect of variable yield strength and variable fractal dimension on flocculation of cohesive sediment. *Water Res.* **43**, 3582–3592.

Song, Z. Y., Wu, T. T., Xu, F. M. & Li, R. J. 2008 A simple formula for predicting settling velocity of sediment particles. *Water Sci. Eng.* **1** (1), 37–43.

Stokes, G. G., 1850 On the effect of the internal friction of fluids on the motion of pendulums. In: *Transactions of the Cambridge Philosophical Society* **IX**, reprinted in *Mathematical and Physical Papers*, Vol. 3, Cambridge University Press, Cambridge, UK, pp. 1–86.

Stone, M. & Krishnappan, B. G. 2003 Floc morphology and size distributions of cohesive sediment in steady-state flow. *Water Res.* **37** (11), 2739–2747.

Strom, K. & Keyvani, A. 2011 An explicit full-range settling velocity equation for mud flocs. *J. Sediment Res.* **81**, 921–934.

Vahedi, A. & Gorczyca, B. 2011 Application of fractal dimensions to study the structure of flocs formed in lime softening process. *Water Res.* **45** (2), 545–556.

Vahedi, A. & Gorczyca, B. 2012 Predicting the settling velocity of flocs formed in water treatment using multiple fractal dimensions. *Water Res.* **46**, 4188–4194.

van Leussen, W. 1991 Fine sediment transport under tidal action. *Geo-Mar. Let.* **11**, 119–126.

Watson, R. L. 1969 Modified Rubey's law accurately predicts sediment settling velocities. *Water Resour. Res.* **5** (5), 1147–1150.

Winterwerp, J. C. 1998 A simple model for turbulence induced flocculation of cohesive sediment. *J. Hydraul. Res.* **36**, 309–326.

Winterwerp, J. C., Manning, A. J., Martens, C., de Mulder, T. & Vanlede, J. 2006 A heuristic formula for turbulence-induced flocculation of cohesive sediment. *Estuar. Coast. Shelf Sci.* **68**, 195–207.

Xu, F., Wang, D. P. & Riemer, N. 2008 Modeling flocculation processes of fine-grained particles using a size-resolved method: comparison with published laboratory experiments. *Cont. Shelf Res.* **28**, 2668–2677.

Zhao, J., Yang, G., Kreitmair, M. & Yue, Y. 2018 A simple method for calculating in-situ settling velocities of cohesive sediment without fractal dimensions. *J. Zhengjiang Univ. Sci. A* **19** (7), 544–556.