Risk assessment models to investigate the impact of emergency on a water supply system

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ABSTRACT

A water supply system is a critical infrastructure to support industrial and agricultural production and human life. It often operates abnormally in an emergent risk situation, resulting in shortage or suspension of water supply, even health risk and economic losses. In order to reduce negative impacts posed by these potential threats, identifying and evaluating possible risks in a water supply system has been becoming more necessary. For this reason, we establish two risk assessment models in accordance with two different situations based on uncertainty theory in the presence of insufficient historical data. In the proposed models, we first discuss emergency in three respects: the possibility of emergency occurrence, the consequence caused by emergency and system vulnerability. Then the risk to a water supply system is defined by the uncertain measure of loss-positive by incorporating a risk tolerance index and loss function contributed by the above analysis. Moreover, several theorems for calculating the risk index of series and parallel water supply systems are presented. Finally, we illustrate the feasibility and validity of the proposed models by implementing a series of numerical examples and further present some noteworthy observations.

Key words | emergencies, risk assessment, uncertainty theory, water supply system

HIGHLIGHTS

- We attempt to investigate the impact of emergency on a water supply system.
- We build a new loss function and define risk as the possibility of loss-positive.
- Risk tolerance (i.e. the degree of acceptable risk) is considered in the models.
- We are the first to use uncertainty theory to assess risk for a water supply system.

INTRODUCTION

Nowadays, integrated systems, such as water supply systems, are more than ever under the threat of emergent risk events, including natural disasters and man-made attacks, due to the deterioration of natural environment and intensification of contradictions. Different from common accidents with high likelihood and low negative effects, emergencies occur relatively infrequently, but cause destructive social and economic consequences with a long duration, such as the 2008 Wenchuan earthquake and tsunami, 2011 Tohoku earthquake and tsunami, 2012 Northern Italy earthquakes, 2015 Gorkha earthquake. This fact necessitates full attention to extract lessons from tragedies that have already happened (Smith et al. 1999; Ma et al. 2007; Zhang et al. 2012; Hirsch et al. 2015; Massalou et al. 2019). In addition, effective identification of potential risks as well as appropriate preventive measures for emergency have become more attractive. In this regard, water supply systems are amongst the most concerned subjects, playing a vital role in industrial activities, agricultural production and human life (Wang et al. 2013; Pagano et al. 2018).

More specifically, a water supply system, a massive and complex system, usually consists of many interconnected...
components, such as headwaters, intake structure, water purification plant, booster pumping station and pipeline network, etc. Meanwhile, it is an accessible system due to its long-term exposure to the natural environment. As a consequence of the complexity of strategic components and accessibility of spatially diverse subsystems, it is subject to natural and man-made threats. Generally, the whole water supply system often operates abnormally whichever subsystem fails, usually resulting in negative consequence and even more serious loss in an emergency situation. Given this, Lindhe et al. (2011) emphasised the necessity of effective risk assessment for water systems to identify and prepare for underlying risks and further reduce possible damage. Moreover, WHO (2008) and many other researchers echoed the identification of potential risks during the process of water supply from source to tap.

Motivated by the above opportunities and challenges, we attempt to investigate the impact of emergency on water supply systems in this paper. Since the historical data about emergency is relatively rare, or non-existent, it is hard to identify the probability that an emergent risk event will occur and estimate the probability distribution of consequences posed by such an event. To tackle these difficulties, in this study we adopt a new mathematical tool: uncertainty theory (Liu 2007), which has been proven to be an effective approach to deal with the uncertainty arising from the lack of historical data. With the aid of this theory, we adopt uncertain measures to measure the possibility that an emergency will occur. Then the function of water shortage is developed to quantify the consequence by such event. Moreover, system vulnerability is calculated by combining Markov latent effects modeling with uncertain reliability analysis. On the basis of these analyses, we present a new loss function of a water supply system, and then define the risk index as the uncertain measure of loss-positive to propose a novel risk assessment model. Later, this model is modified for the situation in which the emergency results in uncertain supply and uncertain demand of water simultaneously. The coherent framework of the presented models is shown in Figure 1.

The remainder of the study is organized as follows: the next section reviews works related to risk assessment about water supply systems and provides our main contributions. The Preliminaries section introduces the basic knowledge of uncertainty theory necessary to better inform our presented models. The following section elaborates a novel risk assessment model for a water supply system and proposes some theorems for calculating the risk index of series and parallel water supply systems, followed by a modified risk assessment model in the subsequent section. The Model implementation and numerical results section considers Handan City as a study area and conducts several

Figure 1 | The framework of risk assessment models by using uncertainty theory.
numerical examples to illustrate the feasibility and validity of the models. The final section concludes the paper and discusses the directions for future research.

LITERATURE REVIEW

This research is directly related to risk assessment of water supply systems. In recent years, a variety of methods to evaluate risk of water supply systems have sprung up. Among them, a semi-quantitative risk matrix is a feasible approach to identify potential hazards and rank them by quantifying the likelihood of occurrence and the severity of consequences (Davison et al. 2005; WHO 2008; Bartram et al. 2009). However this straightforward method hardly plays its full part in more complex water systems inflicted by interactions between various components and possible events.

An effective tool to remedy this limitation is fault tree analysis. Li (2007) developed a fuzzy fault tree analysis approach to identify and quantify the contribution of lower elements to the top research object in a water supply system. However, like the semi-quantitative risk matrix approach, this paper ignored aggregate risk and only ranked influencing factors. Lindhe et al. (2009) and (2011) not only analyzed the cause-effect relationship between whole system failure and key subsystem failure, but also defined the risk of a drinking water system as the customer minutes lost by using the fault tree method. However, sometimes designing an impeccable fault tree may be challenging in a more complicated water system due to its sophisticated structure and intricate environment.

