

Multivariate drought risk analysis based on copula functions: a case study

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ABSTRACT

Drought is asserted as a natural disaster that encompasses vast territories for a long time and affects human life. Indicators are powerful tools for understanding this phenomenon. However, in order to get more information about the drought, multivariate indices were introduced for simultaneous evaluation of multiple variables. In this study, a combined drought index (CDI) based on three drought indices, the Standardized Precipitation Index (SPI), Streamflow Drought Index (SDI), and Standardized Water-level Index (SWI), is defined. Then, the Entropy method is used to determine the weight of each indicator. Among the calculated weights, SDI and SPI had the highest and lowest weight, respectively. The CDI is utilized to identify drought characteristics, such as duration and severity. In addition, the joint distribution function of drought characteristics is formed by copula functions and consequently the probability of different droughts is calculated. For the study area, data and information from eight regions located in Golestan province in the northern part of Iran are used to evaluate the performance of the proposed index. Four categories of drought were defined and their return period calculated. The shortest return period of severe drought was observed in the east and then in the west. In the south and center, the return period of severe drought was longer. Over the course of 30 years, all parts of the province experienced all drought categories.

Key words | drought index, drought return period, entropy method, Golestan province, multivariate copula functions

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HIGHLIGHTS

- Development of combined drought index.
- Calculation of drought characteristics using combined index.
- Calculate the joint probability of drought characteristics.
- Calculation of drought return period in two probabilistic states.

INTRODUCTION

In the midst of various natural disasters that result in damage every year, drought is a unique disaster, because it can affect vast areas for a long time. Unlike different drought definitions, there is a quadruple accepted category for it. (1) Meteorological drought, rainfall reduction; (2) hydrological drought, streamflow or other water resources reduction; (3) agricultural drought, soil moisture reduction; and (4)

socio-economic drought, which reflects the social and economic effects of drought (Wilhite & Glantz 1985). Iran, like other Middle Eastern countries, has a warm and dry climate. The central regions of Iran have witnessed decreasing minimum river flows and increasing duration of droughts (Nasri & Modarres 2018). Precipitation decline has been observed in northwest and low altitude areas of Iran

(Darand & Sohrabi 2018). Their proposal is long-term planning for reducing drought effects based on drought risk analysis. The annual rainfall analysis of 145 rain gauges in Iran showed that over most parts of the country, especially in the west and northwest, rainfall has decreased over the past 100 years (Modarres & Sarhadi 2009).

For drought monitoring and analysis, several indices have been introduced by researchers. Indicators are drought estimation tools selected based on the type of data and drought. Several studies have used various indicators of drought. In the review of both the Standardized Precipitation Index (SPI) and Reconnaissance Drought Index (RDI) by Zarch *et al.* (2015), drought-identified areas vary with each indicator at different times. Before 1998, the number of dry regions by SPI exceeded the RDI, whereas it has been the opposite since then. In addition, the estimates of these two indices also showed some other inconsistencies. Bonaccorso *et al.* (2015) compared SPI and the North Atlantic Oscillation index (NAO): the probability of changing drought levels was different in the same conditions. The Mediterranean Palmer Drought Severity Index (MedPDSI) was introduced by changing the calculation of water volume in the soil and increasing the effect of evapotranspiration on the Palmer Drought Severity Index (PDSI) calculations (Paulo *et al.* 2017). Although the proposed index had behavior similar to PDSI, it was more accurate in changing the levels of drought, especially in moderate, severe and extreme conditions. The standardized Palmer drought index (SPDI) was presented on the basis of a comparison of the standardized precipitation evapotranspiration index (SPEI) and PDSI, which had better results in drought estimation (Ma *et al.* 2014).

According to the studies conducted on the advantages and disadvantages of various indices, the combination of drought indices has become popular recently. The Multivariate Standardized Precipitation Index (MSPI) was presented by combining precipitation data in different time steps (3 to 24 months). The proposed index complied well with the estimates from other indicators (Bazrafshan *et al.* 2014). The Multivariate Standardized Drought Index (MSDI) was proposed by combining the SPI and Standardized Soil Moisture Index (SSI) (Hao & Aghakouchak 2013). The proposed index has these benefits: (1) drought estimation based on SPI and SSI; (2) it reports the onset of the drought, such as the SPI and drought persistence,

like SSI; (3) while SPI and SSI indicate drought, the proposed index shows more difficult conditions. MSDI was calculated based on the common probability function of SPI and SSI that was formed by copula functions.

The multivariate index was presented based on entropy theory; the input data were precipitation, runoff, soil moisture and evapotranspiration (Rajsekhar *et al.* 2015). This index has the ability to understand all types of drought and can be used in different time periods, it declares the onset and continuity of the drought better than single indices. The Multivariate integrated drought index (MIDI) was proposed by combining four drought indicators (Chang *et al.* 2016). Entropy method was used for weighting indices, the joint probability function of drought characteristics was obtained by applying the copula functions and then the drought return period was calculated. The composite meteorological drought index was presented by combining precipitation and soil moisture indices (Zhang *et al.* 2015). Using this index and Run theory, the duration and severity of the droughts were calculated and the copula functions were used to estimate the joint distribution function. The SPI was utilized to determine the drought characteristics (Shiau & Modarres 2009), then the joint distribution function was calculated using the copula functions for two regions in the north and south of Iran.

In the current research, the drought combined index (CDI) was derived from combining the Standardized Precipitation Index (SPI), Streamflow Drought Index (SDI) and Standardized Water-level Index (SWI) to calculate drought characteristics. The joint distribution function of intensity and duration of droughts was calculated with copula functions and the drought return period was obtained.

STUDY AREA AND INPUT DATA

As stated, Golestan province in northern Iran has suffered repeatedly from drought. It has a warm and humid climate on the west coast, is warm and semi-arid in the central regions and has a mountainous climate in the south. The province, with an area of about 20,000 km², is located between 53°50'E to 56°20'E and 36°24'N to 38°8'N. The western border of Golestan province is the Caspian Sea, its southern border is Alborz mountains and its northern

border is Turkmenistan. Its altitude varies from -30 to $3,820$ m and the average precipitation and temperature are 484.6 mm and 16.6°C , respectively. The temperature ranges from 7°C at high altitudes up to 19°C in the central area and precipitation ranges from 200 mm to $2,000$ mm (Mosaedi et al. 2007). In the past 30 years, no point in the province has been invulnerable to drought (Bazrafshan et al. 2011). Over the past three decades, at least 13 percent of the province has been affected by drought annually (Eivazi & Mosaedi 2011). Figure 1 shows the location of the Golestan province elevation and gauges considered in this study.

