Modelling of the coupling of desalination plants with the thermal solar energy system

Sara Irki, Nachida Kasbadji-Merzouk, Salah Hanini and Djamel Ghernaout

ABSTRACT

This work aims to develop models able to describe and predict the thermal solar collector coupled with vacuum membrane distillation (VMD) unit to determine the growth evolution of the temperature in the solar collector and the membrane module of the VMD. These models will be used to evaluate the distillate flow rates during the year. The global solar radiation data recorded on an inclined surface as well as ambient temperature data can be included in a program. First, the models were validated with some experimental data studies from appropriate literature and then they were used to predict the unit’s performance under different operating conditions in the City of Bouzaréah (Algiers). The results obtained from the proposed models showed that the distillate flux water reached 10.5 kg/h·m² in September and around 5.6 kg/h·m² in December during midday.

Key words | desalination, modelling, seawater, solar collector, vacuum membrane distillation (VMD)

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>$A_m$</td>
<td>Membrane area (m²)</td>
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<td>$A_c$</td>
<td>Area of the solar collector (m²)</td>
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<td>$A', B'$</td>
<td>Constant coefficients</td>
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<td>$B$</td>
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<td>$C_p$</td>
<td>Specific heat (J/kg·K)</td>
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<td>$E_{average}$</td>
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<td>$F_R$</td>
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<td>$G_{R}$</td>
<td>Global solar radiation (W/m²)</td>
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<td>$G_{β}$</td>
<td>Global solar radiation measured on an inclined surface (W/m²)</td>
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<td>$G_d$</td>
<td>Diffuse radiation on a horizontal surface (W/m²)</td>
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<td>$G_b$</td>
<td>Beam radiation on a horizontal surface (W/m²)</td>
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<td>$K_B$</td>
<td>Boltzmann constant (J/K)</td>
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<td>Knudsen number</td>
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<td>$L$</td>
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<td>$P$</td>
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<td>$P_2$, $P_3$, $P_4$</td>
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<td>$P_f$</td>
<td>Vapor pressure at the feed side (Pa)</td>
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<td>$P_p$</td>
<td>Vapor pressure at the permeate side (Pa)</td>
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<td>$Q_{abs}$</td>
<td>Rate of heat of the absorbed solar energy (W/m²)</td>
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<td>$Q_{losses}$</td>
<td>Rate of heat loss of the collector (W/m²)</td>
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<tr>
<td>$Q_{max}$</td>
<td>Maximum heat exchange (W/m²)</td>
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<td>$Q_{process}$</td>
<td>Heat flow, W/m²</td>
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<tr>
<td>$Q_u$</td>
<td>Actual heat transfer rate (W)</td>
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The scarcity of water resulting from climatic changes may have an unsettling effect on regional, and even on the national economy (DeNicola et al. 2015). The Arab world is mainly located in an arid to very arid region. These countries are home to 5% of the world’s population, and occupy 10% of the Earth’s space, with only 1% of world water resources (Badran et al. 2014). For example, Jordan, Libya, Kuwait, Qatar, Saudi Arabia and the United Arab Emirates are already suffering from water scarcity. Besides, several countries are facing this major problem, which is predicated for Algeria, Tunisia, Morocco, Egypt, and Syria by 2050 (Elliott & Breslin 2014). According to the World Bank Report for Algeria, the water sector spending as a percentage of gross domestic product (GDP) increased from 1.3% in 2001 to 1.9% in 2005 (World Bank 2010), only 50% of the total amount of freshwater is stored in reservoirs (World Bank 2007). Considered as a semi-arid country, Algeria is facing a growing problem of water shortage (Jeffreys 2012). The reasons for such a situation are multifaceted. It could be due to the inefficient management of water, the increasing demand for energy in the agriculture sector, or the significant deterioration of the environment.
caused by the pressure of a relatively rapid industrial development (Giordano et al. 2019).

The 2004 data show that Algeria’s agriculture absorbs about 64.9% of the withdrawal of water compared to the domestic use of water (21.9%) and industrial use (13%) (World Bank 2007). Water demand for all sectors is predicted to increase to 3.5 billion m³ by 2030 (Hamiche et al. 2015).

Algeria can be classified as having an absolute scarcity of water because it has a natural supply below 500 cubic meters per capita per year. The water quality management plans could be prepared and established if there were new comprehensive, included means for accessing new water resources, which could address many problems of the water shortage (Jeffreys 2012).

The government has launched a seawater desalination program to offer potential and significant water resources. However, the development of these demands on water resources is closely related to high energy consumption (Naimi Ait-Aoudia & Berezowska-Azzag 2016).