Some other researchers presented rigorously mathematical methods to circumvent such predicament. Roozbahani et al. (2013) established an integrated hierarchical framework to identify relationships among a water supply system, its subsystems and hazards with respect to water quantity and quality. And he also provided equations to calculate risk for the whole system and its components, respectively. García-Mora et al. (2015) analyzed potential risk factors and defined reliability function to calculate failure probability of a water distribution network. Pietrucha-Urbanik & Tchorzewska-Cieślak (2017) assessed failure risk of a water supply system in eastern Poland by calculating Mean Time Between Failures and Mean Time To Repair by virtue of real data. Similar to Roozbahani et al. (2013), Orojloo et al. (2018) also presented the hierarchical structure of a whole water system and identified the relationships between each component. They first calculated risk of every layer and further yielded the whole system risk. In their paper, the risk calculation is the product of probability, consequence, and vulnerability which are obtained from experts’ opinion by using fuzzy set theory. All of these works are universal to evaluate risks triggered by different threats.

There are also studies focusing on a specifically trending topic. For example, Wang et al. (2010), Wang (2015) and Yoon et al. (2018) analyzed probabilistic performance of a water supply system affected by earthquake by virtue of Monte Carlo simulation. Whateley et al. (2015) applied a web-based decision support tool to assess potential risk of small water utilities posed by future climate changes which are predicted by General Circulation Model (GCM) projections. Gaffney et al. (2015), Wee & Aris (2017) and Blokker et al. (2018) focused on evaluating contamination risk of a water supply system to human health in terms of three different threats: pharmaceutical, endocrine disrupting compound and microorganism.

All these studies focused more on the identification of risk posed by various threats, but ignored the performance of the local water sector or users while evaluating the risk to the water supply system. In reality, people usually recognize an acceptable value for a bad situation which varies from person to person. For example, a stockholder takes action to stop loss while it approaches or passes his or her acceptable loss. So, to be realistic, we consider this acceptable value, in other words, risk tolerance in our study. In addition to this, the most commonly used tools in the aforementioned achievements are probability theory and fuzzy set theory. In the models which adopted probability theory, the likelihood that a common threat will occur and the resultant consequence are quantified by analyzing related historical data. However, it is imprecise to identify the possibility of an emergent risk event, and estimate the distributions of the consequence by using probability theory, since the historical data about emergency is relatively rare, or sometimes non-existent. To handle such a problem, other scholars choose fuzzy set theory and modeled uncertain parameters as fuzzy numbers. Nevertheless, fuzzy set theory does not emphasize the law of the excluded middle and the law of contradiction, compared with
uncertainty theory (Liu 2007), a ramification of using mathematics for handling uncertainty in the absence of historical data, which is frequently referenced in diverse domains.

To address these challenges, we benefit from uncertainty theory and propose two risk assessment models for a water supply system under emergency. In the presented models, we introduce risk tolerance and present a loss function, contributed by possibility, consequence and vulnerability analysis. Then the risk is defined as the uncertain measure that the loss will occur. The main contributions of our paper are reflected in the following aspects:

1. Most of the aforementioned studies put more emphasis on the identification of risk posed by common and frequent threats. And the works about risk assessment of emergency are still limited. So we attempt to investigate the impact of emergency on a water supply system in this study.

2. To be more realistic, we introduce risk tolerance, namely the degree of acceptable risk into our risk assessment models. In reality, this index can be identified by the damage estimation of unexpected events, political requirements from authority, public perception to uncertain consequences, etc.

3. To the best of our knowledge, this is the first study that applies uncertainty theory to assess the risk of a water supply system, which is conducive to the verification and application of this theory.

4. We consider a new loss function and then define the risk to a water supply system by the uncertain measure of loss-positive, differing from most studies. A bigger risk index indicates that the loss is more likely to occur. This new definition provides an alternative approach to quantify risk from a new perspective.

**PRELIMINARIES**

In the absence of historical data, it is impossible or hard to estimate the probability distribution of indeterminate parameters. In this case, consulting domain experts to evaluate the degree of belief that each event will occur is an available option. In order to rationally deal with personal degrees of belief Liu (2007) presented a new mathematical tool: uncertainty theory. Further, we expounded some fundamental concepts of this theory to better inform our presented models.

Uncertain measure is the most basic concept in uncertainty theory. It is a set function satisfying a normality axiom, duality axiom and subadditivity axiom, which is used to indicate the possibility that an uncertain event will occur.

Another fundamental concept is the uncertain variable. It is a measurable function from an uncertainty space to the set of real numbers. And uncertainty distribution is presented to describe such types of variable. Linear, zigzag, normal and lognormal uncertainty distributions are the most common types in uncertainty theory. Besides, the regular uncertainty distribution is accompanied with a corresponding inverse distribution.

On the basis of these concepts, Liu attempts to evaluate risk and system reliability in 2010 (Liu 2010a, 2010b). He first defined a loss function, and then regarded risk as the possibility that accidental loss will happen. Similarly, in reliability analysis, the structure function indicating system status is presented first, and the reliability is identified as the possibility that the system is working.

The definitions and theorems concerning the above are presented in the Appendix.

