Numerical simulation model of vertical velocity distribution in a channel with artificial floating bed

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ABSTRACT

Artificial floating bed (AFB), as a novel type of ecological drainage ditch, is extensively used worldwide. To more effectively design the structure of the project, an accurate velocity model is required. In this study, a two-dimensional Lattice Boltzmann method (LBM) was employed for the simulation of the vertical velocity in a channel with AFB. The large eddy simulation (LES) was conducted to simulate turbulent flows, while the drag force of AFB was discretized with a centered scheme. Two sets of experimental data were used to verify the model, the mean value of root mean square error (RMSE) and coefficient of determination ($R^2$) are 0.93 and 1.84, respectively. This proved that the proposed model is more effective to simulate the vertical velocity in a channel with AFB.

Key words | artificial floating bed, ecological drainage ditch, large eddy simulation, Lattice Boltzmann method, vertical velocity

HIGHLIGHTS

- The innovation of this paper is the discretization of N-S equation in LBM form under the action of AFB.
- LBM is more conducive to simulate complex boundary problems, and more complex conditions (such as vegetation roots of AFB) can be added to it.
- The water structures also play an important role in the transport of sediment and nutrients, and the LBM method can more easily reflect the water structures and combine with transport of sediment and nutrients.

INTRODUCTION

Agricultural non-point pollution has imposed a heavy burden on China (Yan & Cai 2015), surplus water in farmland mixed with considerable sediment enters drainage ditches under substantial rainfall (Vidon et al. 2012). The conventional drainage ditch generally complies with the optimal cross-section design for water conservancy, and often exhibits too high a velocity, taking nitrogen, phosphorus and other elements adsorbed on the sediment and rushing to the downstream. As a result, soil erosion of farmland is caused, while water quality deterioration is accelerated in the downstream (Dolan & Mcgunagle 2005). Numerous researchers have proved that the ecological drainage ditch is one of the solutions to solve this problem with energy saving and environmental protection ensured (Kumwinba et al. 2017).

The artificial floating bed (AFB) is considered a type of ecological drainage ditch, and the AFB system exhibits an innovative variety of wetland. It will elevate the water level in the ditch and prolong the retention time of sediment and nutrients (Zhou & Wang 2013). AFB is composed of aquatic or terrestrial plants, growing through hydroponics and floating on the surface of the water body. The whole underwater surface of the plants underpins the microorganisms’ attaching process, and is conducive to decomposing organic matter and retaining suspended solids. Moreover, since plants are not rooted in any substrate, they should
obtain nutrients directly from the water column, probably elevating the absorption rate of nutrients into the biomass (Li et al. 2020). AFB will more effectively retain the sediment, while the vegetation in AFB will absorb a larger amount of nutrients (Zhu et al. 2011; Zhao et al. 2012).

Numerous researchers have studied the velocity characteristics in AFB channels (Xavier et al. 2018; Liu et al. 2019). The layout and shape of the AFB significantly impact the velocity (Figure 1). For a simple AFB channel, AFB is approximated as a form of rootless floating vegetation or ice convection velocity. Tsai & Ettema (1994) characterized the vertical distribution of the average velocity in accordance with the double power law expression (DPLE), which is a continuous gradient function as expressed below:

$$u = K_0 \left( \frac{y^m}{h} \right)^{1/m_v} \left( 1 - \frac{y^m}{h} \right)^{1/m_p}$$

(1)

where $u$ denotes velocity, $K_0$ is a coefficient correlated with discharge, $h$ is water depth, $y$ is the ordinate, $m_v$ and $m_p$ refer to coefficients correlated with the friction effect of the channel bed and AFB, respectively.

The solution just assesses the velocity distribution, and rare research has been conducted to build the relationships between the DPLE to sediment concentration distribution and retention of nutrients in a channel with AFB. Thus, a numerical model is considered a more effective way as it can simulate the velocity while simulating the retention of sediment and nutrients. It is a challenging work to ascertain the size of an AFB and a suitable AFB system. Sometimes experimental research can be conducive to selecting the coverage of AFB, whereas there are more factors (e.g. flow rates) impacting the velocity distribution as well. A good numerical model can act as a preliminary tool to support decision-making (Wang & Sample 2013). The establishment of the numerical simulation model helps simulate the transport of sediment and phosphorus in the channel, which can be theoretically referenced for the design of the channel.