The membrane distillation (MD) process constitutes another key technology able to develop non-conventional water resources, due to the production of high-purity distillate from the desalination of brackish and seawater. Comparing with other membrane processes (reverse osmosis, nanofiltration, and electrodialysis), MD is a genuinely environmentally friendly process (Belessiotis et al. 2016). There are various types of MD configuration processes, including direct contact membrane distillation (DCMD), vacuum membrane distillation (VMD), air-gap membrane distillation (AGMD) and sweeping gas membrane distillation (SGMD) (Khayet & Matsuura 2011). The VMD configuration coupled with a low-temperature is more suitable for desalination than the MD process, especially when it is combined with low-temperature solar energy systems.

The fact that the VMD exhibits high-thermal performance at the low operating temperature range and operates at relatively low pressures can lead to more energy efficient and economic processes for water desalination (Khayet & Matsuura 2011; Belessiotis et al. 2016). Under the same conditions, Li et al. (2005) revealed that the permeate flux of VMD was significantly higher at 3,500 mL/m²·h, whereas lower flux of DCMD was obtained with 750 mL/m²·h. The experiments in desalinating seawater were performed in a VMD pilot plant using hollow fiber module membranes with an inner diameter of 327 mm and a thickness of 53 mm. In this system, the source of heating for seawater is heat generated from the vessel engine. A salt rejection factor of up to 99.99% can be maintained after five months of operation with a permeate flux of 5.4 kg/m²·h at a feed temperature of 55 °C and downstream pressure of 93 kPa (Xu et al. 2006). Wang et al. (2009) carried out an experimental investigation on the effect of solar energy collectors of 8 m² coupled with a VMD desalination module for brackish water with an electrical conductivity of more than 250 mS/cm. It was found that the daily permeate flux could be higher than 173 kg/m², in October 2006 under good weather conditions, and more than 50 kg/m² in cloudy weather conditions in Hangzhou, China. The simulation results reported that the production of potable water with treatment by the membrane module submerged in a salt gradient solar pond was around 17 L/day compared to that obtained when using solar collectors at a vacuum pressure (VP) of 5 kPa, which was 617 L/day. It has been successfully applied to the high concentration of salts for the production of distilled water (Mericq et al. 2011).

Frikha et al. (2015) developed a model based on the balance equations of mass and heat on the different units (membrane, exchanger, condenser, and field of solar collectors), to evaluate the distillate flow rate produced by a membrane desalination unit powered by solar energy. The result of the simulation shows that the average production of distilled water reached 21 kg/h·m² and 8 kg/h·m² on 21st June and 21st December, respectively.

Zuo et al. (2014) performed a two-dimensional numerical model for heat and mass transfer in both the flow channel and the membrane matrix of hollow fiber VMD modules. They optimised the operation parameters including vacuum pressure, feed volume flow rate, hollow fiber length, and feed temperature vacuum pressure and concluded that water production cost was decreased from 0.13 to 0.21 $/ton, for an optimized system and non-optimized VMD process, respectively. Naidu et al. (2014) presented a mathematical model to evaluate the feasibility of the novel VMD system with high NaCl concentration and analysed the influence of operation parameters on VMD performance. They reported that the model could predict the
permeate flux under high salt concentration. Deng et al. (2020) applied response surface methodology to investigate the effect of operating conditions on the permeate flux and the energy consumption during the solar thermal-photovoltaic vacuum fiber membrane distillation operation. They concluded that the experimental results of the permeate flux deviate by 3.35% from the predicted value. Lee & Kim (2015) employed one-dimensional simulations to study the effect of operating conditions on specific energy consumption and productivity in the hollow fiber VMD. They found excellent agreement between the numerical results and experimental data reported in the literature. They noticed that the augmentation of the feed temperature, velocity and total module length caused a diminution of specific heat energy consumption.

However, a few papers have considered the modelling of the thermal solar collector coupled with VMD with a view to understanding the different mechanisms and optimisation of design. The objective of this work is to establish the estimating model of potable water production that could be exploited from non-conventional water resources by using a thermal solar energy system coupled with a VMD unit. The obtained results are validated using the results published by Wang et al. (2009) and Frikha et al. (2013). The empirical formulas of global solar radiation and ambient temperatures established by Hakem et al. (2014) are used in our models and introduced to estimate the produced distillation flow.