**RISK ASSESSMENT MODEL CONSIDERING UNCERTAIN SUPPLY**

In the traditional risk assessment model, the risk index is denoted as the product of probability of the threat, possible consequence and probability of the system failure. The mathematical equation is given as follows:

$$ R = P_a(1 - P_e)C $$

where $P_a$ is the probability of threat, $1 - P_e$ is the probability of the system failure, and $C$ is the consequence posed by the threat. This formula was proposed based on probability theory in which the values for $P_a$, $P_e$, and $C$ are given by analyzing enough historical data. However, if there is a lack of sufficient samples, these parameters can be difficult to estimate. Under this condition, consulting experts to obtain the degree of belief is an advisable alternative for the
quantification of the above parameters. In order to tackle degree of belief, uncertainty theory is introduced in this section. And drawing from formula (1), we present two new risk assessment models, which are applicable for uncertain environments.

**The possibility of emergency**

In Equation (1), \( P_a \) is the probability of a common attack which is quantified by the historic records. In reality, historic records about emergency are relatively rare or non-existent. Without sufficient historic data, using the frequency which an emergency has happened to define its probability is imprecise and unsuitable in academic research. And it is almost impossible to estimate the probability of an emergency which never happened. Under this circumstance, consulting domain experts to evaluate the degree of belief that an emergency will occur is a feasible alternative. Then uncertain measure \( M \) is adopted to quantify the above degree of belief. Considering the extent of the damage posed by an emergency, we divide the emergency into \( n \) levels in this paper. The possibility of the \( i \)-th level of emergency is denoted as \( M_i \). For different emergent risk events, the classification criterion are also different. For example, an earthquake is commonly divided into 9 levels according to the Richter Scale and a hurricane is divided into 5 levels according to the Saffir-Simpson Hurricane Wind Scale. Actually, decision-makers can classify a specific emergency in accordance with its domain criterions. Further, this approach is also appropriate to evaluate the comprehensive risk posed by various emergent risk events. Thus \( n \) means emergent risk events considered and \( M_i \) defines the possibility that the event \( i \) will occur.

**Water shortage function with uncertain supply**

The immediate consequence of the failure of a water supply system is that the user demand for water cannot be satisfied. So the associated consequence posed by emergency can be expressed as the rate of water shortage:

\[
C = \frac{Q_d - Q_s}{Q_d}
\]

(2)

where \( C \) is the rate of water shortage, \( Q_d \) is the total demand for water, \( Q_s \) is the available supply of water. Here, we model water supply as an uncertain variable, since a water supply system often operates abnormally when an emergency happens which can result in uncertain supply. Thus the associated consequence is denoted as:

\[
C_i = \frac{Q_d - \xi_i}{Q_d}, \quad i = 1, 2, \ldots, n
\]

(3)

where for each level of emergency, \( C_i \) is the rate of water shortage and \( \xi_i \) is the available supply of water. And we assume that \( \xi_1, \xi_2, \ldots, \xi_n \) are independent uncertain variables with uncertainty distributions \( \Phi_1, \Phi_2, \ldots, \Phi_n \), respectively.

The demand of water \( Q_d \) is not affected by emergency in this equation. In reality, the value of \( Q_d \) can be obtained according to official statistics or official forecast. As for water supply (\( \xi_1 \) as an example), in order to determine its distribution, we can invite domain experts to obtain a set of experimental experimental data \((x_1, a_1), (x_2, a_2), \ldots, (x_n, a_n)\), where \((x_i, a_i)\) means the degree of belief that water supply \( \xi_1 \) is less than or equal to \( x_i \) is \( a_i \). It’s worth noting that \( x_1 < x_2 < \cdots < x_n \), \( 0 \leq a_1 \leq a_2 \leq \cdots \leq a_n \leq 1 \). Thus the uncertainty distribution of \( \xi_1 \) can be obtained based on these data:

\[
\Phi_1(x) = \begin{cases} 
0, & \text{if } x < x_1 \\
n_1 + \frac{(a_{i+1} - a_i)(x - x_i)}{x_{i+1} - x_i}, & \text{if } x_i \leq x < x_{i+1}, 1 \leq i < n \\
n, & \text{if } x_n \leq x
\end{cases}
\]

Repeating the above process, we can determine the distributions of other uncertain variables \( \xi_i \). Thus, the distribution of \( C_i \) can be obtained by Equation (3).

**Water supply system vulnerability assessment**

The various studies on vulnerability appeared as a consequence of diverse perspectives (Watts & Bohle 1993; Pelling 2003; Villagrán De León 2006; Chambers 2010), and a uniform definition elaborating the content of this term was still not settled. In terms of a water supply system, benefiting from these contributions, we consider...
system vulnerability as the failure possibility of a system due to exposure to a hazard. Recently, some research has focused on vulnerability assessments to evaluate the failure possibility caused by potential threats. Thereinto, Hashimoto et al. (1982) constructed a mathematical index, the weighted sum of system performance variables multiplying by its probability to calculate the vulnerability of a water resource system. Cooper et al. (2005) assessed vulnerability of a water supply system under willful attack by proposing a Markov latent effects model. Yazdani & Jeffrey (2012) modeled a water distribution system’s weighted and directed networks and adopted demand-adjusted entropic degree to quantify system vulnerability. Zohra et al. (2012) presented a vulnerability index method to evaluate the vulnerability of water pipelines affected by seismicity. Zhang et al. (2016) first constructed a criteria framework for a water supply system, then the weight of every component was determined by integrating stochastic analytical network process and game cross evaluation to calculate the weighted vulnerability values.

Among these references, Cooper et al. (2005) catered to our expectation for addressing uncertain condition without sufficient data. Cooper evaluated asset security of water utilities under willful attack by using Markov latent effects model (Cooper 2004), the combination of the concept of latent effects (Reason 1997) and a chained subjective analysis methodology. In his work, a water supply system was decomposed into decision elements which represent factors affecting the possibility that an event will impact on system security (Figure 2). Specifically, each bottom decision element is accompanied by a subjectively direct input (in the range of 0 to 1, 0 for weak and 1 for strong). For each input, a weighting factor is assigned and the sum of all these weighting factors acting on a decision element must equal 1. Thus the linear weighted sum of inputs and weighting values is the attribute value of the superior decision element.
element. Eventually the output, measuring system security, is obtained by reiterating this process (see Cooper et al. (2005) for a more detailed explanation).