The data used in this study are rainfall, runoff and groundwater levels in the eight stations distributed over the study area from 1982 to 2012. Selected stations are named Tamer, Arzokuse, Ramian, Gorgan dam, Aghghala, Gonbad, Naharkhoran, and Taghi-abad.

METHODOLOGY

Drought indices

The first step in this study is to calculate drought indices. Three indicators, Standardized Precipitation Index (SPI) (Mckee et al. 1993), Streamflow Drought Index (SDI)

(Nalbantis & Tsakiris 2009) and Standardized Water-level Index (SWI) (Bhuiyan 2004), were computed. The most widely used indicator is the SPI. The SPI is an accurate tool for estimating drought in Golestan province (Bazrafshan et al. 2011). To calculate this index, rainfall is compared with historical data at any time step. The Gamma distribution function is fitted to the time series of the rainfall. Because this function is not fitted to zero, it changes as follows:

$$H(x) = q + (1 - q) * G(x) \quad q = P[x = 0] \quad (1)$$

Then, the SPI values are obtained by the inverse normal distribution function. In addition, the SDI is calculated based on the following equations in given reference period (k) and streamflow data.

$$V_{i,k} = \sum_{j=1}^{3k} Q_{i,j} \quad i = 1.2 \dots$$

$$j = 1.2 \dots 12 \quad k = 1.2.3.4 \quad (2)$$

$$SDI_{i,k} = \frac{V_{i,k} - \bar{V}_k}{S_k} \quad i = 1.2 \dots \quad k = 1.2.3.4 \quad (3)$$

where i and j are the year and month counter, respectively. Q is streamflow volume, V is cumulative streamflow volume and \bar{V}_k and S_k are its average and standard deviation.

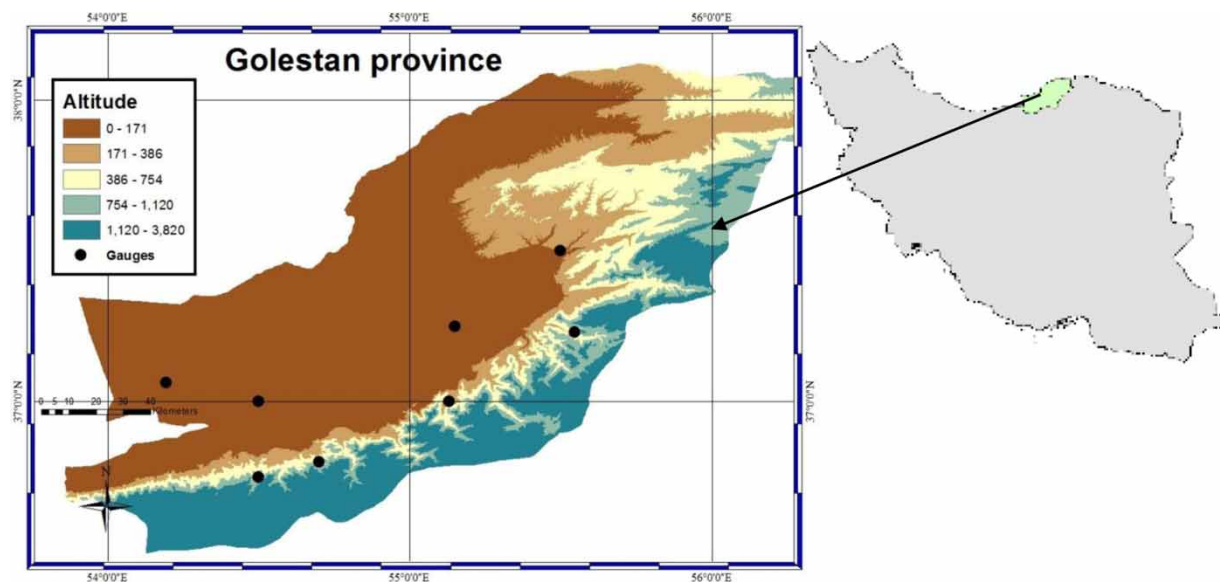


Figure 1 | Location of Golestan province in Iran.

The SWI monitors the hydrological drought by checking the groundwater level. The SWI equation is as follows:

$$SWI = \frac{W_{ij} - W_{i.m}}{\sigma} \quad (4)$$

where i and j are wells and observation counters, respectively. W is seasonal water level, W_m is the average seasonal water level and σ is its standard deviation. Contrary to SPI and SDI, the positive values of the SWI indicate the dryness and its negative values indicate wetness, because the depth of the water surface is measured downward. Therefore, its values are multiplied by '-1' to be comparable with two other indicators.

Entropy weighting method

Entropy is a method for weighting indices. First, the matrix derived from the indices is standardized; the next step is to calculate the entropy of each index according to the following equation.

$$H_j = -\frac{\sum_{i=1}^m f_{ij} * \ln f_{ij}}{\ln m} \quad (5)$$

wherein:

$$f_{ij} = -\frac{r_{ij}}{\sum_{i=1}^m r_{ij}} \quad (6)$$

where, r_{ij} is the index matrix value. Finally, the weight of each indicator is calculated according to the equation below (Li et al. 2011).

$$w_j = \frac{1 - H_j}{n - \sum_{i=1}^n H_j} \quad (7)$$

The calculated value 'w' is a measure of the applied information of each indicator, the higher values of the 'w' indicate the more useful information obtained from that index (Li et al. 2011). The combined index is proposed based on the entropy weighting method and its equation is as follows:

$$CDI = W_{SPI} * SPI + W_{SDI} * SDI + W_{SWI} * SWI \quad (8)$$

W_{index} is the weight of each index in the combined index. In the entropy method, further variations represent a more precise criterion. Therefore, that criterion is more important and higher weight is allocated to it.

Run theory

Guerrero-Salazar & Yevjevich (1975) and Yevjevich (1967) proposed the use of Run theory to determine the drought characteristics and presented the calculation method. According to this theory, a threshold is defined; if the index value is less than that, a drought has occurred. The duration of the drought is the time interval between the onset and termination of drought. In other words, the interval from when the indicator crosses its threshold value until it returns to normal. Its severity is the sum of the values of the index during the drought. The definition of drought with this theory requires a drought index (Raziei et al. 2003; Mishra et al. 2009; Mosaedi et al. 2017).

Copula functions

The advantage of copula functions is their non-dependence on the probability distribution of the input variable. From the first applications of the copula functions in drought, Shiau (2006) used them in calculating the joint distribution function of drought characteristics. The major use of copula functions in drought researches is to formulate common distribution functions of drought indicators or drought characteristics (She & Xia 2018) (Vaziri et al. 2018) (Zuo et al. 2018) (Ayantobo et al. 2018) (Hangshing & Darbral 2018) (Van de Vyver & Van den Bergh 2018) (Kavianpour et al. 2018). If the best probability distribution of x and y are $F_1(x)$ and $F_2(y)$, then the copula function is calculated according to the equation below:

$$P[X \leq x, Y \leq y] = C(F_1(x), F_2(y)) \quad (9)$$

The input and output values of the copula are in the [0,1]. The calculation steps are (a) a scatter plot of x and y , (b) transfer data to [0,1], (c) fit copula function, and d) transfer the copula output to the initial range of variables.