**DESIGN OF THE PILOT PLANT**

A schematic diagram of the desalination unit for the treatment of brackish water or seawater is given in Figure 1. The desalination unit consists of two loops. The first loop includes a powered system composed of the flat plate solar collector system, a circulating pump, a valve, and a counter-current heat exchanger. A control system actuates the opening of the three-way valve to the exchanger when the temperature is higher than the inlet one and recirculates the fluid into the solar collectors’ system when it is lower than the inlet temperature to the exchanger. The second loop consists of the hydrophobic hollow fiber membrane, a condenser, and two collecting tanks.

The solar collector absorbs the incident solar radiation and converts it to useful heat. A heat transfer fluid such as water is circulated through the solar heat collector and carries heat away from the collector for use elsewhere. The heat exchanger uses heat from the fluid coolant in the first loop to heat water in the secondary circuit. The salt water, heated during its passage through the exchanger, passes into the vaporization portion due to the vacuum applied to the permeate side of the membrane module and condenses at the external condenser to produce the distilled water. The retentate is returned to the tank for recycling through the membrane module.

**MODELING**

**Mathematical modeling of the first loop**

Flat plate collectors are usually employed for low-temperature applications up to 100 °C; these applications include distillation of seawater. We propose in this work to use a configuration of coupling a flat plate collector to the desalination unit.

The energy balance of the solar collector is presented in Figure 2. In steady state conditions, the useful energy gain by the circulated fluid into the system is equal to the absorbed solar energy ($Q_{abs}$) minus the rate of overall heat
loss of the collector \( (Q_{\text{losses}}) \) to its surrounding (Hamdan et al. 2014):

\[
Q_u = Q_{\text{abs}} - Q_{\text{losses}} \quad (1)
\]

The useful energy gain is given by (Hamdan et al. 2014):

\[
Q_u = F_R(\tau_c\alpha_p)e A_c G_\beta - F_R U_L A_c (T_{ic} - T_a) \quad (2)
\]

The first term represents the amount of solar energy transmitted by the transparent cover, absorbed by the plate and then transferred to the fluid. The solar rays transmitted in the visible range of the solar spectrum are trapped between the cover and the absorber, which reflects them in the infrared range. The multiple reflection and absorption between the glass and the absorber reveal the transmittance-absorbance product \( (\tau_c\alpha_p)e \) which represents the sum of the energies absorbed by the plate. \( F_R \) is a collector heat removal factor that represents the efficiency factor for the transfer of energy from the absorbing plate towards the coolant, and it is a function of the physical properties of the materials and the capacitance rate \( (mC_p)_c \). In addition, \( G_\beta \) is the global solar radiation measured on an inclined surface, and \( U_L \) is the overall loss coefficient to the surroundings.

For commercial flat plate collectors, the optical yield \( F_R(\tau_c\alpha_p)e \) and the coefficient of overall losses \( F_R U_L \) are determined from the instantaneous efficiency curve of the flat solar collectors provided by the manufacturer. The useful energy recovered by the fluid can be obtained from a simple heat balance in function of the capacitance rate \( (mC_p)_c \) and between outlet fluid and inlet temperature (Duffie & Beckman 2013):

\[
Q_u = (mC_p)_c(T_{oc} - T_{ic}) \quad (3)
\]

where \( T_{oc} \) is the outlet temperature of the collector. By using Equations (2) and (3), we obtain:

\[
T_{oc} = T_{ic} + \frac{1}{(mC_p)_c} \left[ F_R(\tau_c\alpha_p)e G_\beta - F_R U_L (T_{ic} - T_a) \right] \quad (4)
\]

Note that the outlet temperature of the solar collector \( (T_{oc}) \) is equal to the temperature of the heat exchanger, where the heat loss between exchanger and collector to the atmosphere is negligible. This fact implies that the actual heat rate received by the solar collector \( (mC_p)_c \) is directly transferred to the second loop via the counter flow heat exchanger \( (mC_p)_{hot} \). So the exchanger efficiency is given by (Kakaç et al. 2002):

\[
\xi = \frac{Q_u}{Q_{\text{max}}} = \frac{(mC_p)_{cold}(T_P - T_R)}{(mC_p)_{min}(T_{oc} - T_R)} = \frac{(mC_p)_{hot}(T_{oc} - T_{ic})}{(mC_p)_{min}(T_{oc} - T_R)} \quad (5)
\]

where \( (mC_p)_{min} \) is the minimum capacitance rate of hot capacity rate \( (mC_p)_{hot} \) and the cold capacity rate \( (mC_p)_{cold} \) of fluid coming from the desalination water tank at a cold brine water temperature \( T_R \). So, if the effectiveness \( \xi \) of the heat exchanger is also known then the actual heat transfer rate \( Q_u \) can be determined as:

\[
(mC_p)_{hot}(T_{oc} - T_{ic}) = \xi(mC_p)_{min}(T_{oc} - T_R) \quad (6)
\]

\[
T_{ic} = T_{oc} - \xi(mC_p)_{min} (T_{oc} - T_R) \quad (7)
\]