As we know, the whole supply system is a complex system composed of different subsystems. In our work, we first calculate the security of subsystem $j$ under the $i$-th level of emergency (denoted by $a_{ij}$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$) by using the above Markov latent effects model. Then, with the aid of uncertain reliability analysis, the security of the whole water supply system under the $i$-th level of emergency (denoted by $S_i$, $i = 1, 2, \ldots, n$) can be obtained. Naturally, we may calculate the system vulnerability under the $i$-th level of emergency (represented by $V_i$, $i = 1, 2, \ldots, n$) according to the following equation

$$V_i = 1 - S_i, \quad i = 1, 2, \ldots, n$$

### Novel risk assessment model for water supply system

Contributed by the above analysis, Equation (1) can be modified as follows:

$$R_i = M_i C_i V_i, \quad i = 1, 2, \ldots, n$$

where for each level of emergency, $R_i$ is the risk, $M_i$ is the possibility, $C_i$ is the possible consequence and $V_i$ is the system vulnerability. In this equation, $C_i$ is a measurable function with respect to uncertain variable $\xi_i$, respectively. So $R_1, R_2, \ldots, R_n$ are also independent uncertain variables with uncertainty distributions $Y_1, Y_2, \ldots, Y_n$, where

$$Y_i(x) = M(R_i \leq x)$$

$$= M\left\{ M_i V_i \frac{Q_d - \xi_i}{Q_d} \leq x \right\}$$

$$= M\left\{ Q_d - \frac{Q_d \xi_i}{M_i V_i} \leq x \right\}$$

$$= 1 - M\left\{ \xi_i \leq Q_d - \frac{Q_d \alpha}{M_i V_i} \right\}$$

$$= 1 - \Phi_i \left( Q_d - \frac{Q_d \alpha}{M_i V_i} \right)$$

(6)

Further, taking several factors into account such as the damage estimation of unexpected events, pre-existing precautionary measures, public perception of uncertain consequences etc, we introduce the acceptable risk index, namely the risk tolerance of water supply system $R_0$. The loss occurs when maximum value of $R_i$ exceeds given value $R_0$ and then we have a loss function:

$$f(R_1, R_2, \ldots, R_n) = R_1 \lor R_2 \lor \cdots \lor R_n > R_0$$

(7)

**Theorem 1.** Let the risk under a different level of emergency $R_i$ be the product of the possibility $M_i$, system vulnerability $V_i$, consequence $C_i$ which refers to the ratio of the difference between water demand $Q_d$ and water supply $\xi_i$ with regular uncertainty distribution $\Phi_i$ to water demand, $i = 1, 2, \ldots, n$, respectively. The loss occurs when maximum value of $R_i$ is greater than risk tolerance $R_0$. Then the risk index of a water supply system is

$$R = \sqrt[n]{\prod_{i=1}^{n} \Phi_i \left( Q_d - \frac{Q_d \times R_0}{M_i \times V_i} \right)}$$

(8)

**Proof.** According to Definition 8 (see Appendix), the risk index can be expressed as:

$$R = M\{f(R_1, R_2, \ldots, R_n) > 0\}$$

$$= M\{R_1 \lor R_2 \lor \cdots \lor R_n > R_0\}$$

It is obvious that the function $f(R_1, R_2, \ldots, R_n)$ is strictly increasing with respect to $R_1, R_2, \ldots, R_n$. According to Risk Index Theorem, the risk index is just the root $\alpha$ of the equation

$$Y_1^{-1}(1 - \alpha) \lor Y_2^{-1}(1 - \alpha) \lor \cdots \lor Y_n^{-1}(1 - \alpha) = R_0$$

Thus,

$$Y_1(R_0) \land Y_2(R_0) \land \cdots \land Y_n(R_0) = 1 - \alpha$$

Namely,

$$R = 1 - Y_1(R_0) \land Y_2(R_0) \land \cdots \land Y_n(R_0)$$

According to Equation (6), we have

$$R = \sqrt[n]{\prod_{i=1}^{n} \Phi_i \left( Q_d - \frac{Q_d \times R_0}{M_i \times V_i} \right)}$$

(8)
Risk analysis of serial system and parallel system

Without loss of generality, the common system is an integrated system composed by a number of important components in different ways, such as series, parallel or serial-parallel. In general, parallel systems are less likely to fail than series systems. Therefore, we denote $\alpha_{ij}$ as the reliability coefficient of the $j$-th component under the $i$-th level of emergency and then discuss the risk of series and parallel water supply systems.