The Lattice Boltzmann method (LBM) was applied in this study. The LBM originated in the 1970s, referring to a mesoscopic scale method between macro scale and micro scale (He & Luo 1997). Nowadays, LBM has aroused worldwide attention from researchers for its simplicity, scalability on parallel computers, and effectiveness in dealing with complex boundary conditions (Luo 2000). It is capable of simulating the velocity, while achieving a good simulation result with sediment and nutrient transport. Liu et al. (2015) presented a two-dimensional numerical model for sediment transport based on LBM, and the model was verified by testing transportation of sediment in a sharp-bended channel. A modified LBM was proposed by Hu et al. (2016) to simulate the convection diffusion equation; the result demonstrated that it applies to simulating both isotopic and anisotropic diffusion processes.

In recent years, the research on AFB has continued unabated. Sun et al. (2017) discussed the role of spinach and glutinous rice in the AFB system of hydroponics. The results showed that the salt tolerance of spinach was higher than that of glutinous rice, indicating that spinach was a more suitable choice for river pollution treatment. Using reed straw as raw material, a new type of AFB on the advanced treatment of wastewater was studied (Huang et al. 2020). Wang et al. (2018) proved that AFB can effectively improve the quality of urban landscape water reuse even in winter. In addition, the effects of depth, buoyancy and vegetation coverage are also represented by Samal et al. (2019). Although more and more literatures have been published in recent years, many design and operation aspects related
to system performance still need more research in order to better understand the water structures and pollutant removal mechanisms in an AFB channel (Colares et al. 2020).

This study aimed to develop a lattice Boltzmann model to simulate the vertical velocity in a channel with AFB, and the AFB was treated as rootless floating vegetation or ice convection velocity. To verify the effectiveness of the proposed model, two laboratory cases were applied by comparing the numerical predictions with experimental results. This study is helpful to analyze the water structures in the channel with AFB and provide a theoretical basis for the design of an AFB channel.

**NUMERICAL MODEL**

**Governing equations**

The vertical velocity in a channel with AFB can be derived by a Navier–Stokes (N-S) equation. Given the AFB interactions, the equations are written as (Rao et al. 2014):

\[
\frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial t} = \left( \nu + \nu_e \right) \frac{\partial^2 u_i}{\partial x_i \partial x_j} - g \frac{\partial z_b}{\partial x_i} - S_{bi} - S_{vi} \tag{2}
\]

where the Einstein summation convention over Latin indices is adopted; \( t \) denotes the time; \( u_i \) is the velocity; \( \nu \) and \( \nu_e \) represent kinematic and eddy viscosity, respectively; \( z_b \) is the bed elevation; \( S_{bi} \) is the bed shear stress term in the \( i \) direction and is expressed as a Manning formula.

\[
S_{bi} = \frac{g n^2}{h^{1/3}} u_i \sqrt{u_i u_j} \tag{3}
\]

where \( n \) refers to the Manning’s coefficient, and \( h \) is the water depth.

\( S_{vi} \) is the coupling source item that expresses the momentum exchange between the AFB and the flow (Wu et al. 2010).

\[
S_{vi} = \frac{1}{2} C_d A_i (u_i - u_{ri})^2 \tag{4}
\]

where \( C_d \) denotes the comprehensive resistance coefficient of the floating bed, \( A_i \) is the projected area on each coordinate plane, and \( u_{ri} \) is the velocity of the floating bed.

**Lattice Boltzmann method**

In this study, the LBM was adopted to simulate the vertical velocity distribution in a channel with AFB. A two-dimensional and nine particle lattice pattern (D2Q9) was employed in the fluid field, and articles moved to the neighboring lattice points in their directions and at their velocities in Figure 2. Moreover, the D2Q9 lattice pattern was adopted as a model to simulate the fluid problem.