By combining Equations (7) and (4), the useful energy transferred \( Q_u \) as well as the output temperature \( T_{oc} \) as a function of the temperature \( T_R \) and the heat capacity rates \( (mC_p)_{min} \), \( (mC_p)_{hot} \), and \( (mC_p)_c \) are determined as:

\[
T_{oc} = \frac{F_R(\tau_c\alpha_p)e A_c G_\beta - F_R U_L A_c \left( \frac{(mC_p)_{min}}{(mC_p)_{hot}} \xi T_R - T_a \right) + \frac{(mC_p)_c}{(mC_p)_{hot}} \xi T_R}{F_R U_L A_c \left( 1 - \frac{(mC_p)_{hot}}{(mC_p)_c} \xi \right)} \quad (8)
\]
Then,

\[ T_P = T_R + \frac{c_2(nC_p)_{\text{min}}}{(mC_p)_{\text{cold}}} (T_{\infty} - T_R) \]  

To capture the maximum amount of solar energy, the flat solar collectors are inclined at the latitude of the location of the distillation unit. The expression of global radiation on an inclined surface is given by (Duffie & Beckman 2013):

\[ G_\beta = G_b R_b + G_d \left( \frac{1 + \cos \beta}{2} \right) + \rho_d G_h \left( \frac{1 - \cos \beta}{2} \right) \]  

where \( G_b \) is the beam solar radiation, \( G_d \) is the diffuse, and \( G_h \) is the global solar radiation measured on a horizontal surface, respectively. Moreover, \( \rho_d \) is the reflection coefficient of the ground, and the two-terms \( \frac{1 + \cos \beta}{2} \) and \( \frac{1 - \cos \beta}{2} \) are the shape factors between the solar collector and the sky and the solar collector and the ground, respectively.

The conversion factor \( R_b \) is defined as the ratio of the solar radiation on an inclined surface to that on a horizontal surface and could be obtained as (Duffie & Beckman 2013):

\[ R_b = \frac{\cos \theta}{\cos \theta_{\beta=0}} \]  

where \( \theta \) and \( \theta_{\beta=0} \) are the incidence angles calculated on an inclined and a horizontal surface, respectively. The incidence angle is the angle between the direction to the sun and normal to the flat plate solar collectors. When the solar collectors are oriented towards the South, the incidence angle takes the following form (Duffie & Beckman 2013):

\[ \cos \theta = \sin \delta \sin (\phi - \beta) + \cos \delta \cos \omega \cos (\phi - \beta) \]  

where \( \omega, \phi \) and \( \delta \) are hour angle, latitude, and solar declination in degrees, respectively. The altitude of the sun at a given location depends on the location of the distillation unit, and the solar declination angle depends on the day of the year. By dividing Equation (11) by the global solar radiation \( G_h \), we obtain:

\[ R = \frac{G_\beta}{G_h} = \frac{G_b}{G_h} R_b + \frac{G_d}{G_h} \left( \frac{1 + \cos \beta}{2} \right) + \rho_d \left( \frac{1 - \cos \beta}{2} \right) \]  

where \( R \) is the conversion factor of global irradiation from the horizontal to the inclined surface. So the general expression for the estimated global radiation \( G_\beta \) on an inclined surface is given by (Duffie & Beckman 2013):

\[ G_\beta = R G_H \]  

In 1976, Perrin de Brichambaut et al. (1982) recommended a formulation to estimate the global radiation on a horizontal surface. Hakem et al. (2013, 2014) have expressed the global solar radiation on a flat surface from the model of Perrin Brichambaut, such as:

\[ G_H = A' ((\sin h))^{B'} \]  

This model is derived from experimental data collected from 2003 to 2006. The measurements of global solar radiation data on a horizontal surface were recorded every 5 min. In Equation (16), \( h \) is the solar elevation angle, and the parameters \( A' \) and \( B' \) are constant coefficients of the model that depend on location, and day of the year; they are determined from experimental data. The global and diffuse radiation on a horizontal surface can also be expressed as follows (Perrin De Brichambaut & Vauge 1982):

\[ G_h = 1080 ((\sin h))^{1.22} \]  
\[ G_d = 125 ((\sin h))^{0.4} \]
Then the beam radiation is obtained by Equation (17):

\[ G_b = 1 - G_d \]  \hspace{1cm} (19)