**Theorem 2.** Let the risk under a different level of emergency $R_i$ be the product of the possibility $M_i$, system vulnerability $V_i$ which is related to subsystem security $\alpha_{ij}$, consequence $C_i$ which refers to the ratio of the difference between water demand $Q_d$ and water supply $\xi_i$ with regular uncertainty distribution $\Phi_i$ to water demand, $i = 1, 2, \ldots, n, j = 1, 2, \ldots, m$, respectively. The loss occurs when maximum value of $R_i$ exceeds risk tolerance $R_0$. Then the risk index of a series water supply system is

$$R = \bigvee_{i=1}^{n} \Phi_i \left( Q_d - \frac{Q_d \times R_0}{(1 - \bigwedge_{j=1}^{m} \alpha_{ij}) \times M_j} \right)$$

**Proof.** Failure of any unit in a series system will cause the entire system to malfunction. Thus, by Theorem 3, the system security is

$$S_i = \bigwedge_{j=1}^{m} \alpha_{ij}$$

By Equation (4), the system vulnerability can be denoted as

$$V_i = 1 - S_i$$

$$= 1 - \bigwedge_{j=1}^{m} \alpha_{ij}$$

According to Equation (8), the risk index is

$$R = \bigvee_{i=1}^{n} \Phi_i \left( Q_d - \frac{Q_d \times R_0}{(1 - \bigvee_{j=1}^{m} \alpha_{ij}) \times M_i} \right)$$

**Theorem 3.** Let the risk under a different level of emergency $R_i$ be the product of the possibility $M_i$, system vulnerability $V_i$ which is related to subsystem security $\alpha_{ij}$, consequence $C_i$ which refers to the ratio of the difference between water demand $Q_d$ and water supply $\xi_i$ with regular uncertainty distribution $\Phi_i$ to water demand, $i = 1, 2, \ldots, n, j = 1, 2, \ldots, m$, respectively. The loss occurs when maximum value of $R_i$ exceeds risk tolerance $R_0$. Then the risk index of a parallel water supply system is

$$R = \bigvee_{i=1}^{n} \Phi_i \left( Q_d - \frac{Q_d \times R_0}{(1 - \bigvee_{j=1}^{m} \alpha_{ij}) \times M_i} \right)$$

**Proof.** As long as one unit works in the parallel system, the entire system works. Thus, by Theorem 3, the system security is

$$S_i = \bigvee_{j=1}^{m} \alpha_{ij}$$

By Equation (4), the system vulnerability can be denoted as

$$V_i = 1 - S_i$$

$$= 1 - \bigvee_{j=1}^{m} \alpha_{ij}$$

According to Equation (8), the risk index is

$$R = \bigvee_{i=1}^{n} \Phi_i \left( Q_d - \frac{Q_d \times R_0}{(1 - \bigvee_{j=1}^{m} \alpha_{ij}) \times M_i} \right)$$
situation and the water demand is also uncertain. For this reason, the water demand and available water supply are assumed to be uncertain variables in this section. And the modified formula to calculate water shortage rate is

\[ C_i = \frac{\eta_i - \xi_i}{\eta_i}, \quad i = 1, 2, \ldots, n \] (11)

where for each level of emergency, \( C_i \) is the ratio of water shortage to total demand for water, \( \eta_i \) is the total demand for water and \( \xi_i \) is the available supply of water. In (11), \( \xi_1, \xi_2, \ldots, \xi_n, \eta_1, \eta_2, \ldots, \eta_n \) are independent uncertain variables with uncertainty distributions \( \Phi_1, \Phi_2, \ldots, \Phi_n, \Psi_1, \Psi_2, \ldots, \Psi_n \), respectively. Here, the distribution of each uncertain variable can be obtained according to a set of experimental data provided by domain experts. After determining these distributions, the uncertainty distribution of \( C_i \) can also be obtained by Equation (11).

**Modified risk assessment model for water supply system**

In Equation (5), \( R_1, R_2, \ldots, R_n \) are independent uncertain variables with uncertainty distributions \( \Upsilon_1, \Upsilon_2, \ldots, \Upsilon_n \), because \( \xi_i \) and \( \eta_i \) are now independent uncertain variables. By (5) and (11), we have

\[ R_i = M_iV_i \left( \frac{\eta_i - \xi_i}{\eta_i} \right) \]

It is noted that this measurable function is strictly increasing with respect to \( \eta_i \) and strictly decreasing with respect to \( \xi_i \). By Theorem 1, we have

\[ \Upsilon_1(x) = \sup_{y_1} (1 - \Phi_1(y_1)) \wedge \Psi_1(x_1) \]

\[ = \sup_{x_1 = \frac{\eta_1 - \xi_1}{\eta_1}} (1 - \Phi_1(y_1)) \wedge \Psi_1(x_1) \]

Let \( y = \frac{M_iV_iy_1}{M_iV_1} - x \), we have

\[ \Upsilon_1(x) = \sup_{y > 0} \left( 1 - \Phi_1 \left( \frac{M_iV_iy - xy}{M_iV_i} \right) \right) \wedge \Psi_1(y) \] (12)

Similarly, we have

\[ \Upsilon_i(x) = \sup_{y > 0} \left( 1 - \Phi_i \left( \frac{M_iV_iy - xy}{M_iV_i} \right) \right) \wedge \Psi_i(y) \] (13)

**Theorem 4.** Let the risk under a different level of emergency \( R_i \) be the product of the possibility \( M_i \), system vulnerability \( V_i \), consequence \( C_i \) which refers to the ratio of the difference between water demand \( \eta_i \) with uncertainty distribution \( \Psi_i \) and water supply \( \xi_i \) with regular uncertainty distribution \( \Phi_i \) to water demand, \( i = 1, 2, \ldots, n \), respectively. The loss occurs when maximum value of \( R_i \) is greater than risk tolerance \( R_0 \). Then the risk index of a water supply system is

\[ R = 1 - \bigwedge_{i=1}^{n} \sup_{y > 0} \left( 1 - \Phi_i \left( \frac{M_iV_iy - R_0y}{M_iV_i} \right) \right) \wedge \Psi_i(y) \] (14)

**Proof.** By Definition 8, the risk index can be expressed as:

\[ R = \mathcal{M}(f(R_1, R_2, \ldots, R_n) > 0) = \mathcal{M}(R_1 \lor R_2 \lor \cdots \lor R_n - R_0 > 0) \]