In this research, three Archimedean bivariate copula functions were used. Their equations are presented in Table 1.

Table 1 | Three copula functions and their equations

Copula name	Copula function	Variables	Considerations
Frank (Nelsen 2007)	$C(u, v) = -\frac{1}{\theta} \ln \left[1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{e^{-\theta} - 1} \right]$	bivariate	$\theta \neq 0$
Gumbel (Nelsen 2007)	$C(u, v) = \exp \left(-[(-\ln u)^\theta + (-\ln v)^\theta]^{\frac{1}{\theta}} \right)$	bivariate	$\theta \geq 1$
Clayton (Nelsen 2007)	$C(u, v) = (\max[u^{-\theta} + v^{-\theta} - 1, 0])^{-\frac{1}{\theta}}$	bivariate	$\theta \neq 0$

The input of copula functions is the best fitted probability distribution function of variables. With two criteria, Kolmogorov–Smirnov and chi-squared, more than 20 probability distributions were checked for drought specifications to select the best one.

The Root Mean Square Error (RMSE) and Kolmogorov–Smirnov statistic were used to select the most suitable copula function. The equation of RMSE is as follows:

$$RMSE = \sqrt{\frac{\sum_1^N (y_o - y_{cf})^2}{N}} \tag{10}$$

wherein, y_o and y_{cf} are respectively the observations and the output of the copula function, and N is the number of data. The observation values are empirical probability of a drought event. The best option has the lowest RMSE. To determine the best copula function next to the RMSE, the Kolmogorov–Smirnov statistic was also used. This statistic reports a p -value based on the vertical distance between the input variable and the selected distribution function. A p -value represents the levels at which the assumption of variable compliance with a particular statistical distribution is accepted.

Droughts risk return period analysis

The return period means the average time between two identical droughts (Cancelliere & Salas 2010). The return period is related to the inverse of probability of the phenomenon and its equation is as follows:

$$T = \frac{E}{P} \tag{11}$$

In this equation, T is the return period, E is the expectation of drought intervals, which is equal to the ratio of

total time to the number of droughts (Hesami Afshar et al. 2016). The probability of drought is calculated in two ways, common and union, equivalent to ‘and’ and ‘or’. Their equations are below:

$$\begin{aligned} T(D \geq d \text{ and } S \geq s) &= \frac{E}{P(D \geq d, S \geq s)} \\ &= \frac{E}{P(D \leq d, S \leq s)^c} = \frac{E}{1 - P(D \leq d, S \leq s)} \\ &= \frac{E}{1 - F(D) - F(S) + F(D, S)} \\ &= \frac{E}{1 - F(D) - F(S) + C(F(D), F(S))} \quad (\text{Shiau, 2006}) \end{aligned} \tag{12}$$

$$\begin{aligned} T(D \geq d \text{ or } S \geq s) &= \frac{E}{P(D \geq d \text{ or } S \geq s)} = \frac{E}{P(D \leq d \text{ or } S \leq s)^c} \\ &= \frac{E}{1 - P(D \leq d \text{ or } S \leq s)} = \frac{E}{1 - F(D, S)} \\ &= \frac{E}{1 - C(F(D), F(S))} \quad (\text{Shiau, 2006}) \end{aligned} \tag{13}$$

With these formulas, the probability and drought return period are obtained and the risk of drought is analyzed.

RESULTS AND DISCUSSION

SPI, SDI and SWI were calculated based on the 12 months of average monthly rainfall, runoff and groundwater level data from 1982 to 2012 for eight stations in Golestan province (Figure 2). Because the calculation is based on different variables, the values of the indices have a significant difference. Variables sometimes have a time lag. For example, a decrease in precipitation does not quickly lead to a decrease in runoff and changing the depth of groundwater does not react to small fluctuations in rainfall.