The LBM consists of two main steps, i.e., Collision step and streaming step. The Bhatnagar–Gross–Krook model exhibits high simplicity and efficiency, and the two steps can be combined into the evolution equation as:

\[
f_{\alpha}(x + E_{\alpha} \Delta t, t + \Delta t) = f_{\alpha}(x, t) - \frac{1}{\tau_t} [f_{\alpha}(x, t) - f_{eq}^{\alpha}(x, t)] + \frac{\Delta t}{N_{e} e^{2} e_{\alpha}} F_{i} \tag{5}
\]

where \( f_{\alpha} \) represents the distribution function of particles; \( f_{eq}^{\alpha} \) is the local equilibrium distribution function; \( x \) is the space vector in Cartesian coordinates; \( e = \Delta x / \Delta t; \Delta x \) is the lattice size; \( \Delta t \) is the time step; \( \tau_t \) is the total relaxation time parameter; \( F_{i} \) denotes the external forces; \( N_{e} \) is a constant and defined by the lattice pattern.

\[
N_{e} = \frac{1}{e^{2}} \sum_{\alpha} e_{\alpha} e_{\alpha} \tag{6}
\]
\( e_{\alpha} \) denotes the particle velocity along the \( \alpha \) direction, as defined for the D2Q9 lattice pattern.

\[
e_{\alpha} = \begin{cases} 
(0, 0), & \alpha = 0 \\
 e \left[ \cos \left( \frac{(\alpha - 1)\pi}{4} \right), \sin \left( \frac{(\alpha - 1)\pi}{4} \right) \right], & \alpha = 1, 3, 5, 7 \\
\sqrt{2} e \left[ \cos \left( \frac{(\alpha - 1)\pi}{4} \right), \sin \left( \frac{(\alpha - 1)\pi}{4} \right) \right], & \alpha = 2, 4, 6
\end{cases}
\]

(7)

The local equilibrium distribution function \( f_{eq} \) can be expressed by Zhou (2004):

\[
f_{eq} = \begin{cases} 
1 - \frac{5g}{6e^2} - \frac{2}{5e^2} u_i u_i, & \alpha = 0 \\
\frac{g}{6e^2} + \frac{1}{3e^2} e_{ai} u_i + \frac{1}{2e^2} e_{ai} e_{ai} u_i u_i - \frac{1}{6e^2} u_i u_i, & \alpha = 1, 3, 5, 7 \\
\frac{g}{24e^2} + \frac{1}{12e^2} e_{ai} u_i + \frac{1}{8e^2} e_{ai} e_{ai} u_i u_i - \frac{1}{24e^2} u_i u_i, & \alpha = 2, 4, 6
\end{cases}
\]

(8)

The large eddy simulation (LES) was conducted to simulate the turbulent flow, and the total relaxation time parameter \( \tau_t \) refers to Liu et al. (2010).

\[
\tau_t = \tau + \sqrt{\tau^2 + 18C_s^2/(e^2)} \sqrt{\prod ij} \frac{\text{ij}}{ij}
\]

(9)

\[
\prod ij = \sum_{\alpha} e_{ai} e_{ai} (f_{a} - f_{eq}^a)
\]

(10)

where \( \tau \) denotes the single-relaxation time, and \( C_s \) is the Smagorinsky constant.

The \( v_i \) is calculated by

\[
v_i = v + v_e = \frac{e^2\Delta t}{6} (2\tau_t - 1)
\]

(11)

The external forces \( F_i \) is expressed as:

\[
F_i = -g \frac{\partial z_b}{\partial x_i} - \frac{gn^2}{1} u_i u_j - \frac{1}{2} C_d p A_i (u_i - u_p)^2
\]

(12)

The external force term is assessed at the mid-point between the lattice point and its neighboring lattice point as:

\[
F_{ai} = F_{ai} \left( x_i + \frac{1}{2} e_{ai} \Delta t, t \right)
\]

(13)

**Boundary conditions**

The boundary conditions of each side are obtained in accordance with Liu et al. (2012). At the inlet, the \( u \) is given, and the \( v = 0 \), the unknown distributions \( f_1, f_2, f_8 \) (Figure 3) are ascertained as:

\[
f_1 = f_5 + \frac{2u}{5e}
\]

(14)

\[
f_2 = f_6 + \frac{u}{6e} + \frac{f_7 - f_5}{2}
\]

