Effects of weir geometry on scour development in the downstream of Piano Key Weirs

Alireza Mosalman Yazdi, S. Abbas Hoseini, Sohrab Nazari and Nosratollah Amanian

ABSTRACT

Scouring in the downstream of all weirs, including Piano Key Weirs (PKWs), can have major safety implications. Since the research on downstream scouring of PKWs is very limited, and the weir geometry is also known to have an impact on downstream scouring, this study investigated scouring in the downstream of PKWs with rectangular and trapezoidal geometries and two different heights. The scour hole measurements showed that in both rectangular and trapezoidal models, scour hole parameters increased both with the increase in discharge rate and the increase in weir height. Under similar discharge conditions, the scour depth downstream from the rectangular model was greater than that downstream from the trapezoidal model. The dimensionless maximum scour depth, the distance of maximum scour depth from the weir toe, and the scour hole length for the trapezoidal PKW were, on average, 6, 13, and 11% lower than the corresponding ones for the rectangular PKW, respectively. However, these differences decreased with the increase in falling height. For both weir geometries, the maximum scour depth was aligned with the outlet keys. In addition, the maximum scour depth under the outlet keys was 13% greater than the one under the inlet keys.

Key words | Piano Key Weir, scour, trapezoidal model, weir geometry

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HIGHLIGHTS

- Comparing the scouring in sedimentary beds downstream of rectangular and trapezoidal PKWs.
- Studying and comparing the location of the scour hole relative to the inlet and outlet keys in PKWs.
- Proposing equations for the estimation of scour hole characteristics downstream of PKWs.

NOTATION

B PKW weir length (m)
B_b PKW base length (m)
W PKW transversal width (m)
T_s PKW sidewall thickness (m)
L developed PKW crest length (m)
R PKW parapet wall height (m)
P vertical PKW height (m)
Q discharge (m^3/s)
q unit discharge (m^3/s)
W_i inlet alveoli width (m)
W_o outlet alveoli width (m)
α PKW sidewall angle (degre)
R^2 coefficient of determination (–)
x horizontal and streamwise coordinate (m)
ΔH head difference between reservoir and tailwater (m)
σ sediment uniformity coefficient (–)

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INTRODUCTION

Weirs are structures typically used in rivers or reservoirs to release excess water and control the discharge rate. They are also commonly used to handle overflow in diversion dams and reduce erosion in river banks. Weirs are also effective in regulating the water level in canals, and they serve as instruments of discharge measurement in water transmission systems (Vischer & Hager 1998; Khatsuria 2004, 2005).

Weirs can generally be classified into three types; that is, straight, curved, and folded. Among folded weirs, labyrinth weirs support a higher discharge capacity by providing a longer total developed crest length for any given channel width. Labyrinth weirs can be constructed with many different geometries (Gebhardt *et al.* 2018; Ghaderi *et al.* 2020a, 2020b).

In recent years, with the increasing attention to the necessity of the restoration of dam structures in order to improve their safety and hydraulic capacity, a certain type of labyrinth weir called the Piano Key Weir (PKW) has become a popular subject of research (Machiels *et al.* 2011; Pfister *et al.* 2017). PKWs can be used to increase the discharge capacity of not only new dams, but also existing dams or dams under construction with a small foundation. In places with limited width or limited space for foundation building, the overhang of PKWs can increase the weir length, and thus its discharge capacity (Erpicum *et al.* 2017).

The majority of previous studies on PKWs have focused on the hydraulic characteristics and the discharge coefficient of these structures. Research on the length of PKWs has shown that increasing the length has a significant impact on the discharge coefficient; however, this impact begins to diminish when the length exceeds a certain threshold (Kabiri-Samani & Javaheri 2012). In addition, it has been shown that under similar conditions, PKWs with slightly larger sidewall angles are more efficient (Cicero *et al.* 2013; Khassaf & Al-Baghdadi 2015; Mehboudi *et al.* 2016; Mehboudi *et al.* 2017). Studies on the discharge coefficient and hydraulic behavior of labyrinth weirs and PKWs with rectangular and trapezoidal geometries have shown that under similar hydraulic conditions, PKWs provide higher discharge rate than labyrinth weirs. These studies have also recommended the following geometric specifications for trapezoidal PKWs: $\frac{W_i}{W_o} = 1.33$, $\frac{B}{P} > 1.6$, and $\frac{L}{W} \geq 5$ (Mehboudi *et al.* 2016, 2017).