To solve Equation (9), it is also necessary to know the daily variation of the ambient temperature. From their measurements, Hakem et al. (2013, 2014) have proposed the ambient temperature model as a function of time \( t \), such as given by:

\[ T_a(t) = T_{\text{min}} + C \left[ \left( \frac{t - 6}{P_2} \right)^{P_4} \exp \left( - \left( \frac{t - 6}{P_2} \right)^{P_3} \right) \right] \]  \hspace{1cm} (20)

where \( T_{\text{min}} \) is the minimum daily temperature; \( P_2, P_3, P_4 \) and \( C \) are constants specific to the measurement sites. This model is derived from experimental data collected between 2003 and 2006. The measurements of ambient temperature were recorded every 30 min.

**Mathematical modeling of the second loop**

The VMD model is proposed for calculating the temperature in the feed side of the membrane based on the heat balance equation. The brine water feed is vaporized at the hot feed side membrane interface; then, the vapor molecules formed are diffused through the membrane pores.

The separation of the vapor molecules from the feed solution is achieved by applying VP on the permeate side of the membrane, which is lower than the saturation pressure at the permeate side. The condensation takes place outside the membrane module (Hamiche et al. 2015). The energy balance equation is described under the following assumptions: (i) the heat loss through the membrane is negligible, (ii) the heat energy generated that is used to boil water to inlet feed temperature is considered, (iii) the transfer resistance in the pores of the evaporated molecules at the brine water feed/membrane interface is negligible.

**Figure 3** presents the schematics of heat transfer through a porous membrane of the VMD system. If we consider that the heat of the process \( (Q_{\text{process}}) \) is generated from the hot feed side through the boundary layer to the hot membrane interface, then we have the rate of heat accumulation equal to the difference between heat input and output rates. The heat balance equation can be written on the feed side as:

\[ - m_f C_p \frac{dT_f}{dt} = (Q_{|z=0} - Q_{|z=}) - Q_{\text{process}} \]

\[ = \frac{dQ_z}{dz} dz - Q_{\text{process}} \]  \hspace{1cm} (21)
The rate of heat flow $Q_z$ between the inside and outside of the feed side can be determined as:

$$\frac{dQ_z}{dz} = m_zC_p \left. \frac{dT_f}{dz} \right|_{z=0}$$  \hspace{1cm} (22)

$$Q_z = m_zC_p(T_{fz} - T_p)$$  \hspace{1cm} (23)

where $m_z$ is the mass flow rate, $C_p$ is the specific heat; at $z = 0$, $T_{fz=0}$ is equal to the process temperature $T_p$; and $T_{fz}$ represents the mean feed temperature, which is given by $T_{fz} = T_p + (T_{fz-L}/2$ (Wang et al. 2009). The heat of the process $Q_{process}$ from the bulk of the feed through the boundary layer to the hot membrane interface is given by the heat transfer coefficient and temperature difference across the boundary layer as (Belessiotis et al. 2016):

$$Q_{process} = h_lA_m(T_{fz} - T_{fzm}) + J_vA_mH_L[T_{f,av}]$$  \hspace{1cm} (24)

where $J_v$ is the mass flux of the vapor through the membrane, $A_m$ is the area of the membrane, $h_l$ is the heat transfer coefficient, $T_{fzm}$ is the feed side of the membrane temperature, $H_L[T_{f,av}]$ is the enthalpy of water at the hot membrane interface. Moreover, $[T_{f,av}]$ is the average temperature between membrane interfaces and the feed side boundary layer, and it is given by $T_{f,av} = (T_{fz} + T_{fzm})/2$. The membrane interfaces temperature $T_{fzm}$ can be calculated from the equation of heat balance equation as follows (Wang et al. 2009):

$$T_{fzm} = T_{fz} - \frac{m_zC_p(T_p - T_{fz-L})}{h_lA_m}$$  \hspace{1cm} (25)

By substituting Equations (24) and (23) into Equation (21), we obtain:

$$m_lC_p \frac{dT_f}{dt} = h_lA_m(T_{fz} - T_{fzm}) + J_vA_mH_L[T_{f,av}] - m_zC_p(T_{fz} - T_p)$$  \hspace{1cm} (26)

One can first express the temperature derivative $dT_f/dt$ as $(T_f(t) - T_R)/\Delta t$ with time increments $\Delta t$. Then, Equation (26) will be arranged in the following form:

$$T_{f(0)} = T_R + \frac{\Delta t}{m_lC_p}[h_lA_m(T_{fz} - T_{fzm}) + J_vA_mH_L[T_{f,av}] - m_zC_p(T_{fz} - T_p)]$$  \hspace{1cm} (27)