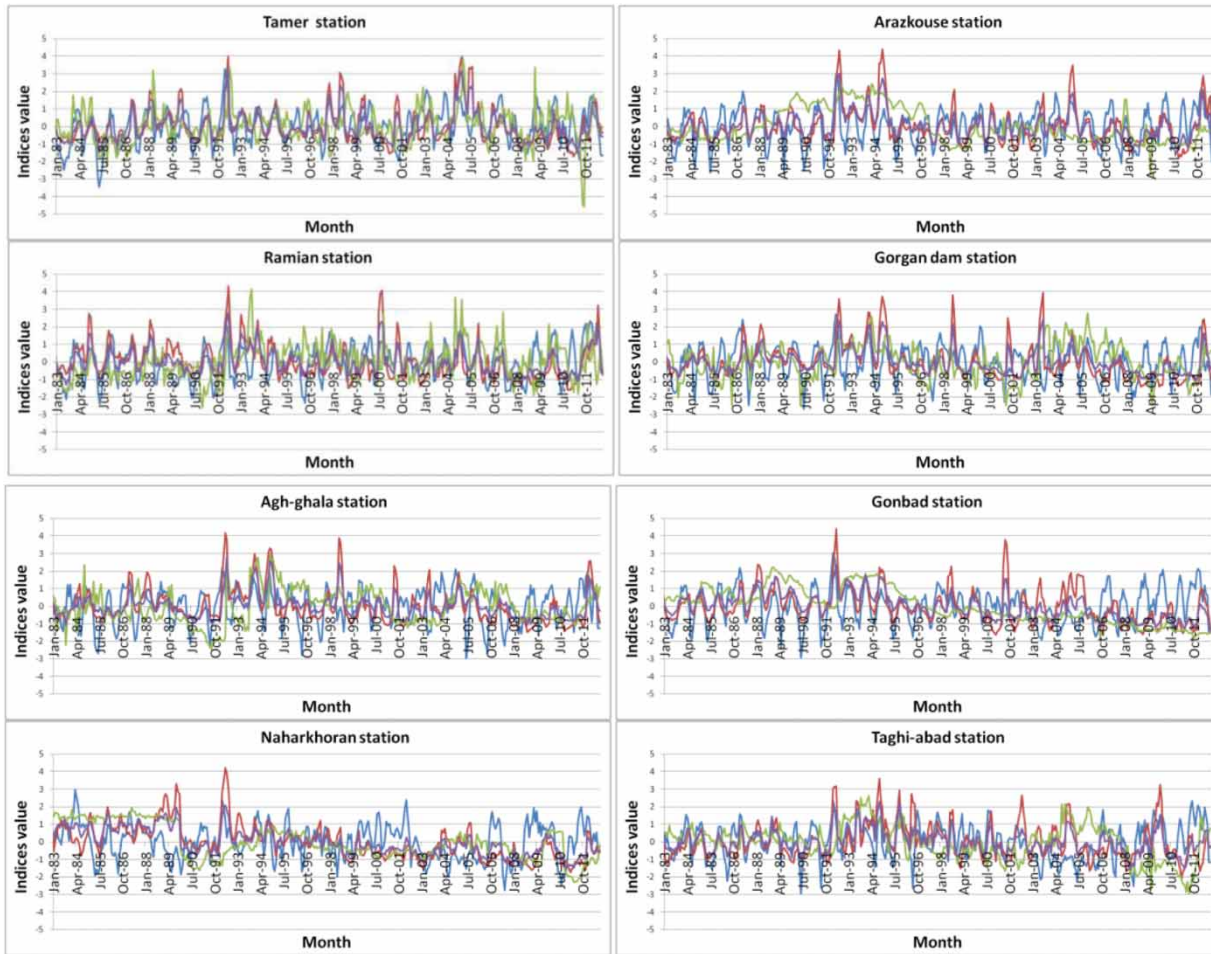


Figure 2 | Drought indicators in Golestan province (blue: 12-months SPI, red: 12-months SDI and green: 12-months SWI). The full colour version of this figure is available in the online version of this paper, at <http://dx.doi.org/10.2166/ws.2020.153>.

In **Figure 2**, there are points where each index reports different conditions, dry, normal or wet. Therefore, a combined index was suggested that contains three indicators information and gives a unique output to decision makers. The combined index is proposed based on the entropy weighting method. The calculated weight for the three indicators is listed in **Table 2**.

The weight of the SDI is always higher than the two other indicators. Therefore, the importance of this index for estimating drought is greater than the other two indices. Except for three stations, Tamer, Arazkouse and Ramian, the SWI weight is higher than the SPI in all the other stations.

In **Figure 3**, the combined index values are plotted against the values of three other indicators at Tamer (maximum SDI weight) and Taghi-abad (minimum SDI weight)

Table 2 | Calculated weight for triple indicators

	SPI	SDI	SWI
Tamer	0.20	0.68	0.11
Arazkouse	0.28	0.49	0.23
Ramian	0.28	0.51	0.21
Gorgan dam	0.20	0.57	0.22
Agh-ghala	0.19	0.56	0.25
Gonbad	0.16	0.47	0.37
Naharkhoran	0.21	0.49	0.30
Taghi-abad	0.28	0.44	0.28

stations. The proposed index has a trend consistent with the trend of other indicators. CDI has well-documented droughts. So, at times when one of the three indices

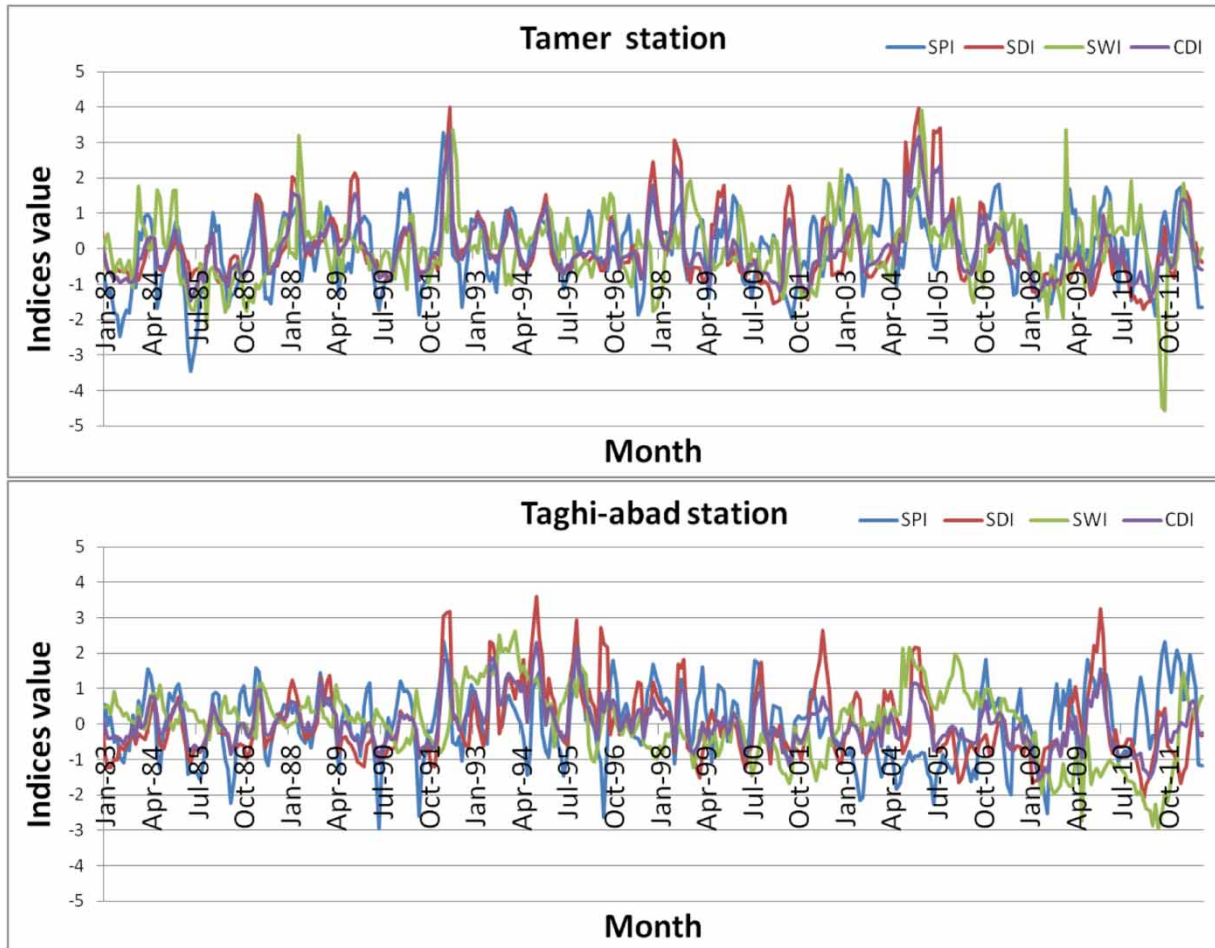


Figure 3 | Combined drought index against 12-months SPI, 12-months SDI and 12-months SWI in Tamer and Taghi-abad.

indicated drought, the CDI also marked the drought, except at times when there was a significant difference between the values of the three primary indicators.