When designing or assessing weirs, it is important to consider not only the hydraulic characteristics, but also the scouring pattern in the downstream, which is a phenomenon that can seriously threaten the stability of the dam structure. Over the past few years, scouring in the downstream of linear weirs has been extensively researched, providing a satisfactory understanding of scouring under these structures (Dehghani *et al.* 2010; Ibrahim 2015). Given the more frequent use of nonlinear weirs in flow control due to their high efficiency, scouring in the downstream of these weirs has also been the subject of multiple studies, especially in recent years. In a study on scouring in the downstream of rectangular and trapezoidal labyrinth weirs, the results showed that rectangular and trapezoidal labyrinth weirs caused 10 and 19% less scouring than linear weirs, respectively (Rajaei *et al.* 2018). In comparison, scouring in the downstream of PKWs has received much less attention. Indeed, only one percent of 135 studies on PKWs have examined scouring in the downstream of these weirs (Oertel 2018).

The studies on scouring in the downstream of PKWs, which have been limited to the rectangular geometry, have shown that the discharge rate has a direct impact on the geometry of the scour hole in a way that the depth of the scour increases with discharge (Justrich *et al.* 2016; Pfister *et al.* 2017). Moreover, since the geometry of the scour hole depends on its depth, previous studies have used the maximum scour depth to estimate the scour hole profile (Justrich *et al.* 2016).

A review of the literature shows that scouring in the downstream of different weirs, both linear and nonlinear, has been the subject of a fairly large number of studies (D'Agostino & Ferro 2004; Rajaei *et al.* 2018; Elnikhely & Fathy 2020). However, not many studies have examined this issue for PKWs, and only a few have investigated scouring in the downstream of trapezoidal PKWs. Furthermore, previous studies have not paid much attention to the position of the scour hole with respect to the inlet and outlet keys in PKWs (Justrich *et al.* 2016; Pfister *et al.* 2017). As several parameters related to sediment scouring may be significant and difficult to understand, comparison studies with different methodologies and experimental setups are required. The present study aims to investigate scour hole dimensions at the downstream of PKWs with different types of rectangular and trapezoidal plan form under different conditions of head and tail water depth. The main objectives of this study focus on the effects of the sidewall angle of PKWs and drop height on the scour hole's characteristics. The obtained measurements were used to formulate a number of equations for predicting the geometric dimensions of the scour hole in the downstream of rectangular and trapezoidal PKWs, which can be used for the reliable design of the foundation of these weirs.

MATERIALS AND METHODS

Dimensional analysis

Based on the angle of their sidewalls, PKWs are divided into two classes; that is, rectangular and trapezoidal. The geometric characteristics of rectangular PKWs are shown in Figure 1(a). In this figure, P is the vertical height, B is the stream-wise length, R is the height of the sidewalls, T_s is the thickness of the sidewalls, and W is the width of the weir. Moreover, subscript i refers to the inlet key and subscript o refers to the outlet key. The experimental models for the geometric rectangular and trapezoidal PKWs are shown in Figure 1(b). As can be seen from Figure 1(b), in the trapezoidal PKWs, the width of the outlet keys increases in the flow direction.

According to previous studies on scouring in the downstream of nonlinear weirs, the scour hole in the downstream of PKWs is influenced by the following geometric and hydraulic parameters (Gebhardt et al. 2018; Noseda et al. 2019):

$$F_1(\varnothing_s, P, \Delta H, d_{50}, q, g, \rho_s, \rho, \mu, \alpha) = 0 \quad (1)$$

In this equation, scour hole geometric parameters ($\varnothing_s$) include s (the maximum scour hole depth), X_s (the stream-wise distance of the scour hole from the weir toe), and X_0 (the scour hole's length), which is the stream-wise distance of the intersection of the scour hole and the initial sediment bed from the weir toe (see Figure 1(c)). In addition, ΔH is the total head difference between the downstream and the upstream of the weir, q is the discharge per unit width, and ρ_s, ρ, g, d_{50}, μ, and α signify the particle density, water density, gravitational acceleration, median sediment size, dynamic viscosity of water, and weir wall angle, respectively.

According to the Buckingham theorem, the dimensionless parameters that affect scouring in the downstream side of the weir can be determined using Equation (2):

$$F_2\left(\frac{\varnothing_s}{\Delta H}, \frac{P}{\Delta H}, \frac{d_{50}}{\Delta H}, \frac{q}{\sqrt{g.\Delta H^3}}, \frac{\rho_s-\rho}{\rho}, \frac{\mu}{\rho.q}, \alpha\right) = 0 \quad (2)$$