The collector is connected to the module VMD containing $m_l$ (kg) of water initially at the temperature $T_R$. The heat transfer coefficient $h_l$ is determined by the Nusselt number as follows:

$$Nu = \frac{h_ld_h}{k_t}$$  \hspace{1cm} (28)

where $d_h$ is the hydraulic diameter, and $k_t$ is the thermal conductivity. The following empirical equation can be used to estimate the heat transfer coefficient. For turbulent flow regime and when the Reynolds number is in the range of $2,500 < Re < 1.25 \times 10^5$ (Mengual et al. 2004), the Nusselt number is given by:

$$Nu = 0.023Re^{0.8}Pr^m$$  \hspace{1cm} (29)

where $m$ is taken to be 0.4 and 0.3 for heating and cooling of the fluid, respectively. In the case of a laminar flow regime ($Re < 2,100$), the Nusselt number is evaluated by (Srisurichan et al. 2006):

$$Nu = 1.86 \left(\frac{Re \cdot Pr \cdot d_h}{L}\right)^{0.33}$$  \hspace{1cm} (30)

where Reynolds $Re$ and Prandlt $Pr$ numbers are given by:

$$Re = \frac{v \cdot d_h \cdot \rho}{\mu}$$  \hspace{1cm} (31)

$$Pr = \frac{C_p \cdot \mu}{k_t}$$  \hspace{1cm} (32)

where $\mu$ is the kinematic viscosity. According to Darcy’s law (Khayet & Matsuura 2011), the mass flux of the vapor through the membrane is proportional to the partial vapor
pressure difference and is written as:

\[ J_v = B \Delta P = B (P_f - P_p) \]  
(33)

where \( P_f \) and \( P_p \) are the vapor pressures at the feed side and the permeate side, respectively; and \( B \) is the membrane permeability coefficient. The partial pressure of water is expressed by the relationship given by Khayet & Matsuura (2011):

\[ P_f = x_w \gamma_w P_s \]  
(34)

where \( x_w \) is the water molar fraction, \( \gamma_w \) is the activity coefficient of water, and \( P_s \) is the saturated vapor pressure at the feed/membrane interface. In the case of dilute NaCl aqueous solutions in the feed side, the partial pressure of the water vapor is described as follows (Khayet & Matsuura 1964):

\[ P_f = (1 - x_{NaCl}) \gamma_w P_s \]  
(35)

where \( x_{NaCl} \) is the molar fraction of the solute. The Antoine equation is widely used for the estimation of \( P_s \), as (Khayet & Matsuura 2011):

\[ P_s = \exp \left[ 23.1964 - \frac{3816.44}{T_{f,aw} - 46.13} \right] \]  
(36)

By substituting Equations (35) and (36) into Equation (33), we find:

\[ J_v = B \left[ \exp \left( 23.1964 - \frac{3816.44}{T_{f,aw} - 46.13} \right) (1 - x_{NaCl}) \gamma_w - P_p \right] \]  
(37)

The activity coefficient of water \( \gamma_w \) in a NaCl solution is often considered by (Lawson & Lloyd 1997):

\[ \gamma_w = 1 - 0.5 x_{NaCl} - 10 x_{NaCl}^2 \]  
(38)

To determine the mechanism of mass transport in membrane operation, the Knudsen number (\( K_n \)) is used. It is a dimensionless number defined as the ratio of the mean free path (\( \lambda_i \)) of the transported molecules to the pore size of the membrane as (Belessiotis et al. 2016):

\[ K_n = \frac{\lambda_i}{d_p} \]  
(39)

where \( \lambda_i \) (\( \mu m \)) and \( d_p \) (\( \mu m \)) are the mean free path traveled by the transported molecules and the pore size, respectively. The mean free path is estimated using (Belessiotis et al. 2016):

\[ \lambda_i = \frac{k_B T_{f,aw}}{\sqrt{2 \pi p \delta_m}} \]  
(40)

where \( k_B \) is the Boltzmann constant, \( p \) is the average pressure in the membrane pores, and \( \delta_m \) is the collision diameter of water. Knudsen diffusion is dominant when collisions with the pore wall occur more frequently than collisions between diffusing molecules (\( K_n > 10 \) and \( r_p < 0.05 \lambda \)) (Khayet & Matsuura 2011). The membrane permeability coefficient may be obtained by:

\[ B = \frac{2}{3} \frac{\zeta r_p}{\tau \delta_m} \sqrt{\frac{8R T_{f,aw}}{\pi M} } \]  
(41)

where \( r_p \) is the mean pore radius assumed to be uniformly applied for all pores, \( M \) is the molecular weight, \( R \) is the gas constant, \( \zeta \) is the membrane porosity, \( \tau \) is the pore tortuosity, and \( \delta \) is the membrane thickness. The transition flow becomes dominant if \( 0.1 > K_n > 0.01 \) and \( 50 > r_p > 0.05 \lambda \), and the transport mechanism is governed by three basic mechanisms known as Knudsen-diffusion. We can also use Poiseuille-flow to express the membrane permeability coefficient, as (Khayet & Matsuura 2011):

\[ B = \frac{1}{\delta_m R T_{f,aw}} \left( \frac{2 \zeta r_p}{\pi M} \sqrt{\frac{8RT_{f,aw}}{\pi M} } + \frac{\zeta^2 r_p^2}{8 \pi n} \right) \]  
(42)

where \( \eta \) is the viscosity. The mass flux of the vapor across the boundary layers adjacent to the membrane surfaces is calculated by using \( T_f \). All the needed simulations were performed using MATLAB software.
MODEL VALIDATION WITH EXPERIMENTAL DATA

Validation condition

In a general manner, the model validity is entirely dependent on the average error to confirm the model proposed. The average error is estimated by:

\[ E_{\text{average}} = \frac{1}{n} \sum_{i=1}^{n} \frac{|X_{\exp,i} - X_{\mod,i}|}{X_{\exp,i}} \]  

where \( n \) is the number of samples, \( X_{\exp,i} \) and \( X_{\mod,i} \) are the simulated and the experimental values. The quality of the non-linear adjustment is provided by the value of the coefficient of determination as expressed by (Nash & Sutcliffe 1970):

\[ R^2 = 1 - \frac{\sum_{i=1}^{n} (X_{\mod,i} - \bar{X}_{\exp,i})^2}{\sum_{i=1}^{n} (\bar{X}_{\exp,i} - \bar{X}_{\exp,i})^2} \]  

where \( \bar{X}_{\exp,i} \) is the mean of the experimental values.

The index of agreement (IA) was originally developed by Willmott (1982), and is given by:

\[ d = 1 - \frac{\sum_{i=1}^{n} (X_{\mod,i} - \bar{X}_{\exp,i})^2}{\sum_{i=1}^{n} (|X_{\mod,i} - \bar{X}_{\exp,i}| + |X_{\exp,i} - \bar{X}_{\exp,i}|)^2} \]  

Data used for model validation

To validate the developed model, we used the experimental results of two VMD units powered with a solar collector. These units, installed in Tunisia and China, were published by Frihka et al. (2013) and Wang et al. (2009), respectively.

Frihka et al. (2013) and Wang et al. (2009) presented the dimensions of various parts of the apparatus (Table 1) and the VMD characteristics (Table 2). We have introduced these data to compare the results of the developed model with the experimental ones at the same radiometric and climatic conditions.

The climatic and radiometric conditions of the tests carried out in China and Tunisia are given by Wang et al. (2009) and Miladi et al. (2017). These experimental results are used to validate the numerical predictions of the experiment and to verify the developed models.

Model validation

A mathematic model for a solar-powered membrane distillation plant was presented by Li & Lu (2020). The major components of the system consists of solar collectors, a
heat exchanger and the membrane module. Each element of the system units was modelled for the simulation and the study of the whole system (Li & Lu 2020). In our work, we have developed a numerical simulation, Equation (9), for calculating the evolution of the temperature output of the solar collector, and then we can use Equation (10) to calculate the temperature output of the heat exchanger. In this model, the solar collector is simulated with a heat exchanger, which supplies the thermal energy according to the intensity of solar radiation, so this model was obtained by combining two complementary phenomena based on the heat balance under such conditions, which is expressed by Equation (1). At the same time, this equation has been used in many such studies to provide information for operation strategies (Avezova 2015; Diego-Ayala & Carrillo 2016; Ma et al. 2018; Li & Lu 2020).

The outlet temperatures of the collector and the heat exchanger, estimated by the developed model, are compared in Figure 4(a) and 4(b) with the experimental temperatures measured by Frikha et al. (2015), for the same meteorological and radiometric conditions. A flow of coolant and the flow of cold fluid are constant and equal to 770 kg/h and 1,200 kg/h, respectively.