It’s obvious that the function \( f(R_1, R_2, \ldots, R_n) \) is strictly increasing with respect to \( R_1, R_2, \ldots, R_n \). By Theorem 2, the risk index \( R \) is just the root \( \alpha \) of the equation

\[ \Upsilon_1^{-1}(1 - \alpha) \lor \Upsilon_2^{-1}(1 - \alpha) \lor \cdots \lor \Upsilon_n^{-1}(1 - \alpha) = R_0 \]

Thus,

\[ \Upsilon_1(R_0) \lor \Upsilon_2(R_0) \lor \cdots \lor \Upsilon_n(R_0) = 1 - \alpha \]

Namely,

\[ R = 1 - \bigwedge_{i=1}^{n} \sup_{y > 0} \left( 1 - \Phi_i \left( \frac{M_iV_iy - R_0y}{M_iV_i} \right) \right) \wedge \Psi_i(y) \]

**Risk analysis of serial system and parallel system**

**Theorem 5.** Let the risk under a different level of emergency \( R_i \) be the product of the possibility \( M_i \), system vulnerability \( V_i \) which is related to subsystem security \( a_i \), consequence \( C_i \) which refers to the ratio of the difference between
water demand $Q_d$ and water supply $\xi_i$ with regular uncertainty distribution $\Phi_i$ to water demand, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, respectively. The loss occurs when maximum value of $R_i$ exceeds risk tolerance $R_0$. Then the risk index of series water supply system is

$$R = 1 - \left( \underset{y>0}{\text{sup}} \left( 1 - \Phi_i \left( y - \frac{R_0 y}{(1 - \bigwedge_{j=1}^{m} a_{ij}) M_i} \right) \right) \right)^{\enspace \Psi_i(y)}$$  \hspace{1cm} (15)

Proof. Failure of any unit in a series system will cause the entire system to malfunction. Thus, by Theorem 3, the system security is

$$S_i = \bigvee_{j=1}^{m} a_{ij}$$

By Equation (4), the system vulnerability can be denoted as

$$V_i = 1 - S_i = 1 - \bigvee_{j=1}^{m} a_{ij}$$

According to Equation (14), the risk index is

$$R = 1 - \left( \underset{y>0}{\text{sup}} \left( 1 - \Phi_i \left( y - \frac{R_0 y}{(1 - \bigvee_{j=1}^{m} \alpha_{ij}) M_i} \right) \right) \right)^{\enspace \Psi_i(y)}$$

**Theorem 6.** Let the risk under a different level of emergency $R_i$ be the product of the possibility $M_i$, system vulnerability $V_i$ which is related to subsystem security $a_{ij}$, consequence $C_i$ which refers to the ratio of the difference between water demand $Q_d$ and water supply $\xi_i$ with regular uncertainty distribution $\Phi_i$ to water demand, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, respectively. The loss occurs when maximum value of $R_i$ exceeds risk tolerance $R_0$. Then the risk index of parallel water supply system is

$$R = 1 - \left( \underset{y>0}{\text{sup}} \left( 1 - \Phi_i \left( y - \frac{R_0 y}{(1 - \bigwedge_{j=1}^{m} a_{ij}) M_i} \right) \right) \right)^{\enspace \Psi_i(y)}$$  \hspace{1cm} (16)

Proof. As long as one unit works in the parallel system, the entire system works. Thus, by Theorem 3, the system security is

$$S_i = \bigvee_{j=1}^{m} a_{ij}$$

By Equation (4), the system vulnerability can be denoted as

$$V_i = 1 - S_i = 1 - \bigvee_{j=1}^{m} a_{ij}$$

According to Equation (14), the risk index is

$$R = 1 - \left( \underset{y>0}{\text{sup}} \left( 1 - \Phi_i \left( y - \frac{R_0 y}{(1 - \bigwedge_{j=1}^{m} a_{ij}) M_i} \right) \right) \right)^{\enspace \Psi_i(y)}$$

**MODEL IMPLEMENTATION AND NUMERICAL RESULTS**

In this section, we consider Handan City, in south China's Hebei Province, as study area and design several examples to illustrate the feasibility and validity of the proposed risk assessment models.

**Study area and data**

The water supply system in Handan City is responsible for the water of local residents and some enterprises, charged by Yangjiaopu Water and Yuecheng Reservoir mainly. More specifically, water is pumped out from aforesaid groundwater and reservoir water through intake structures, then delivered to the Sandi Water Plant and Tiexi Water Plant through water pipes. After purification, water distribution networks transmit the treated water which accords with water quality standard to end users. During this process, interrelated subsystems cooperate with each other to meet water demands. To keep things simple, we break down a water supply system into a few parts: headwaters, intake structure, pipeline network, water purification plant and distribution network mainly. From this point of view, a simplified water supply system in Handan City is described in Figure 3.
Now, we intend to evaluate the risk posed by a public health event. From Equations (8) and (14), we know that the risk index will be calculated easily once main parameters are determined. Generally, this kind of emergency is divided into 4 grades, according to its nature, extent and scope of damage. Thereinto, level I represents the most severe impact, followed by levels II, III, IV. Then we let \( i = 1, 2, 3, 4 \) accordingly and give
\[
M_1 = 0.05; M_2 = 0.15; M_3 = 0.3; M_4 = 0.5
\]

For system vulnerability, weighting factors are drawn uniformly from \([0,1]\) and normalized then to make the sum of all factors acting on the same decision element equal to 1. Further, the attribute values of each input is also drawn uniformly from \([0,1]\), except four decision elements: system barriers, time for attack to impact, response time and likelihood of reporting, whose attribute values are drawn uniformly from \([0,0.25]\), \([0.25,0.5]\), \([0.5,0.75]\) and \([0.75,1]\) corresponding to the four grades. The detailed results of the nine subsystems’ security are shown in Table 1.