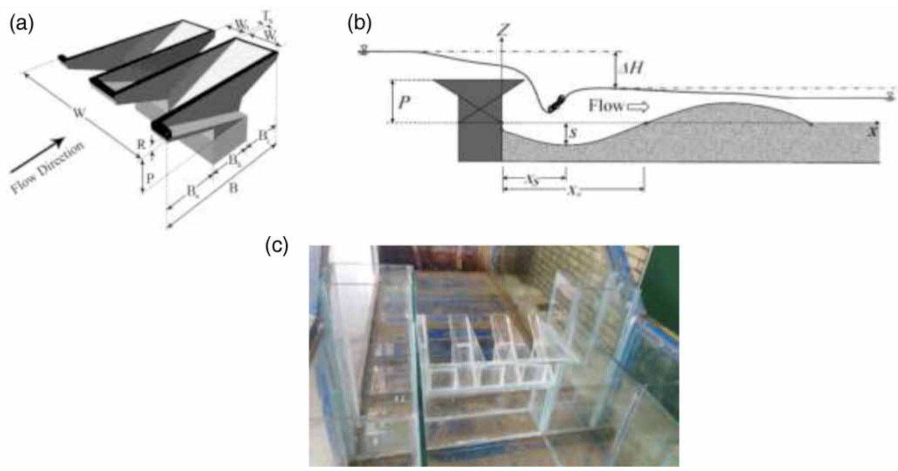

Figure 1 | (a) Geometry of a typical piano key weir (b) longitudinal section of the weir including geometrical and hydraulic parameters (c) picture of experimental set up.

In all the experiments, the Reynolds number or $\frac{\mu}{\rho.q}$ is greater than 10^4, indicating that the flow is turbulent. Therefore, viscosity (μ) has a much smaller effect and can be ignored. The Froude number of the flow can be defined using the dimensionless parameter $\frac{q}{\sqrt{g.\Delta H^3}}$. Combining this with $\frac{d_{50}}{\Delta H}$ and $\frac{\rho_s - \rho}{\rho}$ gives the particle Froude number or F_{r_d} as $\frac{q}{\Delta H\sqrt{g(s-1)d_{50}}}$ (where s is the particle density). Since this study is focused on rectangular and trapezoidal PKWs, α variations are limited to two angles and can be ignored. Therefore, the dimensionless parameters affecting the scouring phenomenon are as follows:

$$\frac{\varnothing_s}{\Delta H} = f\left(F_{r_d},\ \frac{P}{\Delta H}\right) \tag{3}$$

Experimental setup

The experiments were performed in a flume with a length of 2 m, a width of 1 m, and a height of 1.2 m in the upstream side and 0.6 m in the downstream side of the weir. The flume was placed on top of a platform with a height of 1.5 m. The frame of the flume was made of steel, while its walls and floor were made of glass with a thickness of 1 cm.

Discharge was measured and controlled by an electromagnetic flow meter. Calibrated trapezoidal weirs were also placed at the flume outlet to provide a secondary control for discharge measurements.

According to previous studies, the efficiency of a trapezoidal PKW increases with the increase in its sidewall angle, and the recommended geometrical specifications for optimizing the discharge coefficient of this weir are as follows: $\frac{W_i}{W_o} = 1.33$, $\frac{B}{P} > 1.6$, and $\frac{L}{W} \geq 5$ (Mehboudi et al. 2016, 2017). The experiments in this study were performed on a rectangular PKW model and a trapezoidal PKW model with a sidewall angle of 6°. Moreover, the effects of falling height on the scour hole's geometry were investigated by placing the models at heights of $P = 15$ cm and $P = 20$ cm. The geometric specifications of the PKW models according to Figure 1(b) are shown in Table (1). The PKW models were made of plexiglass sheets with a thickness of 4 mm, carefully sealed at the joints to prevent water penetration. The selected weirs had three inlet keys and two outlet keys with two symmetrical half-keys on either side (see Figure 1(b)).

The sedimentary bed in the downstream of the weirs was 2 m long and 25 cm thick. The sediment used in this study was medium-grained sand with a density of 2.65 and an internal friction angle of 35 degrees. The grading characteristics of the sediment were $d_{16} = 5.8$ mm, $d_{50} = 7.8$ mm, and $d_{84} = 10.15$ mm, with a geometrical standard deviation of $\sigma = (d_{84}/d_{16})^{0.5} = 1.32$, which indicates uniform grading.

In the first phase of the experiments, the rectangular PKW model was installed on the platform at a height of 25 cm. Then, water was pumped from an underground tank to an open tank. The experiment started by attempting to create the desired flow rate by adjusting the pump settings. Once the desired flow conditions were created, the scouring process was started in order to reach equilibrium conditions. Then, discharge was checked, and the average flow depth at specific control points was measured by a water level gauge with the precision of ±1 mm. According to Table 2, the experimental runs were performed at three different discharges. In the first stage of the experiment, the flow rate for the M1 model was considered to be 0.0195 m^3/s and the experiment was performed. Then the experiment was repeated for the