As illustrated in Figure 4(a), the two curves have the same shape. However, the comparison shows that the model for the output temperature of the collector presented an average error of 8%. Whereas for the output temperature of the heat exchanger, the excellent agreement achieved between both the calculated model and experimental results, where the average error is lower than 5%, can be observed between 10 and 12 hours. The average error of 8% is achieved between 8 and 16 hours. There is a close relationship between the outlet temperature of the collector and the exchanger with the solar radiation. In addition, there is a good agreement of results around noon when the variation in solar radiation is low.

The evolution feed temperature was predicted from Equation (27); next, we present the result of permeate flux obtained using Equation (37). In Figure 5(a) and 5(b), the permeate flux estimated by the models is compared with the experimental results produced by Frikha et al. (2015) in Tunisia and Wang et al. (2009) in Hangzhou (China). Figure 5(a) shows that the experimental data of permeate flux coincided with the value predicted from the model curve in time between 8 and 13 hours. It can be seen that there is a significant difference between the value predicted from the model and experimental data at an interval of time ranging from 14 to 16 h. Based on the experimental results, it can be deduced that the average error is 19%. Figure 5(b) shows clearly that the model agrees well with the experimental measurements for the same parameter at its initial values. The obtained result for the permeate flux indicates that the model proposed has an average error of 6.6%.

While it has been reported that the vacuum pressure was not maintained for a long period of time, it varies between 10,000 and 15,000 Pa. The variation in the distillate flow rate during operation indicates that the accumulation of salt at the feed membrane interface arises from the ability of the membrane to partially reject the salts in the permeate solution (Frikha et al. 2015).

The model performance indices for the goodness of fit are summarized in Table 3. The simulated values agreed well with the experimental results produced by Frikha et al. (2015) in Tunisia with an index of agreement (d) around unity. However, the coefficient of determination showed low values, indicating low performance of the model for permeate flux at those operating conditions. The predicted values compared with the experimental results produced by Wang et al. (2009) in Hangzhou (China) have a coefficient of determination around unity at 96%, and a computed value of 1 for index of agreement indicates a perfect agreement between the measured and predicted values.

Ma et al. (2018) studied the mass and heat transfer combined with solar radiation modelling in the direct integration of a solar flat-plate collector and VMD in the same module. They investigated various equations and showed that the relative error of measurement became more significant when permeate flux and feed temperatures decreased. The performance of models was lowest under conditions produced by Frikha et al. (2015) in Tunisia that may be associated with the operating conditions and the error of measurement.

APPLICATION, RESULTS, AND DISCUSSION

The proposed model has been applied to the Bouzaréah site, whose geographic coordinates are 36.78° N, 3.01° E. In this
study, the global solar radiation on a horizontal surface and ambient temperature were estimated using models proposed by Hakem et al. (2013, 2014) for the Bouzaréah site. Using Equation (15), one can predict the global solar radiation received on horizontal surfaces for March, June, September, and December at the site of Bouzaréah. In addition, $A'$ and $B'$ parameters are provided in Table 4 for typical days of March, June, September, and December.

Figure 6 presents the variation of the permeation flux for March, June, September, and December. The plant operation can achieve high flux during the September period, at noon with $9.5 \text{ kg/m}^2\cdot\text{h}$, under the same conditions as the plant operated in Tunisia. The permeate flux occurring in December was reduced only slightly; it was around $5.6 \text{ kg/m}^2\cdot\text{h}$. The results show that the same maximum values of the permeate flux reached 6.7 and 6.3 kg/m$^2$·h.
for March and June, respectively. The results indicate that the flux that occurred in September achieved considerably better results than in other months, because the global solar radiation incident varied with the declination and solar elevation angle.

**CONCLUSION**

The present study reveals that the comparison of the model prediction with experimental data allows us to estimate the validity of a proposed thermal model for the solar collector with the heat exchanger. Therefore, the thermal model for
the VMD module may require inaccurate parameter values to predict the model proposed. The coefficient of determination is above 60% with an index of agreement around unity that could be accepted, if we take into account the errors in experimental measurements. In this work, we used the model intended to estimate the distillate flow rate during the year for Bouzaréah City. Moreover, the maximum permeate flux reached 9.5 kg/m² h in September under the following conditions: vacuum pressure 1,000 Pa, and a flow of coolant and flow of cold fluid that are constant and equal to 770 kg/h and 1,200 kg/h, respectively. The temperature of the process is highly sensitive to the day selected for modelling of the distillate flow rate due to variation in weather and in solar irradiation. The models presented can be used to estimate the effect of the impact of geographic location on the distillate flow in order to consider annual operation. These predictive models will be useful for different applications.

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