To view the performance of the proposed index, the Gonbad station is shown in Figure 4 from 2003 to 2012. As seen, the trend and dry and wet times determined by the combined index are in accordance with the triple indicators.

Among the drought characteristics in the present study, its duration and severity have been investigated. In Table 3, the number and average of severity and duration of drought are presented in eight stations in Golestan province.

After determining the characteristics of the drought, it is possible to analyze the probability of occurrence and its return period. In the univariate approach, the probability

of a 6-month or 9-month drought or a drought with a severity of 4 or 8 is calculated separately. However, in a multivariate approach, the probability of a drought occurring with multiple attributes is calculated: for example, a drought that lasts 6 months and has a severity of 2.

Copula functions were used to calculate the joint probability of drought characteristics. Therefore, the best distribution function of each drought characteristic is determined and listed in Table 4.

When the best fitted distribution function was found and the value of the θ parameter was computed, copula functions named Frank, Gumbel, and Clayton were applied between the two drought characteristics of each station. RMSE and p -values for each of the three copula functions at each station are given in Table 5.

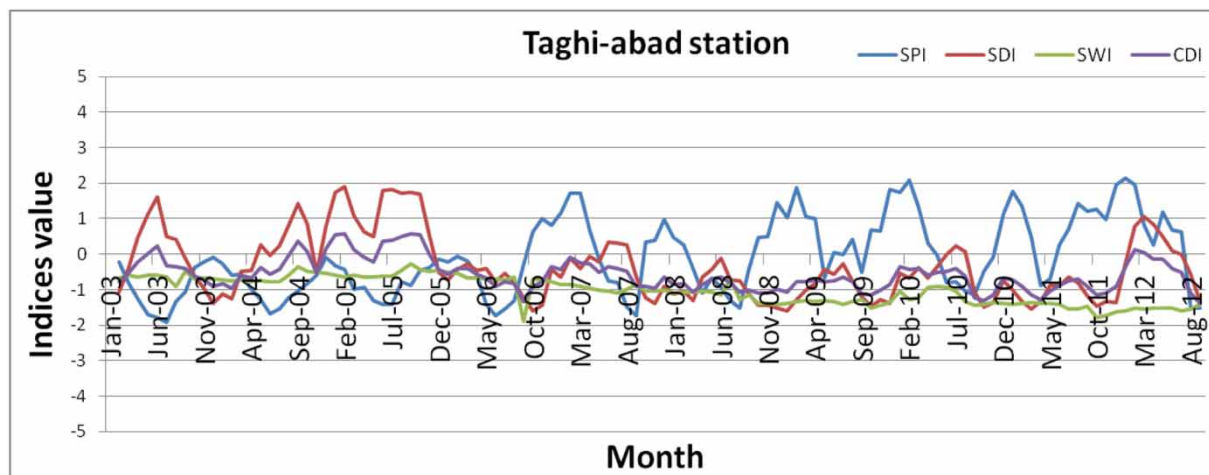


Figure 4 | Drought indices in Gonbad station 2003 to 2012.

Table 3 | Number and characteristics of drought in stations of Golestan province

	Number of droughts	Average duration	Average severity
Tamer	31	6.87	3.55
Arazkouse	32	6.13	3.14
Ramian	38	5.37	2.63
Gorgan dam	29	7.14	3.10
Agh-ghala	24	8.50	3.63
Gonbad	21	9.19	4.96
Naharkhoran	23	8.48	4.73
Taghi-abad	39	5.03	2.31

Table 4 | The best fitted distribution function for drought characteristics at each station

	The best statistical distribution function for drought duration	The best statistical distribution function for drought severity
Tamer	Weibull	Weibull
Arazkouse	Weibull	Generalized extreme value
Ramian	Log logistic	Generalized extreme value
Gorgan dam	Log logistic	Gamma
Agh-ghala	Gamma	Weibull
Gonbad	Weibull	Log normal
Naharkhoran	Log logistic	Log normal
Taghi-abad	Log logistic	Log normal

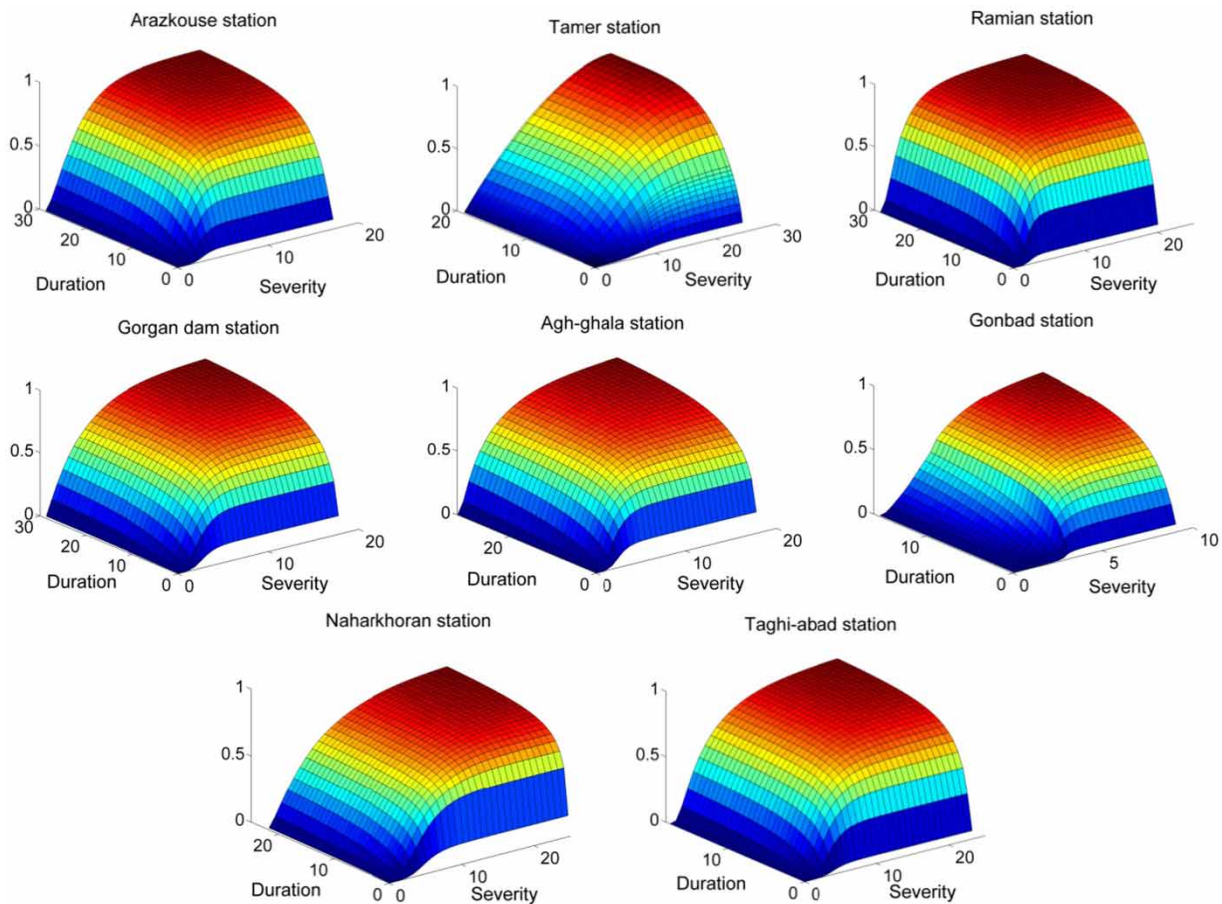
In Table 5, at all stations the RMSE value of the Clayton function is always more than Gumbel and Frank, but the difference between Frank and Gumbel is negligible. The p -value is also better for the Frank function (except for Gonbad and Naharkhoran, with a slight difference). Finally, based on two criteria, the Frank function is selected. In Figure 5, the Frank copula function of drought characteristics has been drawn up at eight stations.