Table 1 | Geometric specifications of rectangular and trapezoidal PKW models

	Parameter value											
Models	**P (cm)**	**α**	**W_i (cm)**	**W_o (cm)**	**W (cm)**	**T_s (cm)**	**B (cm)**	**B_p (cm)**	**B_i (cm)**	**B_o (cm)**	**L (cm)**	**W_i/W_o**	**L/W**
RPKW (M_1)	15	0°	8.75	6.58	50	0.4	35	15	10	10	256	1.33	5.12
RPKW (M_2)	20	0°	8.75	6.58	50	0.4	35	15	10	10	256	1.33	5.12
TPKW (M_3)	15	6°	4.55	3.42	50	0.4	35	15	10	10	239	1.33	4.78
TPKW (M_4)	20	6°	4.55	3.42	50	0.4	35	15	10	10	239	1.33	4.78

Table 2 | Initial conditions of experiments for rectangular and trapezoidal PKWs

Parameter	Rectangular Piano Key Weir						Trapezoidal Piano Key Weir					
Model	M_1	M_1	M_1	M_2	M_2	M_2	M_3	M_3	M_3	M_4	M_4	M_4
Test N.	1	2	3	4	5	6	7	8	9	10	11	12
P (m)	0.15	0.15	0.15	0.2	0.2	0.2	0.15	0.15	0.15	0.2	0.2	0.2
Q (m^3/s)	0.0195	0.0275	0.0323	0.0195	0.0275	0.0323	0.0195	0.0275	0.0323	0.0195	0.0275	0.0323
ΔH (m)	0.0971	0.0890	0.0860	0.1476	0.1425	0.1411	0.0900	0.0862	0.0830	0.1421	0.1389	0.1380
F_{rd}	1.142	1.764	2.164	0.754	1.107	1.280	1.208	1.763	2.161	0.759	1.105	1.319

same model in two other flow rates 0.0275 m^3/s and 0.0323 m^3/s. For other runs, experiments were performed with the three desired discharges.

In the next phases, the experimental process described above was repeated with the rectangular PKW installed at the height of 20 cm, and then with the trapezoidal PKW model installed at the heights of 15 cm and 20 cm. These experiments were performed on the same discharge rates as in the first runs.

At the end of each experiment, the downstream bed and the resulting scour hole were completely drained. Then, the surface height of the sediment bed was measured using a laser measurement system mounted on a trolley, with the precision of ±1 mm.

A summary of the initial conditions of the experiments is given in Table 2. As can be seen, the experiments carried out on the rectangular PKW model with a height of 15 cm (referred to as M_1) are called Tests 1–3, while those conducted on the rectangular PKW model with a height of 20 cm (referred to as M_2) are called Tests 4–6. In addition, the experiments conducted on the trapezoidal PKW model with a height of 15 cm (referred to as M_3) are called Tests 7–9, while those performed on the rectangular PKW model with a height of 20 cm (referred to as M_4) are called Tests 10–12.

In this study, discharge variations ranged from 19 to 33 lit/s. For these discharge rates, the Froude number at the flume inlet ranged from 0.02 to 0.04, indicating that the flow was very slow, and that the flow velocity had a negligible impact on the discharge coefficient.

At the beginning of the experiments, the $Q - \Delta H$ diagrams for all rectangular and trapezoidal weir models were plotted (see Figure 2). The comparison between these plots showed that for a constant discharge rate, the rectangular weir models had a slightly higher upstream-downstream head difference (ΔH) than the trapezoidal models. However, this difference decreased with the increase in the discharge rate, and the values of ΔH for the two models became very close once the discharge rate was increased.

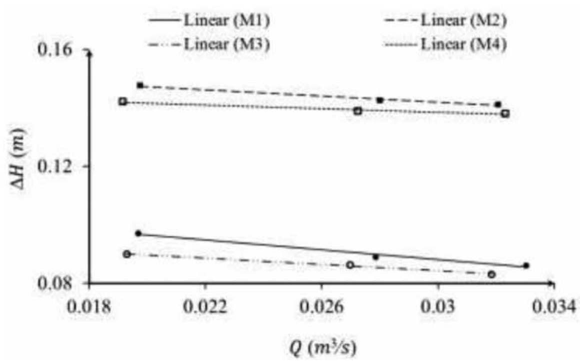

Figure 2 | Upstream-downstream total head difference.

RESULTS

Due to limited previous research on downstream scouring of piano key weirs, the experiments in the current study investigated the score hole and erodible bed changes due to the presence of rectangular and trapezoidal PKWs with different heights in order to determine how they were affected by the weir's height and sidewall angle. As shown in Figure S1 (Supplementary Data), this was achieved by measuring the surface of the sediment bed in a 2 × 0.34 m area (2 m along the flow direction and 0.34 m perpendicular to the flow direction) at 0.01 m intervals at the end of each experiment. The measurements made near the walls were ignored to eliminate the effects of the walls on the results.