**The implementation of the risk assessment models**

In the novel risk assessment model, we assume the total demands for water under every level of emergency. \( \xi_1, \xi_2, \xi_3, \xi_4 \) are uncertain variables with zigzag uncertainty distributions \( \mathcal{Z}(20, 40, 50), \mathcal{Z}(50, 50, 70), \mathcal{Z}(50, 60, 80), \mathcal{Z}(70, 90, 100) \), respectively. Besides, following from theorem 3, we can obtain system security from the following equation
\[
S_i = (\alpha_{i1} \land \alpha_{i2} \land \alpha_{i3} \land \alpha_{i4} \land \alpha_{i9}) \lor (\alpha_{i5} \land \alpha_{i6} \land \alpha_{i7} \land \alpha_{i8} \land \alpha_{i9})
\]

Then, we have
\[
S_1 = 0.4012, S_2 = 0.4424, S_3 = 0.5218, S_4 = 0.5743
\]

Other default parameters are set as \( Q_d = 100 \) and \( R_0 = 0.01 \). According to Equation (8), the risk index of the water supply system is
\[
R = \sqrt{4} \Phi_i \left( 100 - \frac{100 \times 0.01}{M_i \times V_i} \right)
\]
\[
= \Phi_i(66.5999) \lor \Phi_i(88.044) \lor \Phi_i(93.0294) \lor \Phi_i(95.3019) \\
= 1 \lor 1 \lor 1 \lor 0.7651 \\
= 1
\]

The result reflects the loss will happen with the possibility of 1, in other words, the possibility that the risk brought by an emergency greater than risk tolerance 0.01 is 1. Now we change the risk tolerance of water supply system \( R_0 \) to test what happens to the risk index. If \( R_0 \) is

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**Table 1 | Aggregate value calculated for security of the nine subsystems of the water supply system**

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Four levels for public health event</td>
<td>i = 1</td>
<td>0.4701</td>
<td>0.4012</td>
<td>0.4743</td>
<td>0.4375</td>
<td>0.5303</td>
<td>0.5378</td>
<td>0.4318</td>
<td>0.5625</td>
</tr>
<tr>
<td></td>
<td>i = 2</td>
<td>0.5135</td>
<td>0.4424</td>
<td>0.5638</td>
<td>0.5311</td>
<td>0.5843</td>
<td>0.5832</td>
<td>0.5042</td>
<td>0.4366</td>
</tr>
<tr>
<td></td>
<td>i = 3</td>
<td>0.5888</td>
<td>0.5218</td>
<td>0.6052</td>
<td>0.5797</td>
<td>0.6612</td>
<td>0.6600</td>
<td>0.5522</td>
<td>0.4951</td>
</tr>
<tr>
<td></td>
<td>i = 4</td>
<td>0.6532</td>
<td>0.5674</td>
<td>0.6710</td>
<td>0.6321</td>
<td>0.6958</td>
<td>0.7244</td>
<td>0.6391</td>
<td>0.5743</td>
</tr>
</tbody>
</table>

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**Figure 3 | A simplified water supply system in Handan City.**
0.02, the risk index of water supply system is
\[
R = \sqrt[4]{\Phi_1(100 - 100 \times 0.02)} 
= \Phi_1(33.1997) \lor \Phi_1(76.088) \lor \Phi_1(86.0588) \lor \Phi_1(90.6037) 
= 0.16 \lor 1 \lor 1 \lor 0.5302 
= 1
\]

When \( R_0 \) is set to 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, respectively, the risk index of the water supply system is 0.9772, 0.7779, 0.6287, 0.4088, 0.0603, 0, respectively.

As for modified risk assessment model, we also assume \( \zeta_i \) and \( \eta_i \) are uncertain variables with zigzag uncertainty distributions. More concretely, \( \zeta_1 \sim \mathcal{Z}(20, 40, 50), \zeta_2 \sim \mathcal{Z}(30, 50, 70), \zeta_3 \sim \mathcal{Z}(50, 60, 80), \zeta_4 \sim \mathcal{Z}(70, 90, 100), \eta_1 \sim \mathcal{Z}(40, 50, 70), \eta_2 \sim \mathcal{Z}(50, 70, 80), \eta_3 \sim \mathcal{Z}(70, 80, 90) \) and \( \eta_4 \sim \mathcal{Z}(80, 90, 100) \), respectively. Similarly, we have system security as follows:
\[
S_1 = 0.4012, S_2 = 0.4424, S_3 = 0.5218, S_4 = 0.5743.
\]

Now we let \( R_0 = 0.01 \). By formula (14), the risk index of water supply system is
\[
R = 1 - \frac{4}{\sup_{y>0}} \left( \frac{1 - \Phi_1\left(y - 0.01y\right)}{(1-S_i)M_i} \right) \land \Psi_i(y)
= 1 - \Psi_1(54.021) \land \Psi_2(63.8162) \land \Psi_3(75.0776) \land \Psi_4(91.4324)
= 1 - 0.6005 \land 0.3454 \land 0.2539 \land 0.5716
= 0.7461
\]

The result shows that the risk index is 0.7461, indicating that the possibility that the risk brought by an emergency exceeding its risk tolerance 0.01 is 0.7461. As \( R_0 = 0.02 \), the risk index of water supply system is
\[
R = 1 - \frac{4}{\sup_{y>0}} \left( \frac{1 - \Phi_1\left(y - 0.02y\right)}{(1-S_i)M_i} \right) \land \Psi_i(y)
= 1 - \Psi_1(67.5676) \land \Psi_2(68.147) \land \Psi_3(76.9069) \land \Psi_4(92.9112)
= 1 - 0.9392 \land 0.4357 \land 0.3453 \land 0.6456
= 0.6547
\]

Now we set \( R_0 \) to 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, respectively, and the corresponding risk values are 0.5586, 0.4531, 0.2614, 0.0746, 0, 0, respectively.