Then, four categories of drought were defined, from light to extreme severe. These categories are light ($D > 3$, $S > 1.8$), moderate ($D > 6$, $S > 5.1$), severe ($D > 9$, $S > 8.3$) and extreme severe ($D > 12$, $S > 12.5$). The probability of each category in 'AND' and 'OR' cases is given in Table 6.

In Table 6, the probability of mode 'OR' is always more than 'AND'. In mode 'AND' at Taghi-abad station, the probability of light drought occurrence is the lowest among the stations, at 33%, while this amount is 58% in the Tamer station as the highest one. In the moderate drought group, the Tamer station is the most likely to occur, and the least probability is with the Ramian and Taghi-abad stations. At Naharkhoran and Ramian stations, the highest and lowest probability of severe drought was observed, respectively. In the extreme severe drought category, the highest probability of occurrence is at the Naharkhoran station and the lowest is in Ramian and Tamer stations. The Tamer station is most likely to have light drought, while it has the lowest probability of extreme severe drought. At Arazkouse station, the chance of a light drought is 50%, which is almost the

Table 5 | RMSE and p -values of different copula functions

	RMSE values			p -values		
	Gumbel	Clayton	Frank	Gumbel	Clayton	Frank
Tamer	0.065	0.068	<u>0.051</u>	0.82	0.46	<u>0.83</u>
Arazkouse	0.068	0.081	<u>0.065</u>	0.14	0.23	<u>0.26</u>
Ramian	0.067	0.094	<u>0.066</u>	0.21	0.26	<u>0.43</u>
Gorgan dam	0.055	0.079	<u>0.053</u>	0.34	0.3	<u>0.46</u>
Agh-ghala	0.043	0.067	<u>0.041</u>	0.07	0.09	<u>0.12</u>
Gonbad	0.063	0.090	<u>0.055</u>	0.3	<u>0.47</u>	0.43
Naharkhoran	0.050	0.074	<u>0.046</u>	0.24	<u>0.28</u>	0.25
Taghi-abad	0.062	0.096	<u>0.060</u>	0.03	0.05	<u>0.1</u>

**Figure 5** | The Frank copula function fitted on the duration and severity of droughts.

highest. On the opposite side, the chance of an extreme severe drought at this station is almost the lowest among the other stations.

In the 'OR' case, the light, moderate and severe drought groups have the highest and lowest probability of occurrence at Tamer and Taghi-abad stations, respectively.

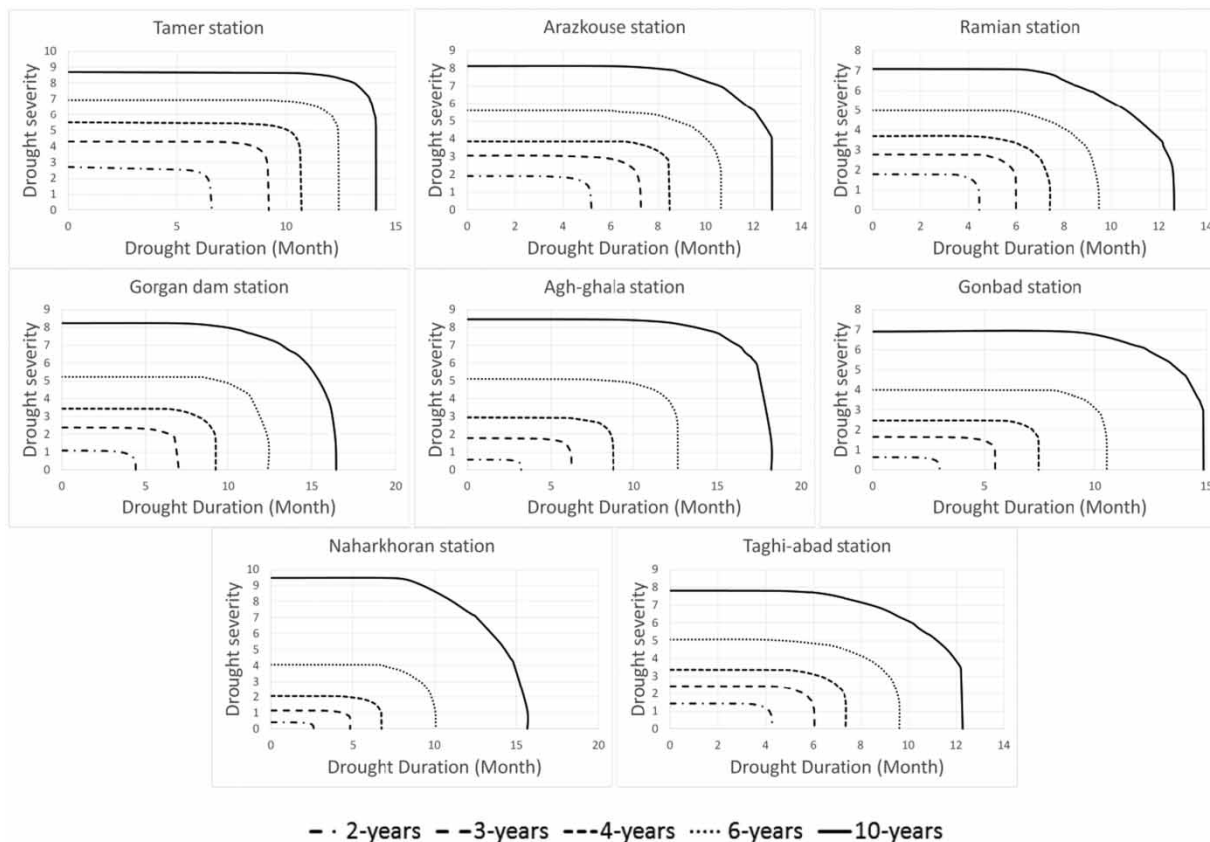
Table 6 | Joint probability of drought categories