The 2D plan and the 3D view of the scouring for one of the experiments is shown in Figure S1. Moreover, the cross-section of the scour profile along the maximum scour depth is also depicted in this figure. As can be seen, in both weirs, the maximum scour depth appears under the outlet keys when the inlet key is in the middle of the width of the weir. The maximum depth of the scour hole created under the outlet key is 13% more than that of the hole created under the inlet key. In addition, the maximum scour depth below the outlet key is 5% closer to the weir toe than the one below the inlet key.

The effects of particle Froude number on the scour hole parameters for rectangular and trapezoidal models are shown in Figure (3). It can be seen that in all models, as the particle Froude number increases, the scour hole depth (s) and length (X_s), and the distance between the maximum scour depth and the weir toe (X_0) increase as well. Previous studies have reported similar findings on the effects of discharge on scouring downstream from linear and nonlinear weirs (Dehghani *et al.* 2010; Justrich *et al.* 2016).

While the height of the weir (P) remains the same, the main reason for the increasing F_{rd} is the increase in the discharge rate. So, Figure (4) compares the profiles of scour holes created below rectangular and trapezoidal PKW models of the same height. It can be seen that in any given discharge, the rectangular models have a slightly higher maximum scour depth than the trapezoidal models of the same height. However, the differences are not significant, and they decrease with the increase in the discharge rate. Moreover, in all discharge rates, the distance between the maximum scour depth and the weir toe and the scour hole's length are lower in the trapezoidal model than in the rectangular model.

Figure (5) presents the effects of ($P/\Delta H$) on the scour profile in the two models at different Fr_d. As shown in Figure (5), in both rectangular and trapezoidal models, the maximum scour depth and the scour hole's length decrease with the increase in $P/\Delta H$; an effect that can easily be attributed to the increase in falling height. In all the experiments, the rectangular and trapezoidal models of different heights only showed negligible difference in the ridge area. This indicates that neither the weir height nor the falling height have a significant impact on the ridge characteristic. Similar results have been reported for typical weirs (Dehghani *et al.* 2010).

Comparing the dimensionless scouring parameters with the weir height ($s/\Delta H$, $X_s/\Delta H$, and $X_0/\Delta H$) for the two models in all experiments shows that the dimensionless values obtained for the maximum scour depth, the maximum scour depth distance, and the scour hole's length for the trapezoidal PKW are, on average, 6, 13, and 11% lower than the corresponding values for the rectangular PKW.

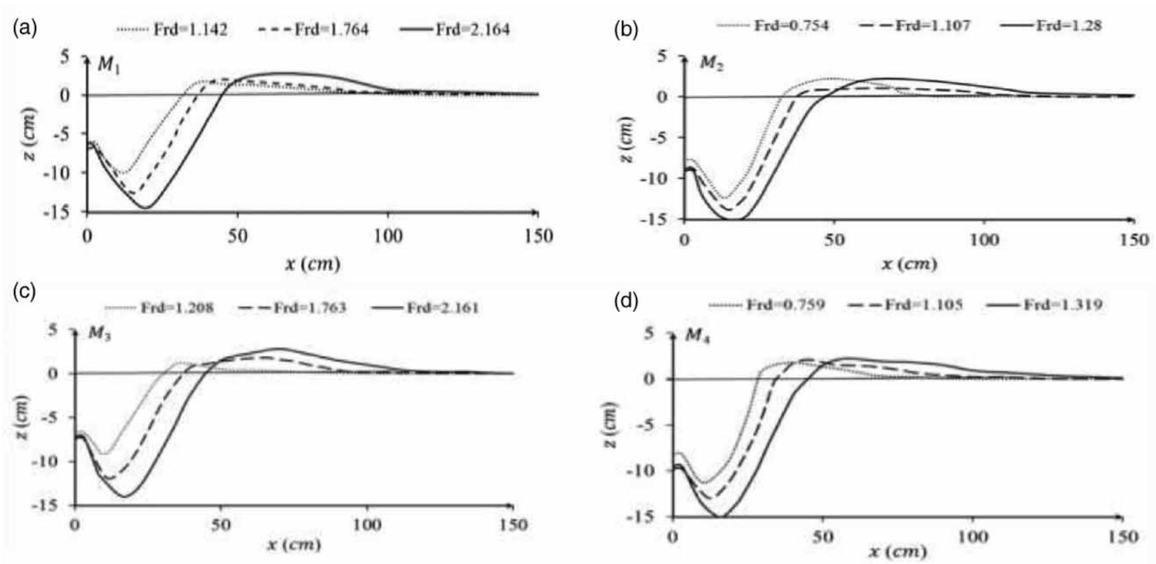

Figure 3 | Effect of particle Froude number on the scour hole profile.