**Result analysis**

The numerical results of two risk assessment models are shown in Figure 4, revealing the relationship between risk tolerance and risk index clearly. From this figure, we know that the risk value \( R \) continually drops to zero finally with the increase of \( R_0 \) in both models. The direct reason for this phenomenon derives from the loss function (7), showing that the larger the risk tolerance is, the lower the possibility that loss will occur is. Such explanation is congenial with common sense that the people with a greater risk tolerance are usually exposed to a much smaller risk than those with lower risk tolerance. Moreover, it is worth noting that the growth of \( R_0 \) is not always followed by a lower risk value. It has no effect on risk index when the risk tolerance are greater than 0.8 in the first model and 0.7 in the second model. These observations indicate that the risk tolerance \( R_0 \) is a crucial factor in the identification of risk posed by emergency. However, as mentioned above, many scholars ignore this fact in their risk assessment models. For example, Lindhe et al. (2009) and Lindhe et al. (2011) adopted a fault tree method to identify the relationship between whole system failure and a key subsystem failure, and defined the
risk by considering customer minutes lost. Roozbahani et al. (2013) and Orojloo et al. (2018) presented a hierarchical risk assessment model for a water supply system and irrigation canal networks, respectively. In these models, scholars first recognized and calculated the risk of each component of the research object. Then the risk of the whole system is the aggregate value of the risk of these subsystems. All of the above only analyzed the critical components related to system failure and provided functions to evaluate risk, but ignored the influence of risk tolerance. To fill this gap, we take this index into account to assess the risk of a water supply system. In reality, to recognize the proper risk tolerance, the decision-makers had better consider multiple factors comprehensively, such as the damage estimation of unexpected events, political requirements from authority, public perception to uncertain consequences, etc.

Further, our risk assessment models are proposed based on the methodology developed by AWWA Research Foundation (2002). This approach identified risk as the combination of probability of attack, probability of system effectiveness (which can be regarded as vulnerability) and consequence, to assess the risk of water utilities. Similarly, Roozbahani et al. (2013) and Orojloo et al. (2018) defined the risk of each subcomponent of water supply systems and irrigation canal networks as the product of probability, consequence and vulnerability. Osikanmi et al. (2020) multiplied probability and consequence to calculate the risk of packaged water production systems posed by 26 hazardous events. However, the possibility of an emergent risk event occurring is very low, but its adverse consequence may be devastating. For example, if we set 0.01 for probability of emergency, 0.9 for its adverse impact (the rate of water shortage in our models), 0.9 for system vulnerability, respectively, the risk value of the water supply system posed by such emergency is 0.0081 by multiplying three parameters or 0.009 by multiplying the first two parameters, which may mislead decision makers to underestimate the potential risk. Our work addresses such challenge and attempts to investigate the impact of emergency on water supply systems. And the obtained results in Figure 4 show that a water supply system is at a higher risk for a public health event under lower risk tolerance, which verifies the effectiveness of risk assessment models in this study.

Furthermore, another remarkable observation is that the first risk assessment model always yields a larger risk index than the second one. Assuming that the water demand and supply are uncertain variables in the latter model while water supply only in the first model is the main reason for this phenomenon, this means that the identification of uncertain variables plays a key role in the risk evaluation. So in reality, decision-makers need to choose adaptive model in accordance with actual situation to assess the risk of a water supply system. Giving top priority to validity of the model and accuracy of the result, a risk assessment model with uncertain supply and demand is a better alternative while emergency conducts negative effects on water supply system and user behavior simultaneously, otherwise a risk assessment model with uncertain supply is more applicable. For example, we aim to evaluate the risk posed by a public health event in this section. Such an emergent risk event may result in abnormal water supply, but not affect the behaviour of users. Therefore, the water demand remains known, as usual, and the supply of water can be regard as an uncertain variable. In this case, the results of the first risk assessment model are more accurate.

**CONCLUSIONS**

In this study, we presented two risk assessment models to investigate the impact of emergency on a water supply system. Considering that the historical data about emergency is really limited, we introduced uncertainty theory to assess the risk of water supply system. In the presented models, we integrated possibility analysis, consequence analysis and system vulnerability analysis to present a new loss function for a water supply system. Then the risk index was defined as the uncertain measure of loss-positive. Finally, we tested the performance of the two models and verified the influence of risk tolerance on risk index by implementing a series of numerical examples.

For future research, we intend to extend our work in several directions. First, we consider water shortage as the direct consequence posed by emergency in this study. In the following work, some other elements such as economic loss and social impact are under consideration to identify the consequence posed by emergency, so as to perfect our models. Second, in
order to obtain results easily, we simplify the water supply system as the integration of headwaters, intake structure, pipeline network, water purification plant and distribution network in numerical experiments. Further study may attempt to assess risk of a more complex water supply system. Finally, risk tolerance is regarded as a known value in the presented models. However, people are more likely to describe it as an interval rather than a precise number in reality. So another direction for further research can be the consideration of risk tolerance as an interval number to improve our models and then investigate the differences between these models.

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COMPLIANCE WITH ETHICAL STANDARDS

The authors declare that they have no conflict of interest.

DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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