	Light		Moderate		Severe		Extreme severe	
	'AND'	'OR'	'AND'	'OR'	'AND'	'OR'	'AND'	'OR'
Tamer	0.58	0.75	0.26	0.52	0.11	0.33	0.02	0.18
Arazkouse	0.49	0.69	0.17	0.40	0.09	0.23	0.04	0.13
Ramian	0.39	0.59	0.12	0.28	0.05	0.15	0.02	0.10
Gorgan dam	0.41	0.64	0.18	0.41	0.10	0.27	0.05	0.19
Agh-ghala	0.41	0.65	0.21	0.43	0.13	0.30	0.08	0.22
Gonbad	0.45	0.71	0.20	0.44	0.12	0.29	0.07	0.20
Naharkhoran	0.35	0.61	0.19	0.37	0.14	0.25	0.10	0.19
Taghi-abad	0.33	0.53	0.12	0.26	0.06	0.15	0.03	0.10

In extreme severe droughts, the highest probability of occurrence is at Agh-ghala station and the lowest probability is for Taghi-abad and Ramian stations. At the Naharkhoran station, there was the highest probability of severe drought occurrence among all other stations in 'AND' mode, but

the likelihood of a severe drought in the 'OR' case is less than for the Tamer, Gonbad, Gorgan dam and Agh-ghala stations.

In the next step, the return period is calculated with Equations (11) and (12). In Figure 6, the drought

**Figure 6** | Drought characteristics in the 'AND' state with different return periods.

characteristics are displayed in the 'AND' state with a return period of 2, 3, 4, 6, and 10 years.

In Figure 6, Tamer's droughts, with different return periods, except ten years, have always had the highest continuity and severity. Conversely, in Naharkhoran, droughts with different return periods have always had the lowest intensity and duration among other stations except in the ten-year return period. The duration of the drought with a 4-year return period at Agh-ghala station is the highest in the whole study area, while its severity is almost the lowest. In general, Agh-ghala droughts have a longer duration and less severity than other parts of Golestan province. Drought with different return periods in Gonbad usually have a shorter duration and lower severity than other areas. Arazkouse's droughts, similar to Tamer, have almost the highest severity and duration, but the drought with the ten-year return period in Arazkouse has the least amount of drought characteristics. The return period in

the 'OR' state has been calculated by Equation (12) and has been shown in Figure 7.

In Figure 7, at Tamer station, droughts that are more severe have a shorter return period. As seen, Naharkhoran's droughts with different return periods have less duration and severity than other areas. Each defined drought category has a certain return period calculated in two modes 'AND' and 'OR', and are shown in Figure 8.

In Figure 8, the return periods in the 'OR' state are always shorter than the 'AND' state. Approximately, the return periods of all drought conditions in the eastern part of Golestan province are the shortest. It means that the time interval between two droughts in this area is less than other areas. The droughts in the center and western part of the province have longer periods of return, which means that it takes more time to repeat a given drought in these areas. Arazkouse has the shortest return period in light drought, but in an extreme severe drought, it has a longer return period than

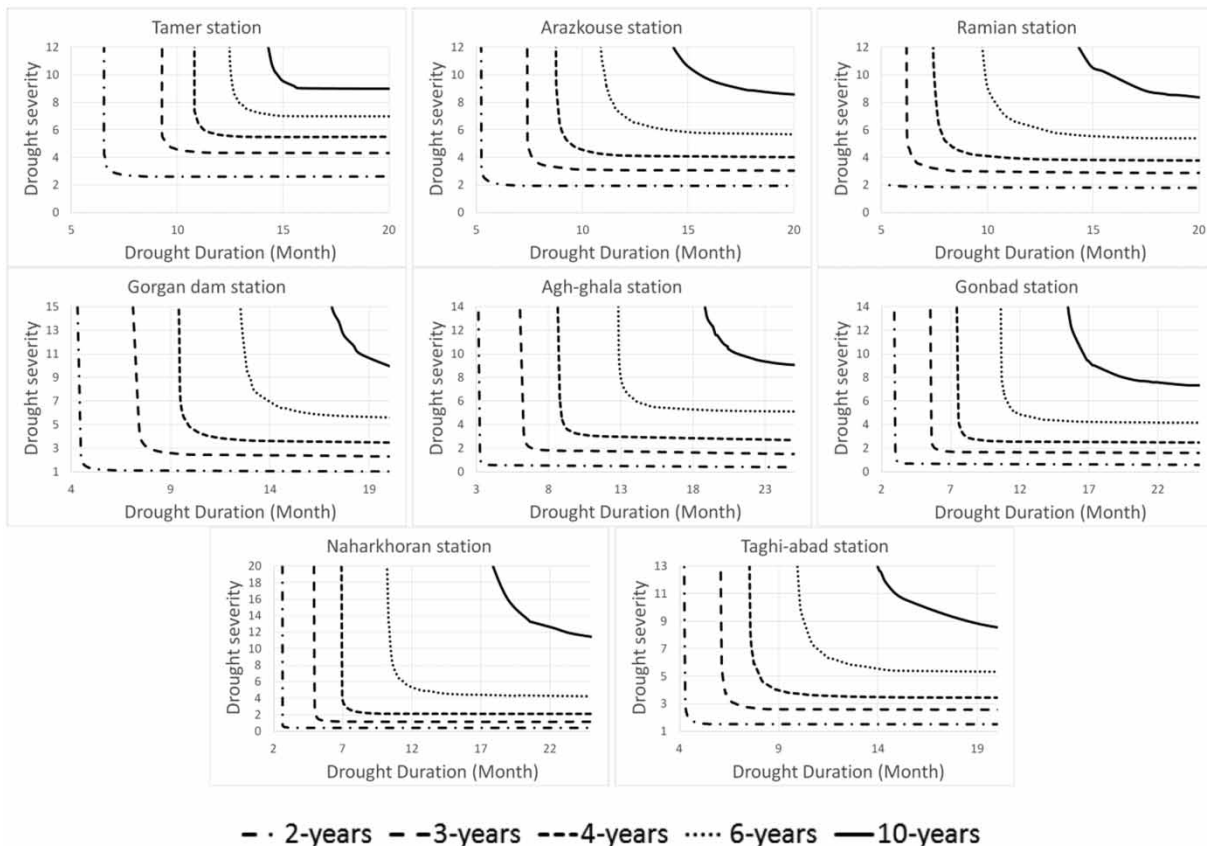


Figure 7 | Drought characteristics in the 'OR' state with different return periods.