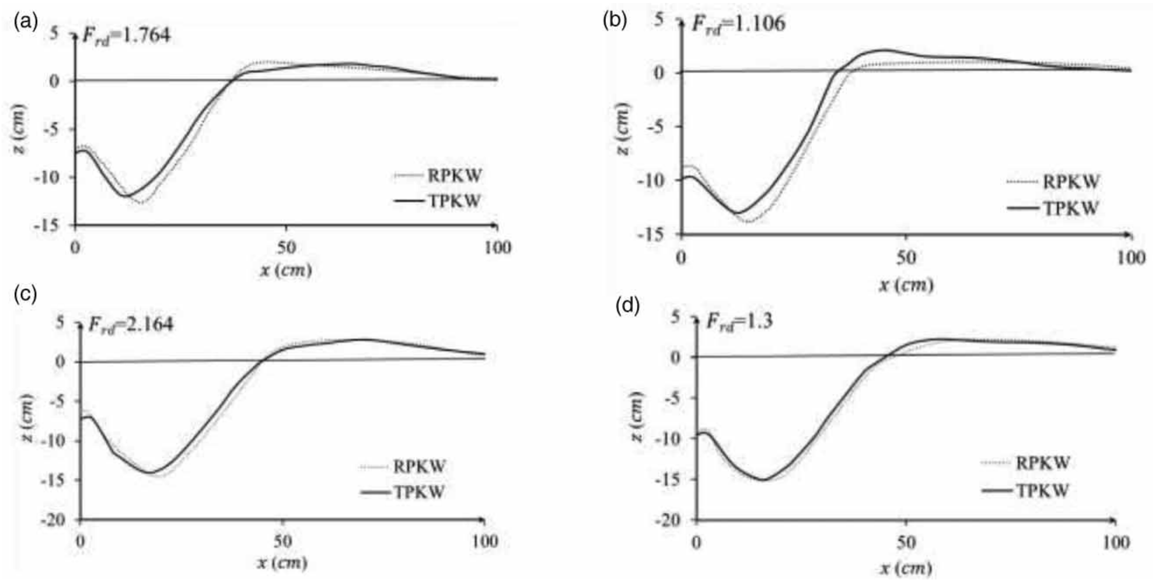

Figure 4 | Comparison of the effect of discharge on the scour hole profile.

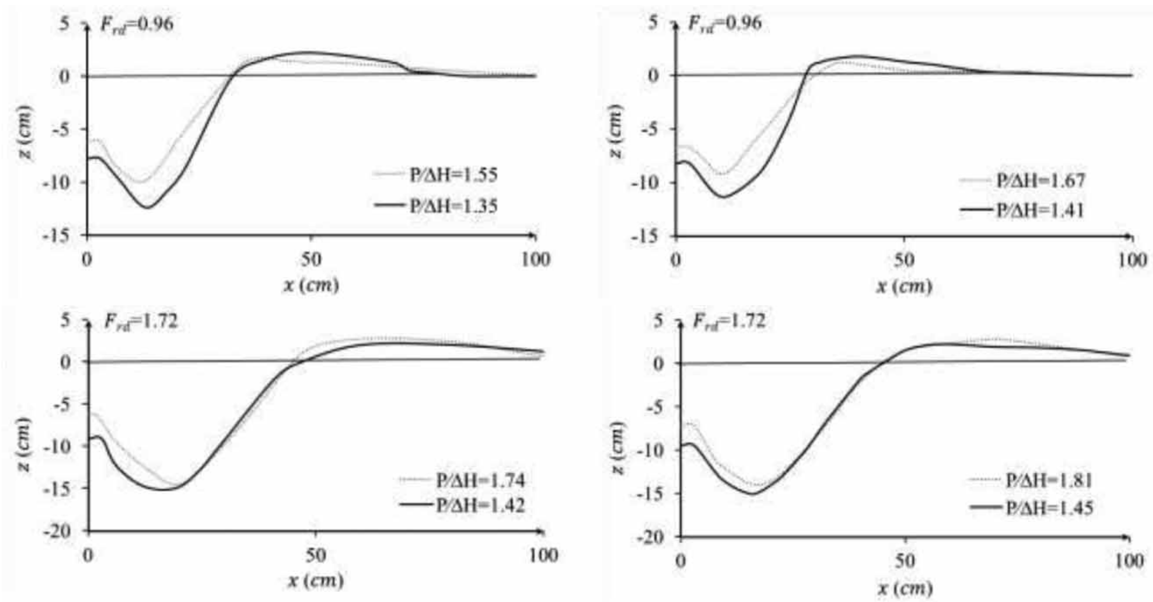

Figure 5 | Comparison of the effect of falling height on the scour hole profile.