(15)

\[
f_8 = f_4 + \frac{u}{6e} + \frac{f_7 - f_5}{2}
\]

(16)
At the outlet, the unknown distributions $f_4, f_5$ and $f_6$, are expressed as (Mohamad 2012):

$$f_4 = f_8 - \frac{u}{6e} - \frac{f_7 - f_5}{2}$$

$$f_5 = f_1 - \frac{2u}{3e}$$

$$f_6 = f_2 - \frac{u}{6e} - \frac{f_7 - f_5}{2}$$

The AFB and channel bed boundaries can be modelled by Yang et al. (2017). At the AFB boundary, the unknown distribution functions $f_6$, $f_7$ and $f_8$ are expressed as:

$$f_6 = rf_4 + (1 - r)f_2$$

$$f_7 = f_3$$

$$f_8 = rf_6 + (1 - r)f_4$$

where $r$ refers to a constant ($0 \leq r \leq 1$).

At the channel bed boundary, the unknown distribution functions $f_2$, $f_3$ and $f_4$ can be ascertained as (Mohamad 2012):

$$f_2 = rf_8 + (1 - r)f_6$$

$$f_3 = f_7$$

$$f_4 = rf_6 + (1 - r)f_8$$

The procedures of simulation can be split into several steps in Figure 4.

### NUMERICAL TESTS AND DISCUSSION

To verify the accuracy of the model, two sets of data were taken for simulation analysis. The experiments were performed by Han et al. (2018) and Wang et al. (2011) to characterize the flow characteristics of channels with AFB. The data sets are listed in Table 1. The data consist of different slopes, different water depths and different discharges. Experiments were carried out in a rectangular circulating water tank. A stabilizer was installed at the inlet of the tank to promote the steady flow in the tank. At the end of the channel, a variable tailgate is set to control the water depth. The sampling frequency is 50 Hz for 160 s, and the vertical sampling interval was almost 1 cm.

The accuracy difference between DPLE and LBM solutions were compared by error analysis root mean square error (RMSE) and coefficient of determination ($R^2$). Error analysis was conducted to ascertain the difference between the predicted and measured data. The RMSE and $R^2$ are calculated by:

$$R^2 = 1 - \frac{SSE}{SST}$$

$$SST = \sum_{i=1}^{N} (Y_i - meanY)^2$$

$$SSE = \sum_{i=1}^{N} (Y_i - X_i)^2$$

$$meanY = \frac{1}{N} \sum_{i=1}^{N} Y_i$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N} (X_i - Y_i)^2}{N}}$$

where $N$ denotes the number of lateral measuring points; $X$ and $Y$ are the calculated and measured values; $SST$ is total sum of squares; $SSE$ represents error sum of squares.

The accuracy of different models is listed in Table 2. The mean $R^2$ and RMSE of DPLE reach 0.81 and 2.73, far lower than those (0.93 and 1.84) of the LBM solution. The analytical model is an empirical model capable of achieving results quickly, whereas the accuracy is relatively low. The LBM is easier to program and exhibits higher accuracy, so this model can be more effectively applied in practice. The simulation results of the DPLE and LBM model are presented in Figures 5–7, the curve of the LBM model is closer to the measured velocity points. This study reveals that the shear stress of the channel bed is higher than vegetation in the experiment of Han et al. (2018), which is in contrast to the results in Wang’s experiment. The LBM model is capable of perfectly simulating two situations.

As shown in Figures 5 and 6, the velocity is large in the form of the upper layer and small in the lower layer since the AFB exhibits larger roughness than the bed surface.
The height of the maximum measured velocity and the height of maximum velocity reach 12.75 cm/s, 14.1 cm/s, 19.8 cm/s, 18.37 cm/s, 19.64 cm/s and 21.47 cm/s, and 11.32 cm, 11.95 cm, 12.61 cm, 12.86 cm, 13.38 cm and 14 cm, respectively. Figures 5 and 6 suggest that DPLE overestimates the velocity of the upper layer while underestimating the velocity of the lower layer. DPLE refers to an empirical model, and the interaction between the AFB and the bed surface is only revealed by coefficients in Equation (1). In fact, the interaction between the two layers is more sophisticated, and
Table 2 | The comparison between DPLE and LBM solutions