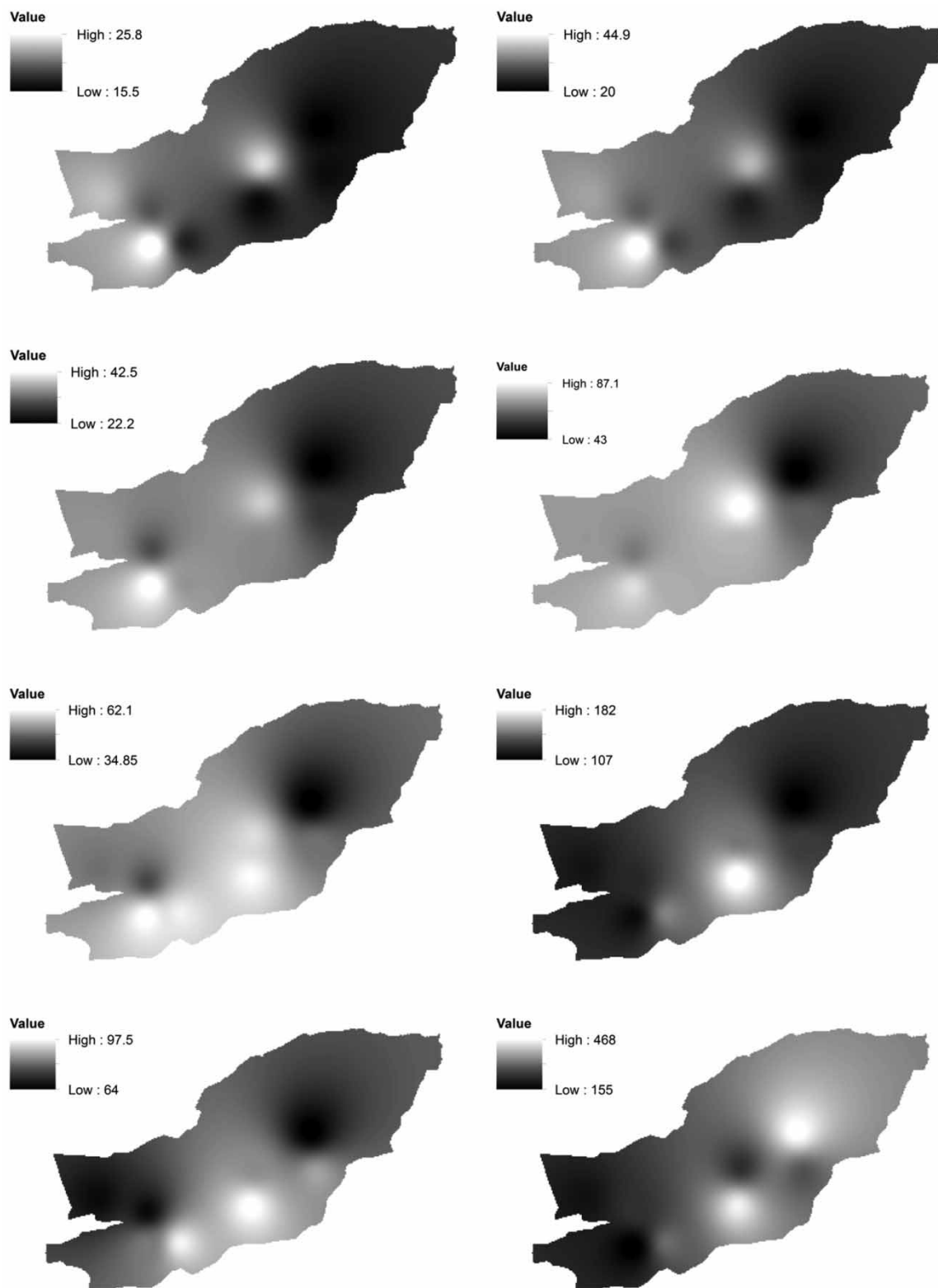


Figure 8 | Map of the return period of different levels of drought in Golestan province (Right: 'AND mode', Left: 'OR' mode) (First row: light, second row: moderate, third row: severe, and fourth row: extreme severe drought).

other areas. Ramian has the longest return period of severe and extreme severe drought and at the same time has the shortest return period of light drought. The drought return period of the Gorgan Dam is normal, but in severe droughts, the interval between repetition of droughts is reduced. The drought return period of Agh-ghala and Gonbad is similar to the Gorgan dam, but their fluctuations are higher. This means that in light and moderate droughts, they have a longer return period, but in severe droughts, they have a shorter return period than other stations. Changes in Taghi-abad drought return times are the reverse of changes in Agh-ghala. Also, for a mild drought, it has a short return period, whereas in severe droughts it has a longer return period than other stations.

Extreme severe 6-month drought has occurred in the western part of the province, according to Bazrafshan *et al.* (2011). All parts of Golestan have experienced all categories of drought, and the severity of the drought in the north is higher than in the center, which is consistent with Lashnizand (2004) and Mosaedi *et al.* (2008). In Gorgan station, 1983–1986 and 2005–2010, drought was observed according to Fathnia *et al.* (2017). There was no specific point for the onset of drought, which is contrary to Nosrati & Azarnivand (2002), who reported the onset of drought from the north.

CONCLUSION

Undoubtedly, drought is the most complex natural disaster due to the extent, long-term impact, gradual occurrence, and the numerous factors involved. Variables and indicators represent a part of the effects of drought, and so the preferred approach to drought analysis is to use a multivariable analysis. Here, by combining three indicators named SPI, SDI, and SWI a combined index based on the entropy weighting method was proposed. The proposed index is more accurate in estimating the onset of drought and indicates a good match, with three primary indicators to estimate extreme events. With the application of hip theory and the suggested index, dry and wet periods were determined in eight areas of Golestan province in the northern part of Iran. The lowest number of drought events was 21 in Gonbad and the highest was 39 in Taghi-abad. Drought characteristics including duration and severity were determined and resulted in the maximum duration of 74 months and the maximum severity of 56.7 in Golestan

province. The simultaneous analysis of the two characteristics of drought is more accurate than a separate survey. In order to calculate the probability of the occurrence of the composition of drought characteristics, the Frank, Gumbel, and Clayton copula functions were used. Then, based on the RMSE criteria, the Frank copula function showed the best performance. In addition, the return period of droughts was calculated with two 'OR' and 'AND' probabilistic approaches. Consequently, four drought categories were defined and their return periods were obtained by the proposed method. It can be concluded that the output maps created in this research are an effective tool for decision makers in the water sector who want to cope with drought challenge.

DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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