DISCUSSION

As shown in the results, the maximum depth of the scour hole created under the outlet key is more than that of the hole created under the inlet key. In addition, the maximum scour depth below the outlet key is closer to the weir toe than the one below the inlet key. Naturally, this difference is caused by the free-falling jets coming from the outlet keys, which create and deepen this scour hole. This effect was studied by examining the parameters of the scour holes created after changing the discharge and ΔH for all four models.

In experimental models, the scour hole's parameters increase with increasing particle Froude number. According to the definition of F_{rd} and Figure (2), the main reason for the increasing F_{rd} involves the increase in the discharge rate while the height of the weir (P) remains the same.

The development of deeper scour holes below the rectangular model than below the trapezoidal model under similar hydraulic conditions can be attributed to the fact that at any given discharge, the rectangular model has a slightly higher upstream-downstream head difference than the trapezoidal model (Figure 2).

Considering the importance of predicting the maximum scour depth, *s*, and its position, X_s, and the scour hole's length, X_0, for the safety of the weirs as well as the estimation of the length of the riprap to be built in their downstream, the experimental results were used to formulate a number of equations for predicting the dimensions of the scour hole below rectangular and trapezoidal PKWs.

Based on the dimensional analysis performed in this study, the geometric characteristics of the scour hole, including the maximum scour depth, the stream-wise distance of the maximum scour depth from the weir toe, and the scour hole's length can be expressed in the following general form:

$$\frac{\boxtimes_s}{\Delta H} = a\, F_{rd}^{b}\left(\frac{P}{\Delta H}\right)^{c} \tag{4}$$

To determine the effects of the geometric shape of the PKW and the falling height, the scour characteristics in the rectangular and trapezoidal models in the conducted experiments were examined and compared based on the configuration presented by Equation (4). Afterward, the non-linear regression method was used to determine the coefficients a, b, and c, and to formulate a number of equations for predicting s, Xs, and X_0 for rectangular and trapezoidal PKWs. These equations are presented in Table 3.

In the equations presented in Table 3, the ranges of coefficients a, b, and c are very different in order to allow formulating a single equation for predicting the geometric characteristics of the scour hole for both rectangular and trapezoidal PKWs. In other words, the maximum scour depth, the location of the maximum scour depth, and the scour hole's length below the PKWs vary substantially with the weir's sidewall angle because this angle affects ΔH as well as the position where the jet falls.

Given the limited number of studies on scouring in the downstream of PKWs, the measured values are compared with a number of previously proposed equations for predicting the geometrical characteristics of scour holes in the downstream of grade-control structures and rectangular PKWs.

In Figure (6), the existing equations for predicting the geometrical characteristics of scour holes are compared with the experimental results (D'Agostino & Ferro 2004; Justrich *et al.* 2016). This comparison shows that the maximum scour depths obtained in the present study are approximately 5% lower than those predicted by the equation presented by Agostino & Ferro (2004); a difference that can be attributed to the difference in weir type. However, in some cases, the obtained maximum scour depths are larger than the estimates obtained from the equation presented by Justrich *et al.* (2016). This difference is due to the changed sidewall angle and the higher weir height in the experiments performed in the present study. The comparisons regarding the location of the maximum scour depth and the scour hole's length were performed using the experimental results obtained from the rectangular and trapezoidal models with a height of 15 cm, as they were more similar to the experimental model presented by Justrich *et al.* (2016).

Table 3 | Proposed equations for estimating scour hole characteristics in the downstream of rectangular and trapezoidal PKWs

RPKW Proposed equations	R^2	**TPKW** Proposed equations	R^2
$\frac{s}{\Delta H} = 0.738\ F_{rd}^{0.532}\left(\frac{P}{\Delta H}\right)^{0.682}$	0.986	$\frac{s}{\Delta H} = 0.847\ F_{rd}^{0.723}\left(\frac{P}{\Delta H}\right)^{0.126}$	0.980
$\frac{X_s}{\Delta H} = 0.337\ F_{rd}^{0.335}\left(\frac{P}{\Delta H}\right)^{2.639}$	0.936	$\frac{X_s}{\Delta H} = 0.447\ F_{rd}^{0.846}\left(\frac{P}{\Delta H}\right)^{1.2}$	0.938
$\frac{X_0}{\Delta H} = 1.525\ F_{rd}^{0.49}\left(\frac{P}{\Delta H}\right)^{1.71}$	0.978	$\frac{X_0}{\Delta H} = 1.823\ F_{rd}^{0.867}\left(\frac{P}{\Delta H}\right)^{0.738}$	0.978