<table>
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<tr>
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<tbody>
<tr>
<td></td>
<td>Run 1 2 3 4 5 6 7 8 9 Mean</td>
<td></td>
</tr>
<tr>
<td>DPLE</td>
<td>$R^2$ 0.82 0.85 0.85 0.8 0.8 0.69 0.81 0.86 0.81</td>
<td>0.81 0.84 0.86 0.81</td>
</tr>
<tr>
<td></td>
<td>RMSE 1.3 1.08 1.47 1.91 2.05 2.57 2.71 5.54 2.73</td>
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<tr>
<td>LBM solution</td>
<td>$R^2$ 0.96 0.98 0.95 0.91 0.91 0.98 0.83 0.94 0.93</td>
<td>0.83 0.94 0.94 0.93</td>
</tr>
<tr>
<td></td>
<td>RMSE 0.69 0.62 0.82 1.44 1.5 0.67 2.41 3.89 4.54 1.84</td>
<td></td>
</tr>
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Figure 5 | The comparison of DPLE and LBM solutions ((a), (b), (c) present Run 1, Run 2 and Run 3, respectively).
LBM model is capable of more effectively simulating such a situation. As shown in Figure 7, the velocity is small in the upper layer and large in the lower layer since the roughness of the AFB is smaller than that of the bed surface. The measured maximum velocity and the height of the maximum velocity are 35.03 cm/s, 67.37 cm/s and 86.62 cm/s, and 6.96 cm, 5.82 cm and 7.53 cm, respectively. As suggested in the figure, the velocity of LBM in Figure 7(a) is less than that simulated by DPLE. In (b) with a maximum velocity of 67.37 cm/s, the LBM model has a velocity that partially exceeds that simulated by DPLE. In (c) with a maximum velocity of 86.62 cm/s, most of the velocity in the LBM model
reaches over the flow rate simulated by DPLE. The LBM model can achieve more effective simulation results under the smaller and larger flow velocity.

As one type of ecological drainage ditch, a more effective prediction of velocity in AFB underpins the simulation of sediment and nutrient transport. The transport equations of sediment and nutrients can refer to the studies by Zhang et al. (2014) and Yuan et al. (2017), and the convection diffusion equations of the LBM model are derived by Mohamad (2012). In this study, only the vertical velocity was simulated and verified. In subsequent research, sediment and nutrient transport will be simulated and verified.

For the study of the water structures in the AFB channel, it is more inclined to experimental research (Xavier et al. 2018; Liu et al. 2019), but it needs more costs. Meanwhile, there are many numerical models for the open channel

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**Figure 7** The comparison of DPLE and LBM solutions (a), (b), (c) present Run 7, Run 8 and Run 9, respectively.
(Hu et al. 2016), but there are less for the AFB channel. The innovation of this paper is the discretization of the N-S equation in LBM form under the action of AFB and the proposed numerous models in this paper can get better results. Although there are many kinds of AFB channel experiments in the outdoors to study the retention of sediment and nutrients, they are mainly quantitative, and the water structures are not reflected (Sun et al. 2017; Wang et al. 2018; Samal et al. 2019; Huang et al. 2020). The water structures also play an important role in the transport of sediment and nutrients, and the LBM method can more easily reflect the water structures and combine with transport of sediment and nutrients (Mohamad 2012). LBM is more conducive to simulate complex boundary problems, and more complex conditions (such as vegetation roots of AFB) can be added to it.

CONCLUSIONS

To more effectively simulate the vertical velocity distribution in a channel with an AFB, a two-dimensional LBM model was adopted. The innovation of this paper is the discretization of the N-S equation in LBM form under the action of AFB. The simulation results reveal that the two-dimensional LBM model outperforms the DPLE in simulating the vertical velocity distribution in a channel with an AFB. LBM is introduced into the velocity simulation of the AFB channel, but only in the form of no-vegetation AFB in this paper. The velocity simulation of an AFB channel with vegetation roots in the future remains to be proposed. At the same time, it should be combined with the transport of sediment and nutrients in our next work. Then, such technology is conducive to research into the mechanism and the design of the AFB, as well as the application of sediment and nutrients in channels with AFB in the future.

REFERENCES


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