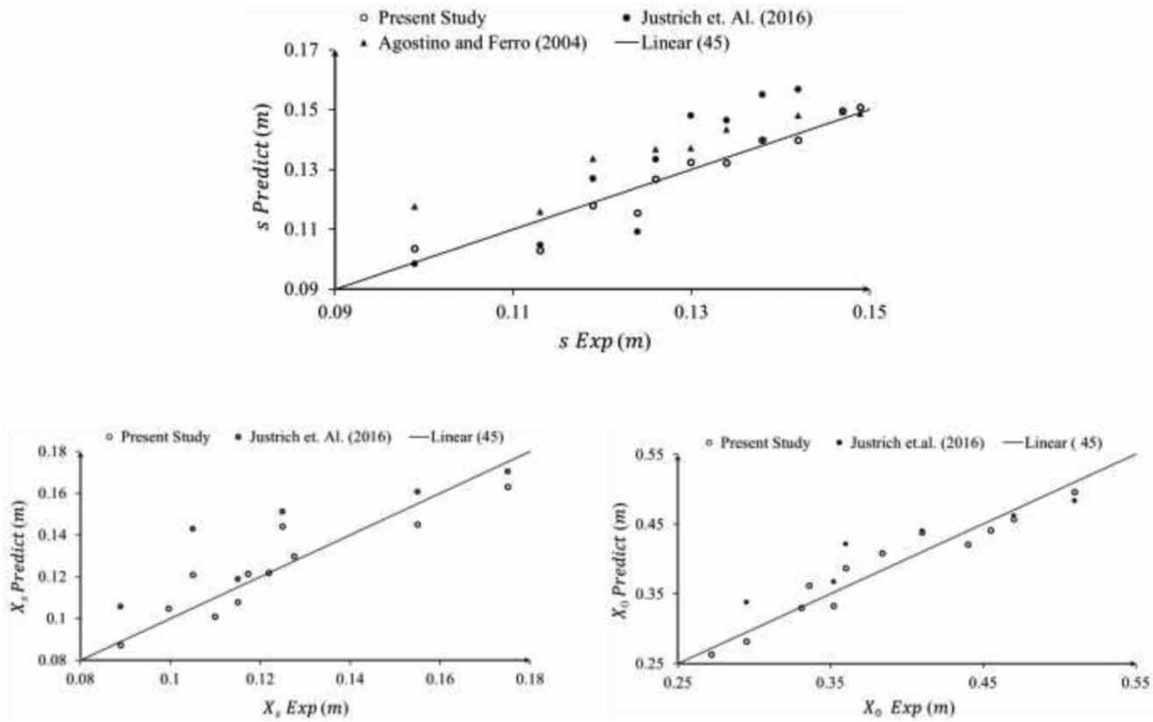

Figure 6 | Comparison of scour hole measurements with the results of existing experimental equations based on the general form of Equation (4).

It was observed that the predictions of Justrich's equation for the location of the maximum scour depth are 9% higher than the obtained experimental results. This difference might be due to differences between the two studies in terms of side-wall angle, weir width, and the number of cycles. In addition, comparing the observed scour hole lengths with the estimates of Justrich's equation showed a 4% difference between the observed and predicted values, showing that the results of the two studies have very little difference.

CONCLUSIONS

The current study examined and compared the characteristics of scour holes in the downstream of rectangular and trapezoidal PKWs. The results show that scour characteristics depend on the weir's geometry and the discharge rate. The findings of the current study can be summarized as follows:

- In both rectangular and trapezoidal PKW models, as discharge rate and upstream water head increased, the maximum scour depth, its distance from the weir toe, and the scour hole's length increased as well. At a constant discharge rate, the maximum scour depth, its distance from the weir toe, and the scour hole's length were greater in the rectangular model than those in the trapezoidal model. However, the differences decreased with the increase in discharge rate.
- The dimensionless ratios obtained for the maximum scour depth, the maximum scour depth distance, and the scour hole's length for the trapezoidal PKW were, on average, 6, 13, and 11% lower than the corresponding values for the rectangular PKW. Generally, it can be said that these differences are not significant
- In both rectangular and trapezoidal PKW models, the maximum scour depth and the scour hole's length increased with the increase in the weir height. At higher discharge rates, weir height had a lower effect on the scour profile.
- In both rectangular and trapezoidal PKW models, the maximum scour depth appeared under the outlet keys when the inlet key was positioned in the middle of the width of the weir. The maximum depth of the hole

created under the outlet key was 13% more than that of the hole created under the inlet key. Moreover, the maximum scour depth below the outlet key was 5% closer to the weir toe than the one below the inlet key.

- New empirical equations were suggested to predict the scour hole characteristics and their results compared with some previous empirical equations for nonlinear weirs and show an acceptable agreement. The present work is helpful for defining a proper foundation depth for a PKW installed on an erodible bed.

DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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